Review of Quantitative Finance and Accounting, 22: 29–38, 2004

Securitizing Accounts Receivable

DARIUS PALIA*AND BEN J. SOPRANZETTI
Rutgers Business School, Rutgers University, 94 Rockafeller Road, Piscataway, NJ 08854, USA
E-mail: dpalia@rci.rutgers.edu

Abstract. This paper provides an optimal contracting framework to determine the equilibrium structure of receivables securitization in the presence of moral hazard. An incentive compatible contract is designed where the seller monitors at an efficient level and retains an equity interest in a portion of the receivable to be sold. The seller retains the riskier tranches and sells the safer ones. The equilibrium proportion of the receivable sold will be increasing with the seller's cost of internal funding. Moreover, for sellers with sufficiently high ex-ante probabilities of solvency, the equilibrium proportion of the receivable sold will be decreasing with the seller's probability of solvency.

Key words: securitization, accounts receivable, factoring, asset sales

Although firms have been factoring their individual accounts receivable for over two thousand years, it is only recently that firms have begun to securitize and sell off the cash flows from their accounts receivable pools. The securitization industry and the factoring industry differ in two important ways. First, receivables securitization allows firms to sell a portion of the payoffs to any given receivable, while factoring typically constrains firms to sell the entire receivable. Second, the buyer in a receivables securitization arrangement commonly does not have the facilities to perform on-going monitoring of the underlying receivable, and instead must rely upon the seller to perform the monitoring functions. In contrast, factoring companies have highly developed credit management departments and may often be better than the seller at monitoring the underlying receivable's credit quality. Since for many firms, such as hospitals and credit card companies, the securitization of accounts receivables has become an important financing tool, a better understanding of the structure of securitization contracts is important.

This paper examines the decision to securitize accounts receivable in a setting where there is moral hazard. As discussed above, in the typical receivables securitization contract, the buyer usually does not the ability to perform ongoing monitoring of the purchased receivable, and instead relies upon the seller to perform the monitoring functions. Since the seller's monitoring efforts are unobservable to the buyer, a moral hazard problem can develop. As will be demonstrated, this moral hazard problem has an important impact upon the structure of the optimal receivables securitization contract. The contract is derived in a one-period model that extends the technology that has been used in the loan sales and factoring literature.[1]

*Corresponding author.

Accounts receivable can be sold either with or without recourse. If the receivable is sold without recourse, then the seller will not be liable for any delinquency in the payment of the underlying receivable.[2] If the receivable is sold with recourse, then the seller may be responsible for a portion or even all of the uncollected amount depending upon the terms of the securitization agreement. The recourse guarantee is similar to a put option, where the strike price is the face value of the receivable. The buyer will be able to put the delinquent receivable back to the seller if the realized payoff is less than the promised amount.

This paper demonstrates that the optimal receivable securitization contract is characterized by the seller monitoring at an efficient level and retaining an equity interest in a portion of the receivable to be sold. The model's implications are observationally identical to what occurs in practice, where it is common for sellers to decompose the payoff to their receivables pool into different tranches. The seller typically retains the riskier tranches and sells the safer ones. In addition, the equilibrium proportion of the receivable sold will be increasing with the seller's cost of internal funding. Further, for sellers with sufficiently high ex-ante probabilities of solvency, the equilibrium proportion of receivables sold will be decreasing with the seller's probability of solvency.

This paper is organized as follows. Section I provides a review of the literature. Section II outlines the model which will be employed throughout the paper. Section III provides a general form for the firm's profit function. Section IV examines the firm's decision to securitize its receivables, and Section V concludes the paper.

I. Literature review

Lewellen and Edmister (1973) examine and discuss the shortcomings of alternative accounts receivable management policies, but provide no specific information on the sale of accounts receivable. Mian and Smith (1992, 1994), provide a thorough exploration of the determinants of accounts receivables policy, but due to a lack of observations, they provide only tangential evidence regarding the determinants of firm's decision to sell or factor its accounts receivable. Smith and Schnucker (1994), document that factoring is much more likely to occur when sellers have little "specialized" investment in their clients and when the their respective cost of monitoring is relatively high. The authors also demonstrate that the factoring of accounts receivable is much less likely to occur when the seller is a wholesaler rather than a retailer. Sopranzetti (1998) examines the optimal factoring contract and finds that the propensity for the factor to require recourse (relative to no recourse) is governed by three factors: the credit quality of the seller's accounts receivables, the seller's solvency, and the seller's reputation. Khaled (2002) also examines the factoring industry and finds that a factoring company's decision to buy accounts receivable also determined by the product market wherein the seller operates. Although the literature has extensively examined the factoring industry, it has not yet examined the increasingly important area of receivables securitization.

II. Assumptions

The model extends the framework used in Sopranzetti (1998) to examine the optimal factoring contract. The primary difference is that in Sopranzetti (1998) the seller was constrained to factor the entire receivable, while in this manuscript, the seller is allowed the flexibility to securitize the receivable, i.e., should the seller wish, he can sell only a portion of it.

Assumption 1. A risk neutral seller (for example a hospital or a credit card company) intends to provide a service to its client for a promised payment of \$L. The cost to provide the service is \$1, and the seller will finance this at its cost internal cost of capital, r_i. The buyer's cost of internal capital is r_f. Sopranzetti (1997) demonstrates, and we assume, that the seller of an account receivable usually has a higher financing cost than the buyer because of costs attributable to financial distress, tax issues, or underinvestment.

Assumption 2. In order for an account receivable to be generated, the seller must extend trade credit to its client, so that on the delivery date, the seller will provide the service, but will not be compensated by the buyer until the end of the trade credit period, which for the purpose of the model is assumed to be τ periods.

Assumption 3. The level of effort that the seller expends monitoring the quality of the underlying receivable is given by c, where $c \in [0, \infty)$. A larger c implies more monitoring. The cost of doing the monitoring is u times c, where $u > 0$. The monitoring cost, u, occurs at the end of the period.

Assumption 4. The seller's probability of solvency is given by p.

Assumption 5. Both the buyer and the seller can perfectly observe the distribution of $\tilde{x}$, the random variable that describes the receivable's actual payoff. The distribution of $\tilde{x}$ is an increasing function of both the receivable's credit quality α, where $\alpha \in [1, \infty)$, and the level of credit monitoring, c.

The distribution for $\tilde{x}$ must satisfy a strict convexity of distribution function constraint with respect to the level of credit monitoring and the level of receivable credit quality. This implies that the distribution function $F(x, \alpha, c)$ satisfies the following:

$$F(x, \alpha, \lambda c + (1-\lambda)c') < \lambda F(x, \alpha, c) + (1-\lambda)F(x, \alpha, c')$$

and

$$F(x, \lambda\alpha + (1-\lambda)\alpha', c) < \lambda F(x, \alpha, c) + (1-\lambda)F(x, \alpha', c)$$

Consequently, the expected payoff to $\tilde{x}$ will be *strictly concave* in both the level of credit monitoring and the level of receivable credit quality, i.e., there are strictly decreasing returns to scale in both of these technologies.

Assumption 6. The market for the sale of receivables is competitive and the seller can sell some fraction b of the receivable to a buyer who is risk neutral. The receivable's credit quality, α, can be costlessly observed by both parties prior to the sale, however the buyer cannot observe, c, the seller's level of credit monitoring.

Assumption 7. Let ζ be an binary variable that equals one if the seller sells the receivable with recourse and zero otherwise. In the event that the seller offers the buyer recourse, the seller will be liable for the full amount that is delinquent.

III. The seller's expected profit function

Using the above assumptions, the general form of the seller's expected profit function is

$$e^{-r_i\tau}\int_0^L (1-b)x - \zeta bp(L-x)\,dF(\tilde{x} \mid \alpha, c)$$
$$-\,e^{-r_i\tau}uc - 1 + e^{-r_f\tau}b\left[\int_0^L x + \zeta p(L-x)dF(\tilde{x} \mid \alpha, c)\right] \tag{1}$$

where the first term is the seller's expected payoff from keeping a fraction $(1-b)$ of the receivable minus the seller's expected liability due to the recourse guarantee; the second term is the seller's monitoring cost; the third term is the initial investment; and the last term is the price that is paid to the seller by the buyer.

As can be seen, there is no closed-form solution to Eq. (1). In order to obtain closed-form results, we choose a tractable functional form for the expected value of receivable's payoff. The expected payoff is equal to $\int_0^L x dF(\tilde{x} \mid \alpha, c) = L(1 - \frac{1}{\alpha}e^{-\beta c})$. This functional form allows the receivable's expected payoff to converge to L as the quality of the receivable increases (as α increases), regardless of the amount of the seller's monitoring effort. The best quality receivables will pay the full face value regardless of whether the seller monitors or not. Moreover, if the sellers does no monitoring at all ($c = 0$), then the lower bound to the expected payoff will be $L(1 - 1/\alpha)$. Note that the functional form mandates that as c increases, the expected payoff converges at the rate β to the face value, L. Lastly, in order for $c \in [0, \infty)$, this functional form necessitates that $L\beta > u$.

IV. The seller's decision to securitize its receivables

It is common in the case of receivables securitization that the seller performs all of the credit management functions after the receivable is sold. The seller must do the monitoring, because the buyer typically is unable to perform ongoing credit monitoring of the underlying receivable. So, a moral hazard problem will develop if the buyer is unable to observe the seller's level of credit management. Because the seller's level of credit monitoring is unobservable to the buyer, a moral hazard problem can develop.

IV.A. The moral hazard problem

Lemma 1. *If the entire receivable is sold, and the seller's level of credit monitoring is unobservable to the buyer, then the seller will monitor at a level* $c = 0$.

Proof: See the Appendix. □

As demonstrated in Sopranzetti (1998), if the *entire* receivable were to be sold, then the seller has no financial incentive to monitor, since the buyer cannot observe the seller's monitoring efforts. This is a typical manifestation of the classic moral hazard problem. Any rational buyer would fully expect the seller to shirk from his/her monitoring responsibilities, and would consequently lower the offering price. In this case, the seller's expected profit when it sells the entire account receivable will be

$$e^{-r_f\tau}\left[\int_0^L x\,dF(\tilde{x},\alpha,0)\right]-1 \tag{2}$$

IV.B. The seller's receivables securitization problem without recourse

The assumptions in this section are identical to the assumptions made above, except that now the seller is permitted to sell a *portion* of the receivable's payoff, b (where $b \in [0, 1]$). Since, in this section, the seller is not offering a recourse guarantee to the buyer, $\zeta = 0$. The seller's problem now becomes one of selecting the equilibrium level of credit monitoring and the proportion of receivable to be sold such that incentive compatibility is maintained and such that the seller's profit is maximized.[3,4]

By substituting $\zeta = 0$ into Eq. (1), the seller's problem becomes

$$\underset{c>0,b}{\text{Max}}\, e^{-r_i\tau}\int_0^L (1-b)x\,dF(\tilde{x}\mid\alpha,c)-e^{-r_i\tau}uc-1+e^{-r_f\tau}\int_0^L bx\,dF(\tilde{x}\mid\alpha,c) \tag{3}$$

such that

$$\int_0^L (1-b)x\,dF_c(\tilde{x}\mid\alpha,c)=u \tag{4}$$

$$b\le 1 \tag{5}$$

Equation (4) is the incentive compatibility constraint, which Hart and Holmstrom (1987) demonstrate can be delineated in this form when the convexity of distribution function condition (Assumption 5) is satisfied.

Letting λ and ν be the Lagrange multipliers for constraints (4) and (5) respectively, the Kuhn-Tucker conditions are

$$\left[e^{-r_i\tau}(1-b)\int_0^L x\,dF_c(\tilde{x}\mid\alpha,c)-e^{-r_i\tau}u+be^{-r_f\tau}\int_0^L x\,dF_c(\tilde{x}\mid\alpha,c)\right.$$
$$\left.-\lambda(1-b)\int_0^L x\,dF_{cc}(\tilde{x},\alpha,c)\right]c=0 \tag{6}$$

and

$$\left[-e^{-r_i\tau}\int_0^L x\,dF(\tilde{x}\mid\alpha,c)+e^{-r_f\tau}\int_0^L x\,dF(\tilde{x}\mid\alpha,c)\right.$$
$$\left.-\lambda\int_0^L x\,dF(\tilde{x},\alpha,c)-\nu\right]b=0 \tag{7}$$

Let $\theta = e^{-r_f\tau} - e^{-r_i\tau}$, $\bar{x} = \int_0^L x dF(\tilde{x} \mid \alpha, c)$, $\bar{x}_c = \int_0^L x dF_c(\tilde{x} \mid \alpha, c)$ and $\bar{x}_{cc} = \int_0^L x dF_{cc}(\tilde{x} \mid \alpha, c)$. Recall from (4) that $\int_0^L (1-b)x dF_c(\tilde{x} \mid \alpha, c) = u$, so $c > 0$. Substituting for u into (6), solving for λ and substituting λ into (7) yields a solution for b.

$$b = \frac{\nu - \theta\bar{x}}{\frac{\bar{x}_c^2}{\bar{x}_{cc}} e^{-r_f\tau} - \theta\bar{x} + \nu}$$

Note that $b \leq 1$, since $\bar{x}_c^2 > 0$ and $\bar{x}_{cc} < 0$, and that $\nu = 0$ if there is an interior solution for b.[5]

IV.C. The seller's problem with recourse

The seller's problem changes slightly when it sells the receivable with recourse: the seller is now made contingently liable for the receivable's payoff, i.e. $\zeta = 1$. When it offers recourse, the seller is effectively offering the buyer a put option to guarantee against the receivable's default risk; consequently, one would expect that the use of recourse would induce the buyer to purchase a greater proportion of the seller's accounts receivable pool in equilibrium.

By substituting $\zeta = 1$ into Eq. (1), the seller's problem becomes

$$\begin{aligned} \operatorname*{Max}_{c>0,b} e^{-r_i\tau} \int_0^L (1-b)x - bp(L-x)dF(\tilde{x} \mid \alpha, c) - e^{-r_i\tau}uc - 1 \\ + e^{-r_f\tau} \int_0^L bx + bp(L-x)dF(\tilde{x} \mid \alpha, c) \end{aligned} \tag{8}$$

such that

$$\int_0^L [1 - b(1-p)]x dF_c(\tilde{x} \mid \alpha, c) = u \tag{9}$$

$$b \leq 1 \tag{10}$$

Note that the incentive compatibility constraint (9) has been modified to reflect the fact that the seller will be holding a greater portion of the receivable in those states of nature where the customer is delinquent.

Letting ψ and φ be the Lagrangian multipliers for (9) and (10) respectively, the first order conditions are

$$\{e^{-r_i\tau}(1-b)\bar{x}_c - \theta pb\bar{x}_c + e^{-r_f\tau}b\bar{x}_c - e^{-r_i\tau}u - \psi\bar{x}_{cc}[1 - b(1-p)]\}c = 0 \tag{11}$$

and

$$\{-e^{-r_i\tau}\bar{x} + \theta p(L-\bar{x}) + e^{-r_f\tau}\bar{x} + \psi\bar{x}_c(1-p) + \varphi\}b = 0 \tag{12}$$

Recall from (9) that $\int_0^L [1 - b(1 - p)]x dF_c(\tilde{x} \mid \alpha, c) = u$, so substitute for u into (11), solve for ψ, and substitute ψ into (12) to obtain the solution for the incentive compatible proportion of receivable to be sold.

$$b = \frac{\theta[\bar{x} + p(L - \bar{x})] - \varphi}{(1 - p)\left\{-\frac{\bar{x}_c^2}{\bar{x}_{cc}}(1 - p)e^{-r_f \tau} + \theta[\bar{x} + p(L - \bar{x})]\right\} - \varphi} \tag{13}$$

Proposition 1. *The equilibrium proportion of receivable sold will be increasing with the seller's cost of internal financing.*

Proof: Take the derivative of the equilibrium proportion sold with respect to theta. It is then easy to demonstrate that the derivative is greater than zero. □

The intuition behind Proposition 1 is straightforward: the greater the seller's cost of internal financing (r_i), the greater the benefit that the seller would derive from letting the buyer finance the outstanding receivable. This increased benefit to selling would entice the seller to securitize a greater proportion of the receivable in equilibrium.

Proposition 2. *If the seller's probability of solvency is greater than* 0.5, *then the equilibrium proportion of receivable sold will be decreasing with the seller's probability of solvency.*

Proof: Take the derivative of the equilibrium proportion sold with respect to p. In order to sign the derivative, it is necessary to invoke the functional form for the receivable's expected payoff. It is then easy to demonstrate that when $p > 0.5$ the derivative is less than zero. □

Sopranzetti (1998) examines the factoring industry and documents that there is a three tiered structure to factoring contracts: only the highest quality receivables are factored without recourse, intermediate quality receivables are factored with recourse, and low quality receivables are not factored. Notice that, our result, unlike that of Sopranzetti (1998) the breakpoint for the credit quality of salable receivables is not an issue. This is because the seller now has the flexibility to retain some of the credit risk by keeping a portion of the receivable's payoff. In this manner every receivable can be made salable by simply varying the equilibrium proportion that will be retained. The poorer the credit quality, the greater the proportion of the receivable (and the credit risk) that will be retained by the seller.

In reality, it is common for a seller to sell strips or tranches of the payoffs to its receivable pool. Typically the seller will sell tranches that are comprised of the less risky cash flows and retain the tranches that comprise the riskier cash flows. The securitization with recourse contract closely approximates this second best equilibrium contract where the seller retains a junior claim while the buyer owns a senior claim to the receivable's payoff. By offering recourse, the seller will bear the brunt of the receivable's credit risk, just as if he had retained the risky tranche of the receivable's cash flow; furthermore, the payoffs and the level of monitoring effort will also be identical under both contracts.

V. Conclusion

This paper provides an optimal contracting framework to determine the equilibrium structure of receivables securitization when the seller is susceptible to moral hazard. An incentive compatible contract is designed where the seller monitors at an efficient level and retains an equity interest in a portion of the receivable to be sold. The model's implications are observationally similar to what is happening in practice, where it is common for sellers to decompose the payoff to their receivables pool into different tranches. The seller typically retains the riskier tranches and sells the safer ones. In addition, the model predicts that that the equilibrium proportion of the receivable sold will be increasing with the seller's cost of internal funding. Moreover, for sellers with sufficiently high ex-ante probabilities of solvency, the equilibrium proportion of the receivable sold will be decreasing with the seller's probability of solvency.

Future research might examine that motivations behind a firm's decision to choose between factoring its accounts receivable, securitizing them, or using them as collateral for a collateralized loan. Each of these three financing choices will generate cash for the firm, however the question of determining which is optimal has yet to be sufficiently answered. Moreover, it would be interesting to learn if some types of financial institutions possess monitoring advantages over others. If so, then the equilibrium proportion of the receivable to be sold might be increased substantially.

Appendix

Proof of Lemma 1: Since the firm is selling the receivable without recourse, $\zeta = 0$. Substituting $\zeta = 0$ into Eq. (1) yields

$$\operatorname*{Max}_{c} e^{-r_i\tau}\left[\int_0^L (1-b)x\,dF(\tilde{x} \mid \alpha, c)\right] - e^{-r_i\tau}uc - 1 + e^{-r_f\tau}\int_0^L bx\,dF(\tilde{x} \mid \alpha, c)$$

such that

$$\int_0^L (1-b)x\,dF_c(\tilde{x} \mid \alpha, c) = u$$

$$c > 0$$

If κ and λ are the Lagrangian multipliers for the first and second constraints respectively, then the Kuhn-Tucker conditions are

$$\left[e^{-r_i\tau}(1-b)\int_0^L x\,dF_c(\tilde{x} \mid \alpha, c) - e^{-r_i\tau}u + be^{-r_f\tau}\int_0^L x\,dF_c(\tilde{x} \mid \alpha, c)\right.$$
$$\left.-\lambda(1-b)\int_0^L x\,dF_{cc}(\tilde{x} \mid \alpha, c)\right]c = 0$$

$$\left[u - \int_0^L (1-b)x\,dF_c(\tilde{x} \mid \alpha, c)\right]\kappa = 0$$

$$[-c]\lambda = 0$$

Since the entire receivable is sold, $b = 1$. Substitute $b = 1$ into the Kuhn-Tucker conditions. Since $u > 0$, κ must equal zero. Substitute $-c\lambda = 0$ into the first equation yeilds $[-e^{-r_i\tau}u + e^{-r_f\tau}\int_0^L x dF_c(\tilde{x} \mid \alpha, c)]c = 0$. The first term in the brackets represents the marginal cost of adding one more unit of monitoring, and the second term in the brackets represents the marginal benefit derived from one more unit of monitoring. The second term equals zero, since the buyer cannot ex-post observe c, thus once the transaction price has been set, the seller's payoff is independent of its credit monitoring effort. Consequently, the derivative of the expected payoff with respect to the level of credit monitoring, c, is equal to zero. So, in order to satisfy the Kuhn-Tucker conditions, c must equal zero. □

Acknowledgments

We would like to thank Charles Kahn, George Pennacchi, Inmoo Lee, and Tim Loughran for valuable comments and suggestions.

Notes

1. See Pennacchi (1988), Sopranzetti (1998), and Gorton and Pennacchi (1995).
2. Securitization contracts are written such that the buyer does not bear the non-payment risk that stems from the sale of faulty or poor quality goods.
3. A parallel problem is analyzed by Pennacchi (1988) who examines the motivations behind banks' loan sales decisions.
4. In this section, the buyer-seller contract is assumed to be an equity-equity contract. In general, this type of contract may not be optimal. However, it will be demonstrated that when the firm is allowed to sell the receivable with recourse, the design of the contract is broadened, and consequently its form will approximate that of an optimal contract.
5. Recall that because Assumption 5 must hold, the expected value of the receivable is strictly concave with respect to the firm's level of credit monitoring, and thus the second derivative will always be negative.

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