

Transfer and descent: some recent results

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A basic tool for studying the transfer of representations is the dual transfer of orbital integrals. Here we report on some recent results for orbital integrals and in particular on a descent theorem [LS3].

Throughout, F is a local field of characteristic zero with algebraic closure \overline{F} and $\Gamma = \text{Gal}(\overline{F}/F)$; G is a connected reductive group defined over F .

§1 AN EXAMPLE

The notions of endoscopy and transfer require some care in formulation, but there is one example where we can give definitions quickly. Namely, suppose that H is the (unique up to F -isomorphism) quasi-split inner form of G . Then H is endoscopic for G in the sense of standard endoscopy (where no twisting by an automorphism is specified; see [LS1], [KS]). Up to isomorphism, it is the unique endoscopic group with dimension as great as that of G .

Fix an inner twist $\psi : G \rightarrow H$. Although ψ is not defined over F (unless G is quasisplit) we may use it to define a correspondence between points of $G(F)$ and points of $H(F)$. Indeed, ψ induces a bijective map ψ_{conj} from the conjugacy classes in $G(\overline{F})$ to those in $H(\overline{F})$ and because $\psi\sigma(\psi)^{-1}$ is inner, $\sigma \in \Gamma$, ψ_{conj} respects the action of Γ . Thus we have a bijection between the classes defined over F . We shall consider just those elements in G (or H) which are strongly regular in the sense that their centralizers are tori. The conjugacy class in $G(\overline{F})$ of a strongly regular element γ_G of $G(F)$ is defined over F . By a converse theorem of Steinberg for quasi-split groups, the image under ψ_{conj} of this class also contains F -rational elements. We say that any such element γ_H is an *image of γ_G* .

Recall that the stable conjugacy class of a strongly regular element in $G(F)$ (or $H(F)$) consists of the F -rational elements in its \overline{F} -conjugacy class. Thus the correspondence (γ_G, γ_H) provides an injective map of the set of stable conjugacy classes of strongly regular elements in $G(F)$ in the stable classes of such elements in $H(F)$ (this map is surjective only if G is quasi-split and thus F -isomorphic to H).

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Because the strongly regular elements in $G(F)$ are dense in $G(F)$ for the topology inherited from F , the correspondence just defined is sufficient to specify *transfer*. Recall that the stable orbital integral of $f \in C_c^\infty(G(F))$ (or $f \in C(G(F))$) at strongly regular $\gamma \in G(F)$ is

$$(1.1) \quad \Phi^{\text{st}}(\gamma, f) = \sum_{\gamma'} \Phi(\gamma', f),$$

where the summation is over representatives γ' for the conjugacy classes in the stable conjugacy class of γ and $\Phi(\gamma', f)$ is the ordinary orbital integral

$$\Phi(\gamma', f) = \int_{G_{\gamma'}(F) \backslash G(F)} f(g^{-1}\gamma'g) d\bar{g}.$$

Here $G_{\gamma'} = \text{Cent}(\gamma', G)$ and $d\bar{g}$ is the quotient of some fixed Haar measure on $G(F)$ by a Haar measure on the Cartan subgroup $G_{\gamma'}(F)$. If $\gamma' = g^{-1}\gamma g$, $g \in G(\bar{F})$, then $\text{Int } g : G_{\gamma'} \rightarrow G_\gamma$ is defined over F : we require that the Haar measures on $G_\gamma(F)$ and $G_{\gamma'}(F)$ be related by transport under $\text{Int } g$. The *transfer problem* for (G, H) is to show that for each $f^G \in C_c^\infty(G(F))$ there exists $f^H \in C_c^\infty(H(F))$ such that

$$(1.2) \quad \Phi^{\text{st}}(\gamma_H, f^H) = \begin{cases} \Phi^{\text{st}}(\gamma_G, f^G) & \text{if } \gamma_H \text{ is an image of } \gamma_G \\ 0 & \text{if } \gamma_H \text{ is not an image,} \end{cases}$$

for all strongly regular elements γ_H in $H(F)$.

Define $\Delta(\gamma_H, \gamma_G)$ for strongly regular $\gamma_H \in H(F)$ and $\gamma_G \in G(F)$ by

$$\Delta(\gamma_H, \gamma_G) = \begin{cases} 1 & \text{if } \gamma_H \text{ is an image of } \gamma_G \\ 0 & \text{otherwise.} \end{cases}$$

Then we may rewrite (1.2) as

$$(1.3) \quad \Phi^{\text{st}}(\gamma_H, f^H) = \sum_{\gamma_G} \Delta(\gamma_H, \gamma_G) \Phi(\gamma_G, f^G)$$

for all strongly regular $\gamma_H \in H(F)$. The summation is over representatives γ_G for the conjugacy classes of strongly regular elements in $G(F)$. If (1.1) or (1.2) is true we say that f^G and f^H have *Δ -matching orbital integrals*.

For F archimedean the transfer problem is solved, at least for the Schwartz functions [S1] or C_c^∞ -functions bifinite under a maximal compact subgroup [CD]. For F nonarchimedean we consider first the problem of *local transfer at the identity*: given $f^G \in C_c^\infty(G(F))$ we are to

find $f^H \in C_c^\infty(H(F))$ such that (1.2) holds for strongly regular γ_H near the identity in $H(F)$. This may be reformulated in terms of Shalika germs. The Shalika germ expansion yields

$$\Phi(\gamma, f) = \sum_{\mathcal{O}} \Gamma_{\mathcal{O}}(\gamma) a_{\mathcal{O}}(f)$$

for γ strongly regular near the identity, where the summation is over unipotent conjugacy classes \mathcal{O} in $G(F)$, $\Gamma_{\mathcal{O}}$ is the Shalika germ for \mathcal{O} and $a_{\mathcal{O}}(f)$ is the orbital integral of f along \mathcal{O} (for some normalization of measure). We set

$$\Gamma_{\mathcal{O}}^{\text{st}}(\gamma) = \sum_{\gamma'} \Gamma_{\mathcal{O}}(\gamma'),$$

where the summation is over representatives γ' for the conjugacy classes in the stable conjugacy class of γ ; $\Gamma_{\mathcal{O}}^{\text{st}}$ is the *stable germ* for \mathcal{O} . For transfer we will use instead the notation γ_G , \mathcal{O}_G , and so on. We write $\Gamma_{\mathcal{O}_G}^{(H)}$ for

$$\gamma_H \mapsto \begin{cases} \Gamma_{\mathcal{O}_G}^{\text{st}}(\gamma_G) & \text{if } \gamma_H \text{ is an image of } \gamma_G \\ 0 & \text{if } \gamma_H \text{ is not an image.} \end{cases}$$

Then it follows readily that the pair (G, H) admits local transfer at the identity if and only if

$$(1.4) \quad \begin{aligned} & \text{for each unipotent conjugacy class } \mathcal{O}_G \text{ in } G(F), \\ & \text{the transferred stable germ } \Gamma_{\mathcal{O}_G}^{(H)} \text{ is a linear} \\ & \text{combination of the stable germs } \Gamma_{\mathcal{O}_H}^{\text{st}} \text{ for } H(F). \end{aligned}$$

For progress on (1.4) see the comments after (2.2).

We shall now assume local transfer at the identity for the centralizers of semisimple elements in $G(F)$ and deduce the full transfer on $G(F)$. The descent argument here is very simple.

First we formulate the assumption precisely. Call a semisimple element ϵ_H in $H(F)$ an *image* of the (semisimple) element ϵ_G in $G(F)$ if there exist a maximal torus T_G over F in G containing ϵ_G and an element x of $H(\overline{F})$ such that ${}_x\psi = \text{Int } x \circ \psi : T_G \rightarrow H$ is defined over F and carries ϵ_G to ϵ_H . For strongly regular elements this coincides with the earlier notion of image. Applying a lemma of Kottwitz we may further choose x so that $H_{\epsilon_H} = \text{Cent}(\epsilon_H, H)^\circ$ is quasi-split. Observe that ${}_x\psi : G_{\epsilon_G} \rightarrow H_{\epsilon_H}$ is an inner twist. It is not uniquely determined by ψ but its inner class is determined by the inner class of ψ , i.e., by

$\{\text{Int } y \circ \psi : y \in H(\overline{F})\}$, and it is only the inner class that matters for the correspondence of F -rational strongly regular points.

Our assumption will be:

$$(1.5) \quad \begin{array}{l} \text{for each semisimple element } \epsilon_H \text{ of } H(F) \text{ the pair} \\ (G_{\epsilon_G}, H_{\epsilon_H}) \text{ admits local transfer at the identity.} \end{array}$$

It is then immediate that:

$$(1.6) \quad \begin{array}{l} \text{we have local transfer for } (G_{\epsilon_G}, H_{\epsilon_H}) \text{ at any} \\ \text{central element of } H_{\epsilon_H}(F) \text{ and, in particular, at } \epsilon_H. \end{array}$$

Now we take $f^G \in C_c^\infty(G(F))$ and set

$$\Phi(\gamma_H, f^G) = \sum_{\gamma_G} \Delta(\gamma_H, \gamma_G) \Phi(\gamma_G, f^G).$$

We have to show that $\Phi(\cdot, f^G)$ is a stable orbital integral on $H(F)$. For this it is sufficient to show that it is *locally* a stable orbital integral on $H(F)$, i.e., for each semisimple $\epsilon_H \in H(F)$ there is $f_{\epsilon_H} \in C_c^\infty(H(F))$ such that

$$\Phi^{\text{st}}(\gamma_H, f_{\epsilon_H}) = \Phi(\gamma_H, f^G)$$

for strongly regular γ_H near ϵ_H [LS3, Lemma 2.2.A].

The Harish-Chandra descent for orbital integrals says that near semisimple ϵ_G in $G(F)$ the stable orbital integral $\Phi(\gamma_H, f^G)$ is a sum of stable orbital integrals on the groups $G_{\epsilon'_G}(F)$, with ϵ'_G stably conjugate to ϵ_G . See [LS3, Section 1.5]. From (1.6) we conclude that near each semisimple ϵ_H in $H(F)$, $\Phi(\gamma_H, f^G)$ is a stable orbital integral on $H_{\epsilon_H}(F)$ and hence one on $H(F)$, and transfer for (G, H) follows.

§2 AN OUTLINE OF THE GENERAL SETTING

An *endoscopic group* H for G is given as part of a *set of endoscopic data* for G (see [LS1]). This set also includes a group \mathcal{H} which is “almost” the L-group ${}^L H$ of H and a suitable embedding of \mathcal{H} in ${}^L G$. The group \mathcal{H} , rather than ${}^L H$, and the embedding arise naturally in two constructions of endoscopic data: the (T, κ) -construction associated with orbital integrals and the S_ϕ -construction associated with representations. In many cases, \mathcal{H} is isomorphic to ${}^L H$ and transfer involves $H(F)$ itself. In general, however, we introduce a suitable central extension H_1 of H ; in particular, $H_1(F) \rightarrow H(F)$ is surjective. Then for transfer we consider representations of $H_1(F)$ which act according to a

character λ of $Z_1(F) = \ker(H_1(F) \rightarrow H(F))$ specified by our data. On the dual side, we consider the stable orbital integrals of those functions on $H_1(F)$ which transform under translation by $Z_1(F)$ according to the character λ^{-1} .

For the problems of local behavior (around a semisimple point) it makes no difference whether we work on $H_1(F)$ or on $H(F)$. Only in patching together the local results is passage to $H_1(F)$ necessary. This passage affects only a single term Δ_2 in the transfer factor which can be handled quite easily. See Section 4.4 of [LS1]. To simplify the exposition and save notation we will assume from now on that \mathcal{H} is isomorphic to ${}^L H$ and so take $H_1 = H$.

We follow the steps in Section 1. First, there is a canonical map of semisimple conjugacy classes in $H(\overline{F})$ to such classes in $G(\overline{F})$ and this map respects Galois action (see [LS1]). The *strongly G -regular* classes of elements in $H(\overline{F})$ are those mapping to the strongly regular elements in $G(\overline{F})$. We have then a simple notion of *image* for stable conjugacy classes of strongly G -regular elements in $H(F)$ using the *inverse* of the canonical map on classes. *Transfer* is again specified by (1.3), i.e., by

$$\Phi^{\text{st}}(\gamma_H, f^H) = \sum \Delta(\gamma_H, \gamma_G) \Phi(\gamma_G, f^G),$$

now for all strongly G -regular elements γ_H of $H(F)$, and where now $\Delta(\gamma_H, \gamma_G)$ is the transfer factor of [LS1]. For *local transfer at the identity* we require (1.3) only for those γ_H near the identity and we may replace $\Delta(\gamma_H, \gamma_G)$ by a locally defined term $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$ which coincides with $\Delta(\gamma_H, \gamma_G)$ up to a constant.

To shorten the exposition we will assume F *nonarchimedean*. Suppose for now that G is *quasi-split*. There are various ways to describe $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$. See for example, [H2], [LS2], [S2]. We follow [S2]. Given $\gamma_H \in H(F)$ strongly G -regular and sufficiently close to the identity we shall fix an admissible embedding of $T_H = \text{Cent}(\gamma_H, H)$ in G together with a set of a -data $\{a_\alpha\}$ for the image T_G of T_H . See [LS1, Sections (1.3), (2.2), (3.1)]. Let $\gamma_G \in T_G(F)$ be the image of γ_H under the embedding. As usual we parametrize the conjugacy classes in the stable conjugacy class of γ_G by $\mathcal{D}(T_G)$. Suppose $\gamma_G(\omega)$ is an element in the class attached to $\omega \in \mathcal{D}(T_G)$. Then

$$(2.1) \quad \Delta_{\text{loc}}(\gamma_H, \gamma_G(\omega)) = \kappa(\omega) \Delta_{\text{loc}}(\gamma_H, \gamma_G)$$

where κ is the character on $\mathcal{D}(T_G)$ determined by the endoscopic data underlying H [LS2]. Thus we have just to describe $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$. It is the product of

- (i) a root of unity determined by the embedding and a -data (Δ_I in [LS1]),
 - (ii) the usual discriminant function (Δ_{IV} in [LS1])
- and
- (iii) a term indexed by orbits of the Galois group Γ in the set R_G of roots of T_G in G .

An orbit \mathcal{O} makes a nontrivial contribution only if it is symmetric, i.e., $\mathcal{O} = -\mathcal{O}$, and it consists of roots outside H , i.e., roots not lying in the image of R_H under the map induced by our embedding of T_H in G . Then take $\alpha \in \mathcal{O}$. For γ_H sufficiently near 1 the term $\alpha(\gamma_G)^{1/2} - \alpha(\gamma_G)^{-1/2}$ is defined in the usual way. Moreover,

$$\frac{\alpha(\gamma_G)^{1/2} - \alpha(\gamma_G)^{-1/2}}{a_\alpha}$$

lies in the fixed field $F_{\pm\alpha} \subset \overline{F}$ of the stabilizer of $\pm\alpha$ in Γ . Let F_α be the fixed field of the stabilizer of α in Γ . Because \mathcal{O} is symmetric, F_α is a quadratic extension of $F_{\pm\alpha}$. Let χ_α be the attached quadratic character of $F_{\pm\alpha}$. Then the contribution to $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$ from \mathcal{O} is

$$\chi_\alpha\left(\frac{\alpha(\gamma_G)^{1/2} - \alpha(\gamma_G)^{-1/2}}{a_\alpha}\right).$$

For general G there is an additional contribution to $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$. First, strongly G -regular γ_H may be the image of no γ_G in $G(F)$. In that case,

$$\Delta_{\text{loc}}(\gamma_H, \gamma_G) = 0$$

for all strongly regular γ_G in $G(F)$. On the other hand, if γ_H is the image of some element γ_G then we may embed $T_G = \text{Cent}(\gamma_G, G)$ in G^* , the quasi-split inner form of G . Let γ_{G^*} be the image of γ_G under this embedding. Then γ_{G^*} is an image of γ_G in the sense of Section 1, and with H regarded as endoscopic for G^* , γ_H is an image of γ_{G^*} . The Local Hypothesis indicates how we are to write $\Delta_{\text{loc}}(\gamma_H, \gamma_G)$ in terms of $\Delta_{\text{loc}}(\gamma_H, \gamma_{G^*})$. See [LS1], [LS2] for details.

To express local transfer at the identity in terms of Shalika germs, let \mathcal{O}_G be a unipotent conjugacy class in $G(F)$ and define

$$\Gamma_{\mathcal{O}_G}^{(H)}(\gamma_H) = \sum_{\gamma_G} \Delta_{\text{loc}}(\gamma_H, \gamma_G) \Gamma_{\mathcal{O}_G}(\gamma_G)$$

with the usual notational conventions. Then local transfer at the identity amounts to showing:

$$(2.2) \quad \begin{aligned} & \text{for each unipotent conjugacy class } \mathcal{O}_G \text{ in } G(F), \\ & \text{the transferred “}\kappa\text{-germ” } \Gamma_{\mathcal{O}_G}^{(H)} \text{ is a linear} \\ & \text{combination of the stable germs } \Gamma_{\mathcal{O}_H}^{\text{st}} \text{ for } H. \end{aligned}$$

Property (2.2) is known for \mathcal{O}_G regular [LS1], \mathcal{O}_G trivial (an easy consequence of well-known results) or, in many cases, \mathcal{O}_G subregular [H1]. There are also complete results for specific groups, e.g., $SU(3)$ [LS2], [H2] and $GS\mathfrak{p}(4)$ [H3].

Transition to globally defined transfer factors, i.e., factors defined on all strongly G -regular elements, is quite subtle. First, we introduce a term (Δ_2 in [LS1]) which is a quasi-character on the Cartan subgroup $T_H(F)$ containing γ_H . This character is an analogue of the “ ρ -shift” for real groups and, similarly, uses classification of the embeddings of the L -groups of Cartan subgroups in ${}^L G$. Data from this analysis also allows us to replace (iii) above with a globally defined term. Moreover, we do not use (2.1) and the Local Hypothesis separately but instead introduce a new unified term (Δ_1 in [LS1]). Now, however, only a *relative transfer factor* $\Delta(\gamma_H, \gamma_G; \bar{\gamma}_H, \bar{\gamma}_G)$ is well-defined (and canonical), for $\gamma_H, \bar{\gamma}_H$ any two strongly G -regular elements, with $\bar{\gamma}_H$ an image of $\bar{\gamma}_G$. But then we fix some such $\bar{\gamma}_H$ and $\bar{\gamma}_G$, specify $\Delta(\bar{\gamma}_H, \bar{\gamma}_G)$ arbitrarily and set

$$\Delta(\gamma_H, \gamma_G) = \Delta(\gamma_H, \gamma_G; \bar{\gamma}_H, \bar{\gamma}_G) \Delta(\bar{\gamma}_H, \bar{\gamma}_G).$$

See [LS1, Sections 3.7, 4.1]. This normalization fits well with adelic considerations [LS1, Section 6] and the problem of handling several inner forms of G simultaneously.

With $\Delta(\gamma_H, \gamma_G)$ defined we continue the program of Section 1. Let semisimple ϵ_H in $H(F)$ be an image [LS1] of, say, ϵ_G in $G(F)$. Again we may assume H_{ϵ_H} quasi-split. Then H_{ϵ_H} is endoscopic for G_{ϵ_G} [LS3, Section 1.4]. We assume the analogue of (1.5), i.e., local transfer at the identity for all pairs $(G_{\epsilon_G}, H_{\epsilon_H})$. To obtain local transfer at ϵ_H we need only observe the following property of transfer factors under translation by central elements. We shall state it for (G, H) . The center Z_G of $G(F)$ is canonically embedded in the center of $H(F)$. If $z \in Z_G$ and γ_H is an image of γ_G then $z\gamma_H$ is an image of $z\gamma_G$. According to [LS1, Lemma 4.4.A] there is a character λ on Z_G such that

$$\Delta(z\gamma_H, z\gamma_G) = \lambda(z) \Delta(\gamma_H, \gamma_G)$$

for all z, γ_H . The proof of this requires a detailed analysis of the term Δ_2 in the transfer factor (see [LS3, Sections 3, 4]).

We now assume the analogue of (1.6), i.e., each pair $(G_{\epsilon_G}, H_{\epsilon_H})$ admits local transfer around ϵ_H . For $f^G \in C_c^\infty(G(F))$, consider the “normalized κ -orbital integral”

$$\Phi(\gamma_H, f^G) = \sum_{\gamma_G} \Delta(\gamma_H, \gamma_G) \Phi(\gamma_G, f^G)$$

for γ_H near ϵ_H . If ϵ_H is not an image then $\Phi(\gamma_H, f^G)$ vanishes near ϵ_H . Otherwise fix ϵ_G with ϵ_H as an image. Each orbital integral $\Phi(\gamma_G, f^G)$ is, by Harish-Chandra descent, locally an orbital integral on $G_{\epsilon'_G}(F)$ for some suitable ϵ'_G stably conjugate to ϵ_G . We need also descent for the transfer factor $\Delta(\gamma_H, \gamma_G)$. The main theorem of [LS3] is that

$$\Delta_G(\gamma_H, \gamma_G) = (\text{const}) \Delta_{G_{\epsilon'_G}}(\gamma_H, \gamma_G)$$

for γ_H near ϵ_H , γ_G near ϵ'_G . (For F archimedean, this is true in the limit). The result is better stated in terms of relative transfer factors. Take $\bar{\gamma}_H, \bar{\gamma}_G$ near ϵ_H, ϵ'_G respectively, with $\bar{\gamma}_H$ an image of $\bar{\gamma}_G$, and set $\Theta = \Delta_G / \Delta_{G_{\epsilon'_G}}$. Then

$$(2.3) \quad \Theta(\gamma_H, \gamma_G; \bar{\gamma}_H, \bar{\gamma}_G) = 1.$$

The proof of (2.3) in [LS3] proceeds as follows. First we reduce easily to the case G quasi-split and $\epsilon'_G = \epsilon_G$ (see Section 3.1). Then we analyse in detail the cohomologically defined factors Θ_I, Θ_2 of Θ (Sections 3, 4). The only other factor in Θ possibly making a nontrivial contribution is Θ_{II} which is given by a simple explicit expression indexed by Galois orbits of roots. See (5.1); Θ_{II} is the right side of (5.1.1) and the final formulas for Θ_I, Θ_2 are also given. We find that only orbits of roots outside both G_{ϵ_G} and H may make a nontrivial contribution to Θ_I, Θ_2 or Θ_{II} . At the same time a long reduction argument allows us to assume all roots in G have the same length and that they take only the values ± 1 on ϵ_G while the coroots take only these values on the endoscopic datum s defining H (Section 5). Under such conditions we are able to compute the product of Θ_I and Θ_2 explicitly (see Theorem 6.3.C) and so compare orbit by orbit with Θ_{II} . We have, in particular, a product formula over all places which allows us to assume odd residual characteristic. The rest is a number theoretical computation (see Section 6.6).

We may now finish. The normalized κ -orbital integral $\Phi(\gamma_H, f)$ is a linear combination of such integrals for the groups $G_{\epsilon'_G}$ (see Sections 1.5,

1.7 of [LS3] for more precise information) and so we follow the argument for the stable case in Section 1. *Our conclusion is that what remains to prove transfer is the local problem (2.2) for Shalika germs.*

This very long analysis has further consequences. We mention two examples. From the study of the regular unipotent contribution we obtain in the p -adic case a formula for regular unipotent germs [S2] and in the real case a proof that the transfer factors defined here coincide with those introduced earlier in [S1]. From descent we verify some conjectures of Kottwitz [K] about extending transfer factors and the matching of orbital integrals from the “most regular” classes to all equisingular semisimple classes [LS3, Section 2].

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