Saturday April 8th

Eric Sedgwick (DePaul University, USA)

Title: What is hard about 3-manifold topology?

Abstract: This will be an introductory talk on the computational complexity of decision problems in 3-dimensional topology. We will start by giving an informal introduction to the complexity classes P, NP, NP-hard, NP-complete and co-NP. We then discuss a variety of decision problems in 3-manifold topology, including problems related to knotting/linking, manifold recognition and embedding. The goal is to compare problems known to have differing complexities as well as to speculate about the complexity of problems about which little is known.

Adele Jackson (University of Oxford, UK)

Title: Triangulations of Seifert fibered spaces

Abstract: To resolve recognition problems in 3-manifolds, we often want to show that some topological feature is of bounded complexity in any triangulation within some class of 3-manifolds. The main tool used for this is normal surface theory. I will discuss some results in this direction including that, given a triangulation of a Seifert fibered space with boundary, all the singular fibres (except those of multiplicity two) are simplicial in the 79th barycentric subdivision.

Kate Petersen (Univesity of Minnesota, USA)

Title: Lens Space co-Recognition

Abstract: I will discuss the computational complexity of determining whether a given closed triangulated 3-manifold is a lens space. I will focus on the case when the input manifold is a Seifert Fiber Space, showing that the recognition problem is in co-NP. This means that there is a polynomial time verifiable certificate that the input manifold is not a lens space. This is joint work with Neil Hoffman.

Nathan Dunfield (University of Illinois, Urbana-Champaign, USA)

Title: Computing a link diagram from its exterior

Abstract: A knot is a circle piecewise-linearly embedded into the 3-sphere. The topology of a knot is intimately related to that of its exterior, which is the complement of an open regular neighborhood of the knot. Knots are typically encoded by planar diagrams, whereas their exteriors, which are compact 3-manifolds with torus boundary, are encoded by triangulations. Here, we give the first practical algorithm for finding a diagram of a knot given a triangulation of its exterior. Our method applies to links as well as knots, and allows us to

recover links with hundreds of crossings. We use it to find the first diagrams known for 23 principal congruence arithmetic link exteriors; the largest has over 2,500 crossings. Other applications include finding pairs of knots with the same 0-surgery, which relates to questions about slice knots and the smooth 4D Poincare conjecture. This is joint work with Cameron Rudd and Malik Obeidin. Based on: arXiv:2112.03251.

Clément Maria (INRIA, France)

Title: Parameterized complexity in low dimensional topology

Abstract: Parameterized complexity is a theory allowing a finer analysis of the complexity of algorithms, which was originally applied to graph problems. In this talk, I will survey recent results on the use of parameters for algorithmic and combinatorial topology, with a focus on knots and 3-manifolds. I will try to motivate and highlight the particular flavor of parameterized complexity when applied to the computation of quantum invariants, at the interface of topology, classical and quantum computational complexity, and combinatorics.

Sunday April 9th

Stephan Tillmann (The University of Sydney, Australia)

Title: Slope norm, crosscap number and complexity of Dehn fillings

Abstract: I will report on joint work with Bus Jaco, Hyam Rubinstein and Jonathan Spreer. We describe an algorithm to compute smallest complexity surfaces spanned by boundary curves in any orientable, compact, irreducible 3-manifold M with incompressible boundary a torus. This is applied to give an algorithm to compute the crosscap number of a knot, and to give complexity bounds for 3-manifolds obtained from M by Dehn filling.

Kristof Huszar (ENS de Lyon, LIP, CNRS, France)

Title: 3-Manifolds without thin triangulations

Abstract: There are several computationally hard problems about triangulated 3-manifolds that admit an efficient algorithmic solution, provided the input triangulation is sufficiently thin in some sense. Finding such triangulations is therefore an important challenge, however, this can be limited by the topology of the underlying 3-manifold.

In this talk I give an overview of several recent results that link the key combinatorial parameters in the above context to classical topological invariants of 3-manifolds in a quantitative way. As a consequence, we exhibit various kinds of 3-manifolds, where the smallest width of any triangulation is arbitrary large. We establish these results through constructions involving generalized Heegaard splittings, layered triangulations, and "complicated" JSJ decompositions.

Joint work with Jonathan Spreer and Uli Wagner.

Niloufar Fuladi (Université Gustave Eiffel, France)

Title: Cross-cap drawings and short decompositions for surfaces

Abstract: A cross-cap drawing of a graph G is a drawing on the sphere with g distinct points, called cross-caps, such that the drawing is an embedding except at the cross-caps, where edges cross properly. A cross-cap drawing of a graph G with g cross-caps can be used to represent an embedding of G on a non-orientable surface of genus g. This simple connection between crossing numbers and graphs embedded on surfaces was first noticed by Mohar who conjectured that any triangulation of a non-orientable surface of genus g admits a cross-cap drawing with g cross-caps in which each edge of the triangulation enters each cross-cap at most once. Motivated by Mohar's conjecture, Schaefer and Stefankovic provided an algorithm that computes a cross-cap drawing with a minimal number of cross-caps for a graph G such that each edge of the graph enters each cross-cap at most twice. In this talk, I will present two of my works around these croscap drawings: firstly, I will provide a pseudo-triangulation for a non-orientable surface of genus 5 that disproves a stronger conjecture of Mohar stated for pseudo-triangulations. Secondly, I will show that the algorithm of Schaefer and Stefankovic can be strengthened as follows: given a graph G embedded on a non-orientable surface N, we can cut N into a disk by cutting along a "short" canonical system of loops, meaning that each loop in the system crosses each edge of G at most a constant number of times. Such canonical decompositions were known for orientable surfaces, but the techniques used to compute them do not generalize to non-orientable surfaces due to their more complex nature. Our proof uses techniques coming from computational biology, specifically from the signed reversal distance algorithm of Hannenhalli-Pevzner. This is joint work with Alfredo Hubard and Arnaud de Mesmay.

Hanh Vo (Arizona State University, USA)

Title: Deciding when two curves are of the same type.

Abstract: Let S be a compact orientable connected surface with negative Euler characteristic. Two closed curves on S are of the same topological type if their corresponding free homotopy classes differ by a mapping class of S. Given two closed curves on S, we propose an algorithm to detect whether they are of the same type or not. This is joint work with Juan Souto.

Cameron Gordon (University of Texas at Austin, USA)

Title: The homeomorphism problem for 4-manifolds

Abstract: In 1958 Markov showed that the homeomorphism problem for manifolds of dimension at least 4 is algorithmically unsolvable. In particular, he showed that for some integer k the recognition problem for the connected sum of k copies of $S^2 \times S^2$ is unsolvable. We will outline a proof that one can take $k = 12$.

Andrew Yarmola (Princeton University, USA)

Title: Towards the Margulis constant for hyperbolic 3-manifolds

Abstract: For a closed hyperbolic 3-manifold M, let M_ϵ be the set of all points in M whose injectivity radius is less than ϵ. A lemma of Margulis implies that there is a universal constant μ_3 such that if $\epsilon < \mu_3$ then M_ϵ is topologically a disjoint union of solid tubes. This constant is useful many finiteness arguments as well as effective Dehn surgery. The true value of μ_3 is unknown, but lower bounds exist. Meyerhoff shows that $\mu_3 > 0.108$, while Culler, Shalen, and others have better lower bounds when the choice of M is restricted. In this talk, we will describe partial progress towards the exact value of μ_3. In particular, we will explain how a symmetric variant of this constant can be computed and how it can be used, in preliminary computations, to suggest that $\mu_3 > 0.5$. This is joint work with David Gabai and David Futer.