Department of Mathematics and Computer Science

Colloquium

How to tell apart the congruence subgroups of $SL(2, \mathbb{Z})$

Vesselin Dimitrov (IAS)

Abstract:

It was already familiar to Klein that $SL(2, \mathbb{R})$ appeared unusual among the Lie groups for having extraordinarily many lattices (discrete subgroups with finite covolume).

Firstly, the lattices themselves are not limited to the arithmetic examples such as $SL(2, \mathbb{Z})$, but come in continuous families rich enough to encompass every compact or finite type Riemann surface: the Fuchsian or group-theoretic point of view. And secondly, even within the arithmetic lattices like $SL(2, \mathbb{Z})$ the overwhelming majority of the sublattices are not described by congruence conditions on the matrix entries. All this is in contrast to higher rank semisimple Lie groups such as $SL_3(\mathbb{R})$, where all the sublattices are the essentially obvious ones given by finitely many congruence conditions inside an arithmetic lattice.

We will focus on $SL_2(\mathbb{Z})$ and explain a recent theorem joint with Frank Calegari and Yunqing Tang that may be read as an arithmetic amendment of the failing congruence subgroup property of that arithmetic lattice. It turns out that taking a natural integral structure into account, the answer to the congruence subgroup property turns from negative to positive. A practical expression of this phenomenon is a criterion for a sublattice to be congruence based on the integrality of the Fourier coefficients of the holomorphic modular forms on that group. But the underlying theme seems broader, and the discussion will lead us to some open questions for further exploration.

4 – 5pm, Wednesday, October 5, 2022

204 Smith Hall