Abstracts of the Talks (Wednesday May 22, 2024)

9:00 – 10:00 **Shari Moskow**, Drexel University

Title: "Nonlinearity helps convergence of the inverse Born series."

Abstract: We investigate the Born and inverse Born series for a scalar wave equation with linear and nonlinear terms, the nonlinearity being cubic of Kerr type. We show conditions which guarantee convergence of the inverse Born series, enabling recovery of the coefficients of the linear and nonlinear terms. Furthermore, we show that if the unknown perturbation is only in the nonlinear term, an arbitrarily strong nonlinearity can be reconstructed for sufficiently small data. Similar convergence results hold for general polynomial nonlinearities. Our results are illustrated with numerical examples. This is joint work with N. Defilippis and J. Schotland.

10:30 – 11:30 **Shixu Meng**, Virginia Tech

Title: "A low rank method for inverse scattering."

Abstract: Inverse problems play important roles in various applications, including target identification, non-destructive testing, and parameter estimation. Particularly challenging is the inverse scattering problem in inhomogeneous media, which aims to estimate unknowns based on available measurement data. Given its inherently ill-posed nature, our aim is to address this challenge by developing a low rank method. In the Born or linearized case, we solve the unknown in a low-dimensional space comprising disk prolate spheroidal wave functions, which are computed efficiently via a Sturm-Liouville problem. The low rank method leads to increasing stability and dimensionality reduction, as demonstrated by numerical examples with potentially noisy and large-scale measurement data. We establish a stability estimate by leveraging the interplay between a Fourier integral operator and a Sturm-Liouville differential operator. Additionally, motivated by the parameter estimation result of this low rank method, we explore the potential of parameter estimation using the linear sampling method for both the Born case and the fully nonlinear case. In particular, we show that the linear sampling indicator converges, depending on the problem formulation, to either a linear or nonlinear transformation of the unknown.

11:30 – 12:30 **Narek Hovsepyan**, Rutgers University

Title: "On the lack of external response of nonlinear media in the second-harmonic generation process."

Abstract: Second Harmonic Generation (SHG) is a process in which the input wave (e.g. laser beam) interacts with a nonlinear medium and generates a new wave, called the second harmonic, at double the frequency of the original input wave. We investigate whether there are situations in which the generated second harmonic wave does not scatter and is localized inside the medium, i.e., the nonlinear interaction of the medium with the probing wave is invisible to an outside observer. This leads to the analysis of a semilinear elliptic system formulated inside the medium with non-standard boundary conditions. More generally, we set up a mathematical framework needed to investigate a multitude of questions related to the nonlinear scattering problem associated with SHG (or other similar multi-frequency optical phenomena). This is based on a joint work with Fioralba Cakoni, Matti Lassas and Michael Vogelius.