## Measuring a Gerrymander

There is no such thing as a fair or non-partisan districting plan. Whether intentionally or blindly, such plans involve political choices and have critical effects on the political parties. The various "proposed public interest criteria for redistricting...are not neutral, they are not grounded in broader principles that command general assent, and in many cases they are incoherent and cannot be made to work. ${ }^{11}$ Because reapportionment is inherently political, some way to identify or measure its political effects is needed in selecting a districting plan. Yet there are few concrete measures of what is often called a gerrymander, "electoral districts [which] result, or are thought to result, in partisan advantage. ${ }^{2}$ The main difficulty in gauging a gerrymander is in finding a coherent basis of comparison; to evaluate a districting proposal's political effects, it is crucial to establish some sort of context in which to do so.

One such context is to locate the natural limits, the upper and lower bounds, within which a gerrymander may take place. Since the gerrymanderer cannot change the way people vote or where they live, these demographic factors constrain his efforts by providing a fixed upper limit, a perfect gerrymander. A perfect gerrymander is defined as
any set of districts such that no other set, or no possible redistricting, could increase the [number of seats won by] the favored party.... A perfect gerrymander is not necessarily unique; there may be infinitely many slightly different sets of districts which allow the favored party to carry the same number of districts. That maximal number, by definition, is unique, and so it provides a standard of comparison for any other set of districts. In order to use this concept, we must be able to calculate that number and show that it is in fact maximal. ${ }^{3}$

Before constructing a perfect gerrymander which takes into account the "spatial distribution of partisan or group support," ${ }^{4}$ however, a number of practical considerations must be addressed.

A database of the region to be gerrymandered must be set up to include population and political information about each of the smallest possible subunits. Although population data comes from the U.S. Census Bureau, the choice of what political data to use in constructing a perfect gerrymander is less clear. At first glance, the precinct-by-precinct returns from the last election would seem to be the best available prediction of how each precinct will vote in the next election. This method, however, is unacceptable because "legislative votes are not 'cross-addable'; that is, it is not accurate to assume that the votes for a

[^0]party's candidate in one district would translate into votes for the same party's candidate in another district. ${ }^{5}$

One option which tries to correct this flaw is to calculate a party loyalty measure for each precinct. This method involves making statistical projections of how candidates might fare in new areas. Suppose that in a particular district, a district candidate did worse than his party's candidates for say governor and attorney general. Based on a similar comparison of another district candidate of the same party and the same statewide party candidates, an "expected vote model ${ }^{\prime 6}$ can statistically relate the information to project how the first district candidate would have done in a precinct outside his district. If all of a party's district candidates in the state are projected into that one precinct, then the average support received "across all the simulated races gives the expected mean for a candidate from a given party. ${ }^{17}$ This procedure, of course, rests on the less than perfect "assumption that the relationship between the district candidates' votes and the statewide candidates' votes would be the same" ${ }^{8}$ across the state, in every precinct.

A second option for choosing the database's political data is to use party registration figures, which "are not biased by candidate, election, and issuespecific factors." ${ }^{19}$ It is a simple matter to see from the last election the lowest party registration percentage at which all, or nearly all, districts or precincts were carried by a party. In fact, data from several election years are easily combined to determine even more accurately what minimum registration level can be considered safe for a party. Despite its easy application, though, this method has several drawbacks. First of all, the registration "figures themselves may be inaccurate and biased, since some of the people who are on the registration rolls for a given area will have since moved or died." ${ }^{10}$ This is particularly a problem in areas with a high level of transiency. In addition, this method does nothing to account for an increasing number of independent and unaffiliated voters. Finally, there is only an inexact and "loose correlation between voting behavior and registration. ${ }^{11}$

Although loyalty measures and registration figures are two of the best available approximations of party strength, they provide no more than a rough estimate of how each precinct will vote in the next election. There may also be any number of legitimate reasons, such as a long-range electoral trend, for slightly raising or lowering the estimate of a party's minimum "safe" percentage. Once the political data is finally assembled, the only remaining step in compiling a database is to combine the political and population data for each precinct. Unfortunately, since there is "no exact correspondence between the precinct units of the political data and the tract or block units of the census data, the merger process [is] tedious and difficult...[and must be] checked and rechecked many times to eliminate inevitable human errors." ${ }^{12}$ Although the job of assembling such a database seems immense, most of the work must be done anyway for the more general task of reapportionment. With access to the
reapportioner's computer database, only a few additional adjustments and calculations may be necessary for the task of constructing a perfect gerrymander. We may now proceed with that task. ${ }^{13}$

Some frequently used variables in the perfect gerrymander model are as follows:
$\mathbf{X}$ the space, usually a state, to be redistricted.
$\mathbf{N}$ the number of districts into which X is to be divided.
$\mathbf{U}$ any given region in $\mathbf{X}$ where there are:
$\mathbf{P}(\mathbf{U})$ people;
F(U) supporters of the favored party; and $\quad \mathbf{B}(\mathbf{U}) \quad$ supporters of both parties combined.

The object is to create for the favored party the largest possible safe region $\mathbf{A}$, with just barely enough support, and to pack the opposition party's supporters into the smallest possible region $\mathbf{X}-\mathbf{A}$. Before the actual district lines can be drawn, the safe region A must be constructed.

Suppose that there is only one precinct in the space $\mathbf{X}$ where the favored party's portion of voter support, $\mathbf{F} / \mathbf{B}$, is at its maximum and also that $\mathbf{F} / \mathbf{B}$ progressively declines when moving away from that precinct. The safe region A should obviously include the maximal precinct and all the other precincts around it which have a level of $\mathbf{F} / \mathbf{B}$ at or above what is considered safe. Although the remaining precincts will all be carried by the opposition party, some of these precincts can be included in $\mathbf{A}$ as long as the overall level $\mathbf{F}(\mathbf{A}) / \mathbf{B}(\mathbf{A})$ does not fall below the favored party's safety mark. Deciding which set of the remaining precincts will maximize $\mathbf{P}(\mathbf{A})$ is best done by using a decision tree which considers every possible combination of contiguous precincts. Each decision "path" on the tree ends when the addition of another precinct to $\mathbf{A}$ would lower $\mathbf{F}(\mathbf{A}) / \mathbf{B}(\mathbf{A})$ below the safety level. The results of each decision path can be compared and the one with the highest value for $\mathbf{P}(\mathbf{A})$ designated as the optimal solution. Unfortunately, since precincts generally contain hundreds of people ${ }^{14}$ and states, millions, such a decision tree, with perhaps 1000 ! solutions, is totally impractical and could not even be computed in the time between censuses. Instead, one must use an algorithm which approximates the maximization of $\mathbf{P}(\mathbf{A})$. Once the region $\mathbf{A}$ produced by the algorithm approaches the safety limit, however, it may be desirable to switch to the remaining portion of the overall decision tree and find the optimal solution at least from that point.

The algorithm for determining the safe region $\mathbf{A}$ is as follows. Suppose again that there is only one relative maximum of $\mathbf{F} / \mathbf{B}$ and include in $\mathbf{A}$ that maximal precinct and all the other precincts around it which have a level of $\mathbf{F} / \mathbf{B}$ at or above what is considered safe. Since districts must be contiguous, consider only the remaining precincts adjacent to a precinct already in $\mathbf{A}$ and add the precinct with the highest $\mathbf{F} / \mathbf{B}$ to $\mathbf{A}$. After checking that this step has not
lowered the overall $\mathbf{F}(\mathbf{A}) / \mathbf{B}(\mathbf{A})$ for the new region $\mathbf{A}$ beneath the safety mark, repeat the procedure until the addition of the precinct with the next highest $\mathbf{F} / \mathbf{B}$ would bring the whole region $\mathbf{A}$ below the safe level. It may be, however, that the remaining precinct with the highest $\mathbf{F} / \mathbf{B}$ has a large $\mathbf{B}$ which dilutes the region $\mathbf{A}$, making it unsafe. In that case, a smaller precinct, with a slightly lower $\mathbf{F} / \mathbf{B}$, could still be added to $\mathbf{A}$, whereas the safer but larger precinct could not. Therefore, consider the remaining precincts adjacent to $\mathbf{A}$ in descending order of $\mathbf{F} / \mathbf{B}$ and add in that order those which do not reduce $\mathbf{F}(\mathbf{A}) / \mathbf{B}(\mathbf{A})$ to below the safe level. If more than one precinct has the same $\mathbf{F} / \mathbf{B}$, and all of these precincts cannot be added without making A unsafe, then the precincts should be added one at a time in decreasing order of population $\mathbf{P}$, checking at each step that the overall $\mathbf{F}(\mathbf{A}) / \mathbf{B}(\mathbf{A})$ has not fallen below the safety mark. In the more general case where the space $\mathbf{X}$ has several relative maxima of $\mathbf{F} / \mathbf{B}$, the algorithm is repeated for each of the separate subregions in $\mathbf{X}$ where $\mathbf{F} / \mathbf{B}$ is at or above the favored party's safety level. If the safe subregions come into contact while expanding, they should be joined.

Once the safe region $\mathbf{A}$ is maximal, it must be made divisible by districts of equal population. Of the total $\mathbf{N}$ number of districts in the space $\mathbf{X}$, each separate safe subregion $\mathbf{A}_{\mathbf{i}}$ is entitled to $\mathbf{N} \cdot \mathbf{P}\left(\mathbf{A}_{\mathbf{i}}\right) / \mathbf{P}(\mathbf{X})$ districts. When this figure is not a whole number, it may be possible to link nearby subregions of $\mathbf{A}$ by corridors through the unsafe region X-A to allow additional districts to be formed; for example, linking a subregion of $\mathbf{A}$ with enough people for 1.6 districts and another for 1.5 districts. The same procedure, using several corridors, can also be applied to three or more safe subregions whose combined populations would yield additional districts. Slightly arbitrary parameters must be set for the minimum width of and maximum region through which a corridor $\mathbf{C}$ may be constructed to connect two safe subregions $\mathbf{A}_{\mathbf{i}}$ and $\mathbf{A}_{\mathbf{j}}$. Within these parameters, the best corridor is the one which minimizes the cost of creating the combined subregion $\mathbf{A}_{\mathbf{i}+\mathbf{j}}$. Since the corridor itself slightly increases the $\mathbf{F}$ number of favored party supporters in the enlarged subregion by

## $\frac{F(C)}{F\left(\mathbf{A}_{\mathbf{i}}\right)+\mathbf{F}\left(\mathbf{A}_{\mathbf{j}}\right)}$,

then, ideally, the corridor should increase the $\mathbf{B}$ number of supporters of both parties by the same fraction, maintaining the enlarged subregion at the minimum safe level. Since a corridor contains extremely unsafe territory, though, many more than the ideal number of supporters of both parties are added by having a corridor. The best corridor, however, minimizes the difference between the number of supporters of both parties a corridor adds and the number it should add to preserve the favored party's level of support,
$\mathbf{B}(\mathbf{C})-\frac{\mathbf{F}(\mathbf{C})}{\mathbf{F}\left(\mathbf{A}_{\mathbf{i}}\right)+\mathbf{F}\left(\mathbf{A}_{\mathbf{j}}\right)} \cdot\left\{\mathbf{B}\left(\mathbf{A}_{\mathbf{i}}\right)+\mathbf{B}\left(\mathbf{A}_{\mathbf{j}}\right)\right\}$.
The best corridor will minimize the amount of precincts from $\mathbf{A}_{\mathbf{i}}$ or $\mathbf{A}_{\mathbf{j}}$ that must be abandoned to bring $\mathbf{F}\left(\mathbf{A}_{\mathbf{i}+\mathbf{j}}\right) / \mathbf{B}\left(\mathbf{A}_{\mathbf{i}+\mathbf{j}}\right)$ back up to the safety mark. If, even using the best corridor, so many precincts must be abandoned that $\mathbf{P}\left(\mathbf{A}_{\mathbf{i}}+\mathbf{j}\right)$ is too small to merit the extra districts sought in the first place, then the two subregions cannot be successfully linked by a corridor. When a subregion $\mathbf{A}_{\mathbf{i}}$, which is already maximal, cannot be linked to any others by a corridor, $\mathbf{A}_{\mathbf{i}}$ must be reduced in size to an exact number of districts. By shedding those precincts with the lowest $\mathbf{F} / \mathbf{B}, \mathbf{A}_{\mathbf{i}}$ can also become somewhat safer in the process.

Once all the safe subregions $\mathbf{A}_{\mathbf{i}}$ have been either linked or reduced to an exact number of districts, it only remains to count how many districts are in the favored party's safe region under the perfect gerrymander. While not essential to the model, it may prove useful to draw a set of district lines to demonstrate that it is possible to divide the maximal region $\mathbf{A}$ such that every one of its districts is at or a little above the safe level. One way to do so is to draw as many lines radiating from the maximally safe precinct as there will be districts in that subregion of $\mathbf{A}$, creating roughly equal pie-shaped districts. Districts with a population at the wrong level add or subtract precincts from neighboring districts within their subregion $\mathbf{A}_{\mathbf{i}}$. To achieve a desired level of party support, districts "trade" precincts of different partisan strengths with neighboring districts to achieve the correct level.

The maximal number of seats won by the favored party in a perfect gerrymander, $\mathbf{N}_{\text {max }}$, provides an upper limit for what a gerrymanderer can do; finding the lower bound is also useful for putting districting proposals in some sort of context. By constructing a perfect gerrymander for the opposition party, using the same database and procedures described above, one can determine the absolute minimum number of districts that the favored party will carry, $\mathbf{N}_{\mathbf{m i n}}$. With the range of possible partisan outcomes of reapportionment thus established, a Gerrymander Index measuring the severity of a gerrymander can be calculated. For any districting plan $\Delta$ in which a particular party may anticipate winning $\mathbf{N}_{\Delta}$ seats, the

## Gerrymander Index $(\Delta)=\frac{\mathbf{N}_{\Delta}-\mathbf{N}_{\text {min }}}{\mathbf{N}_{\text {max }}-\mathbf{N}_{\text {min }}}$.

The measure of the districting plan $\Delta$ ranges from zero to one, where the higher the fraction, the closer the plan comes to a perfect gerrymander. In addition to the Gerrymander Index, the actual values for $\mathbf{N}_{\min }$ and $\mathbf{N}_{\max }$ may be of interest to those evaluating a redistricting proposal.

There is, of course, no ideal value for the index for which a plan should aim. The Gerrymander Index merely provides a framework for discussion of a reapportionment plan and its political effects. What those effects should be is a political question, not a mathematical one. A mathematical measure of a redistricting plan, though, can elevate the political debate to a more informed and meaningful level.

## Notes

1. Daniel Lowenstein and Jonathan Steinberg, "The Quest for Legislative Districting in the Public Interest: Elusive or Illusory?," UCLA Law Review 33 (1985): 75.
2. Philip Musgrove, The General Theory of Gerrymandering, Sage Professional Papers in American Politics, Vol. 3, Series Number 04034 (Beverly Hills and London: Sage Publications, 1977), p. 5.
3. Ibid., pp. 6-7.
4. John K. Wildgen and Richard L. Engstrom, "Spatial Distribution of Partisan Support and the Seats/Votes Relationship," Legislative Studies Quarterly 3 (1980): 433.
5. Charles Backstrom, Leonard Robins, and Scott Eller, "Issues in Gerrymandering: An Exploratory Measure of Partisan Gerrymandering Applied to Minnesota," Minnesota Law Review 62 (1978): 1128.
6. Bruce E. Cain, The Reapportionment Puzzle (Berkley and Los Angeles: University of California Press, 1984), p. 139.
7. Ibid., p. 145.
8. Ibid., p. 139.
9. Ibid., p. 141.
10. Ibid., p. 142.
11. Backstrom, Robins, and Eller, "Issues in Gerrymandering," p. 1132.
12. Cain, The Reapportionment Puzzle, p. 83.
13. The perfect gerrymander model is taken in large part from pages 8-12, 18-20, 26, and 46-7 of Philip Musgrove's The General Theory of Gerrymandering, but with several substantive changes. In addition to employing a different set of variables, this paper provides a more precise treatment of the case where data are available only in discrete units, i.e., by precinct. The Gerrymander Index at the end of this paper is distinct from the one proposed by Musgrove.
14. Cain, The Reapportionment Puzzle, p. 85.

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Winter 1988

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