Greedy Routing in Wireless Networks

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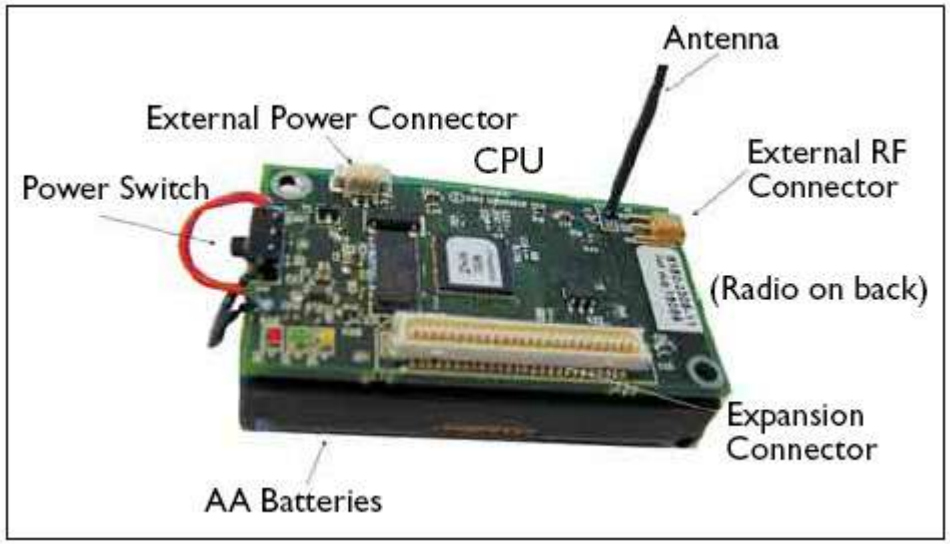
A generic sensor node

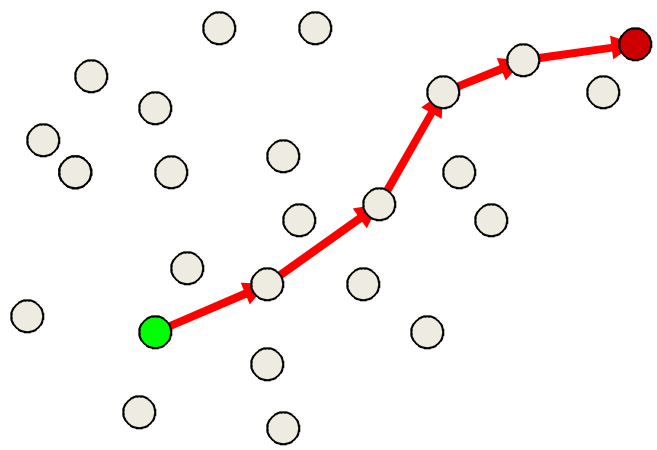
- CPU.
- On-board flash **memory** or external memory
- Sensors: thermometer, camera, motion, light sensor, etc.
- **Wireless** radio.
- **Battery**.



- **Environment Monitoring**
- **Cyber Physical Systems**
- **Participatory Sensing**

Greedy Routing

- Assign coordinates to nodes
- Message moves to neighbor closest to destination
- Simple, compact, scalable



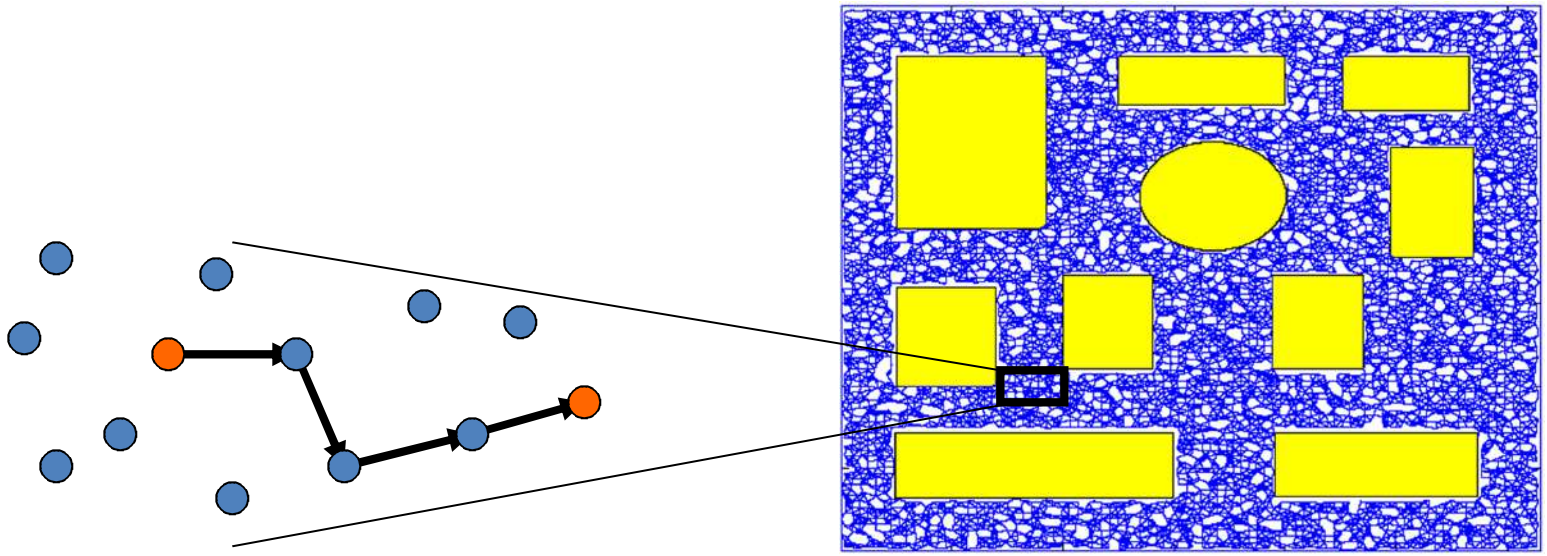
Our objective

1. Examine a realistic setting of dense networks with holes
2. Greedy routing w/ nice properties
   - Guaranteed delivery
   - Load balancing
   - Multipath routing
   - Link dynamics
   - Node mobility

- Tool: computational geometry.

Large-scale sensor field

- large-scale, dense deployment.
  - Sufficient sensor coverage.
- w/ holes/obstacles.
- Prominent feature: **shape** of the network.



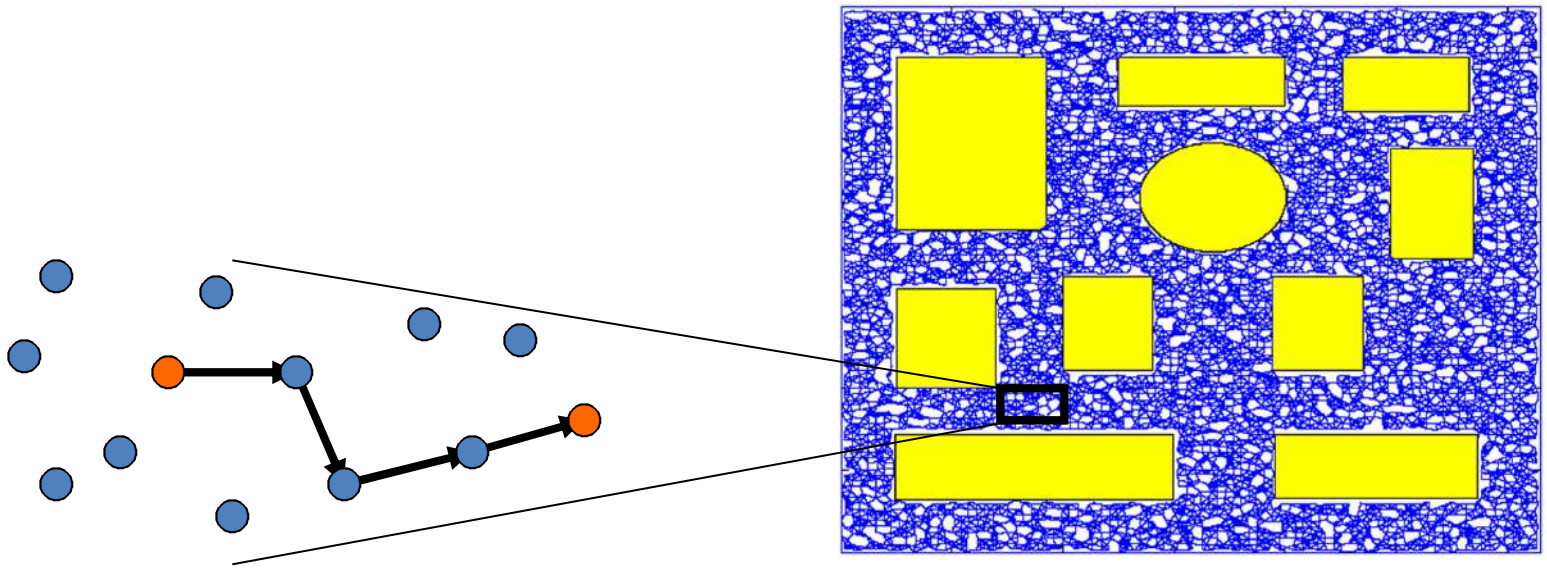
Local view vs. global view

Local view

- dense graphs
- links are dynamic

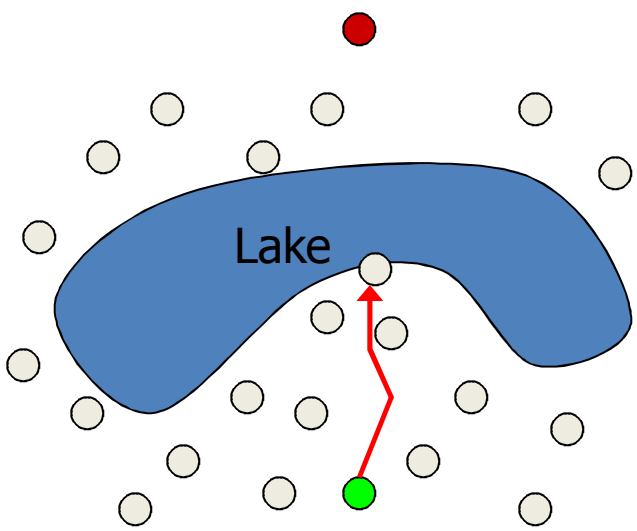
Global view

- geometric model
- topology is stable



Part I: Guaranteed delivery

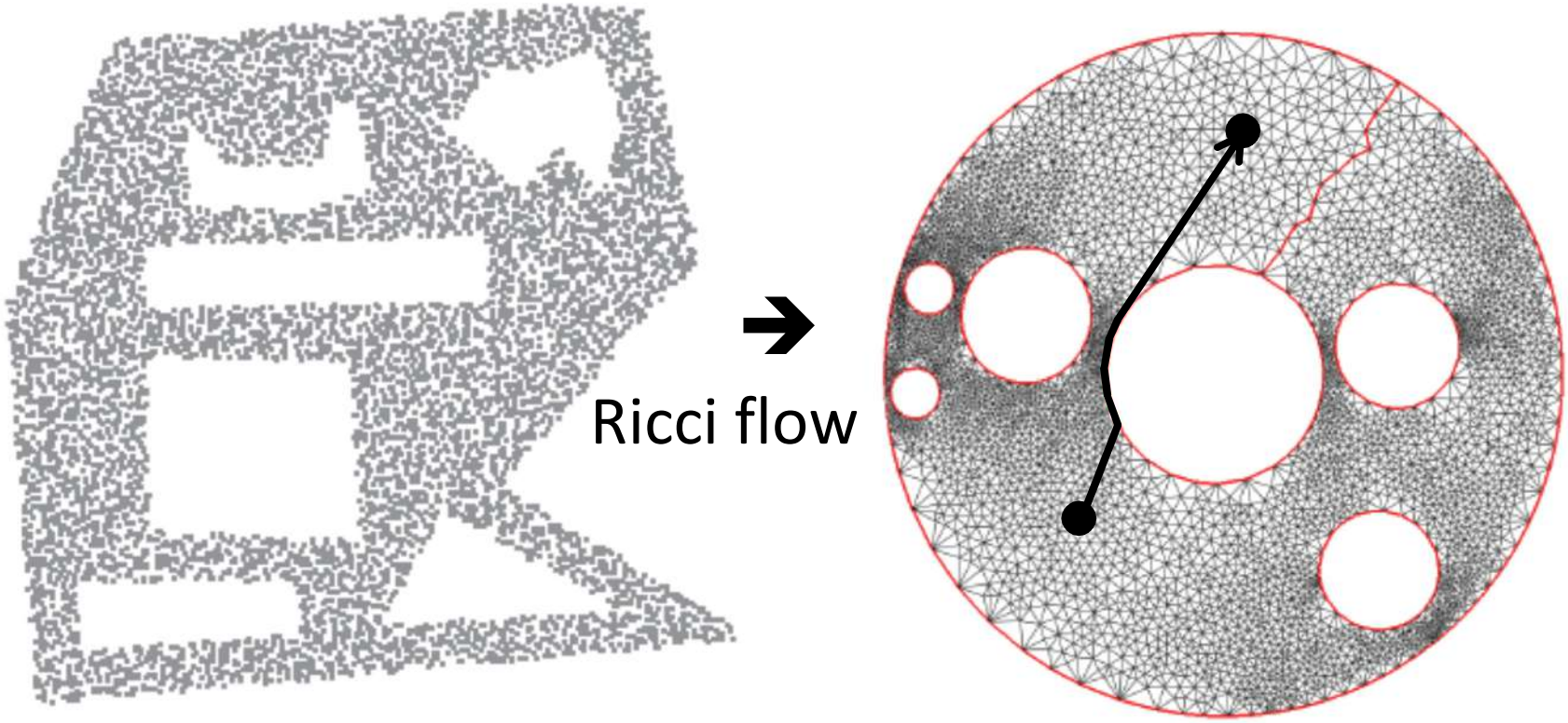
- Can get stuck



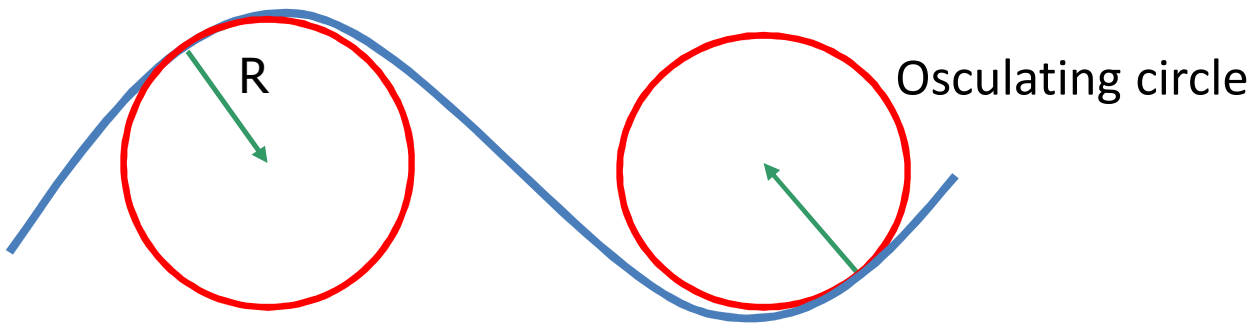
- 1st idea: take detour to get around the local minimum.
- 2nd idea: assign new coordinates & **remove** local minima.

Deform the holes to be circular

- Greedy routing does not get stuck at circular holes.



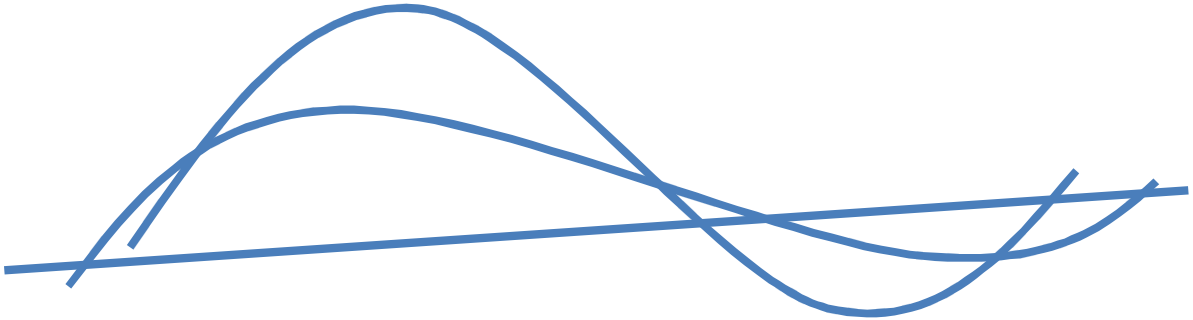
Curvature



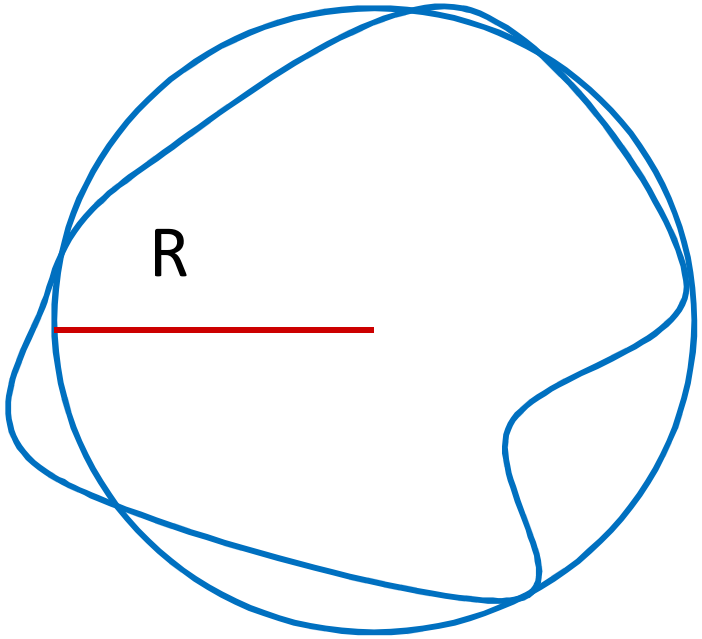
Curvature: $\kappa = 1/R$

Flatter curves have smaller curvature.

Changing the Curvature

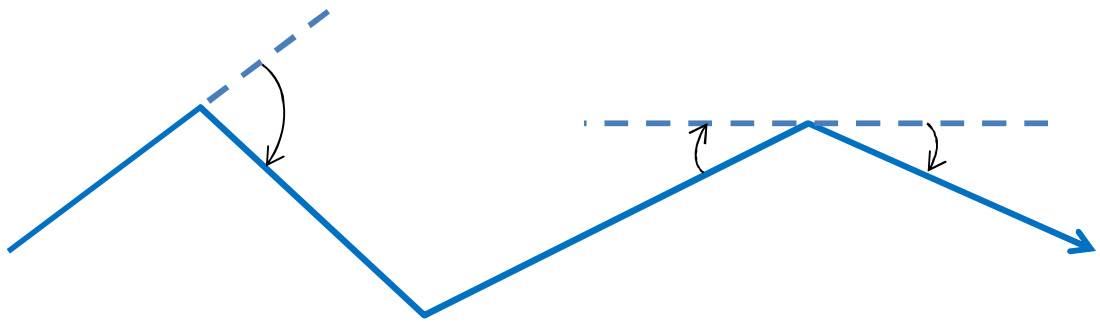


Flatten a curve:
$\kappa \to 0$

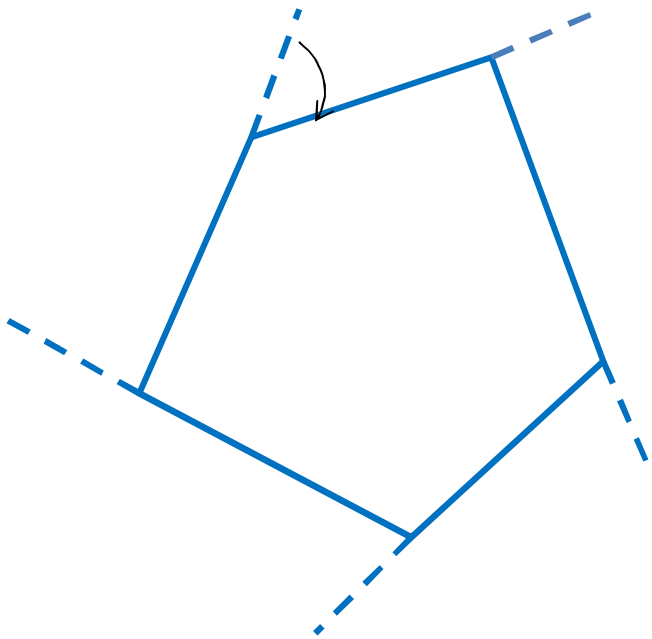


Make a cycle round
$\kappa \to 1/R$

Discrete Curvature

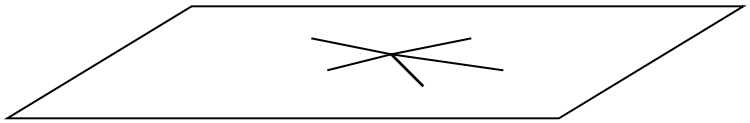


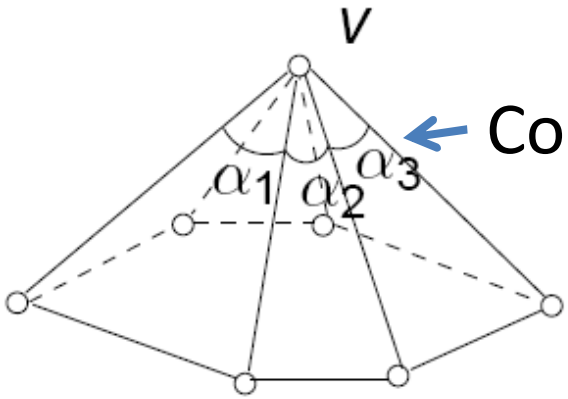
Turning angle



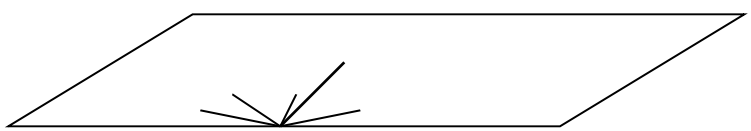
Sum of curvature = 2π

Discrete Curvature in 2D Triangulated Surface





Corner angle



For an interior vertex

Deviation from the plane

$$\kappa(v) = 2\pi - \sum_i \alpha_i$$

For a vertex on the boundary

Deviation from straight line

$$\kappa(v) = \pi - \sum_i \alpha_i$$

What they look like

Negative Curvature

Positive Curvature

Discrete Curvature in 2D Triangulated Surface

- Gauss Bonnet Theorem: total curvature of a surface M is a topological invariant:

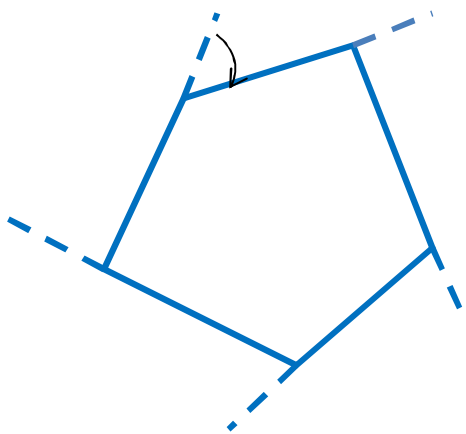
$$\sum_{v_i \in V} K_i = 2\pi\chi(M)$$

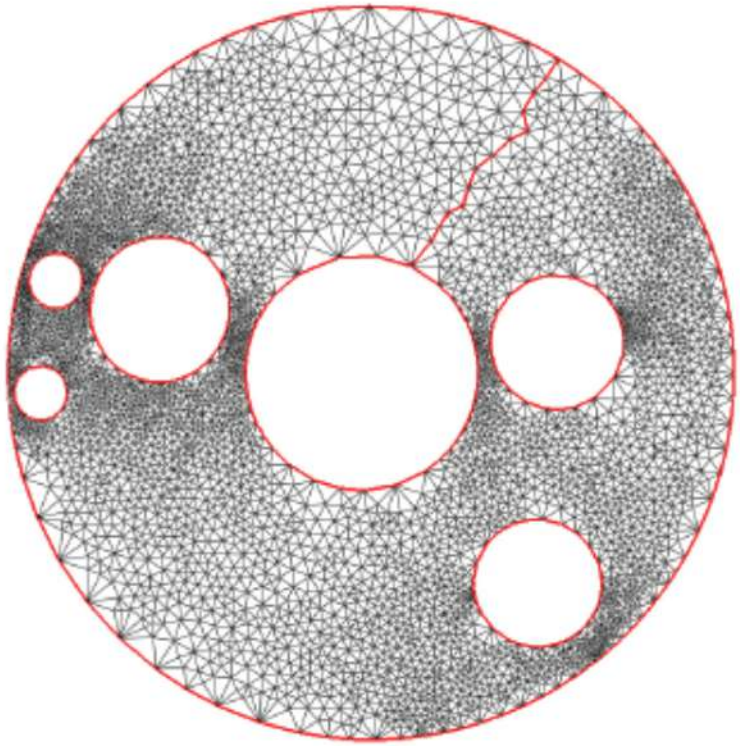
Euler Characteristic: 2 – 2(# handles) – (# holes)

- Ricci flow: diffuse uneven curvatures to constant

Sanity check: total curvature

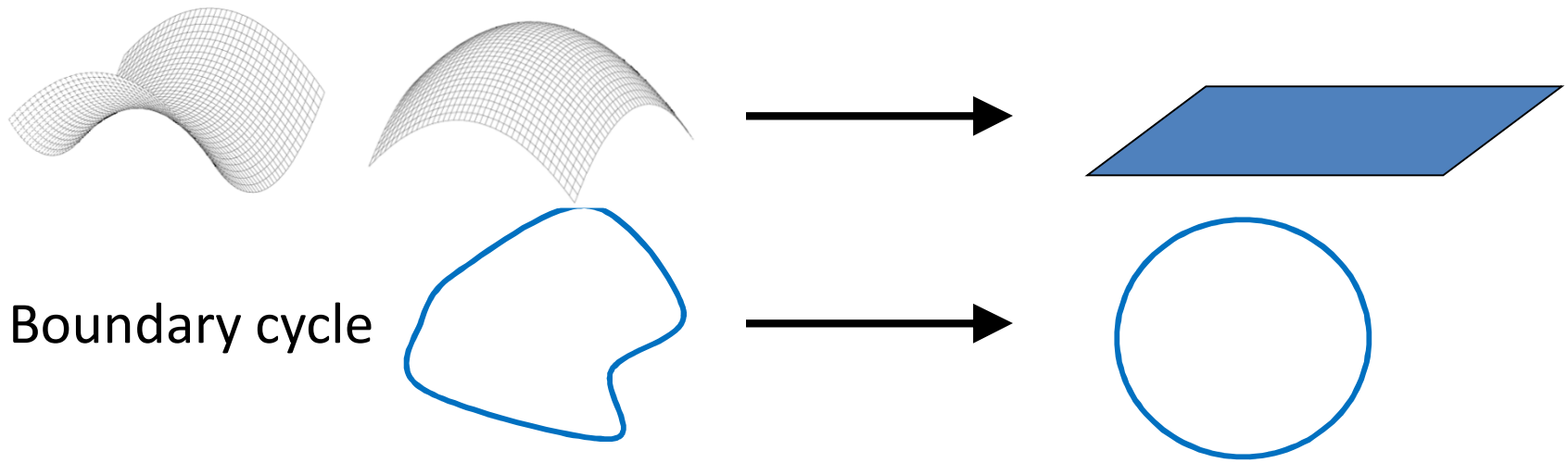
- h holes, Euler characteristics = 2-(h+1)
- Outer boundary: curvature 2π
- Each inner hole: -2π
- Interior vertex: 0
- Total curvature: (1-h)2π





Ricci Flow in our case

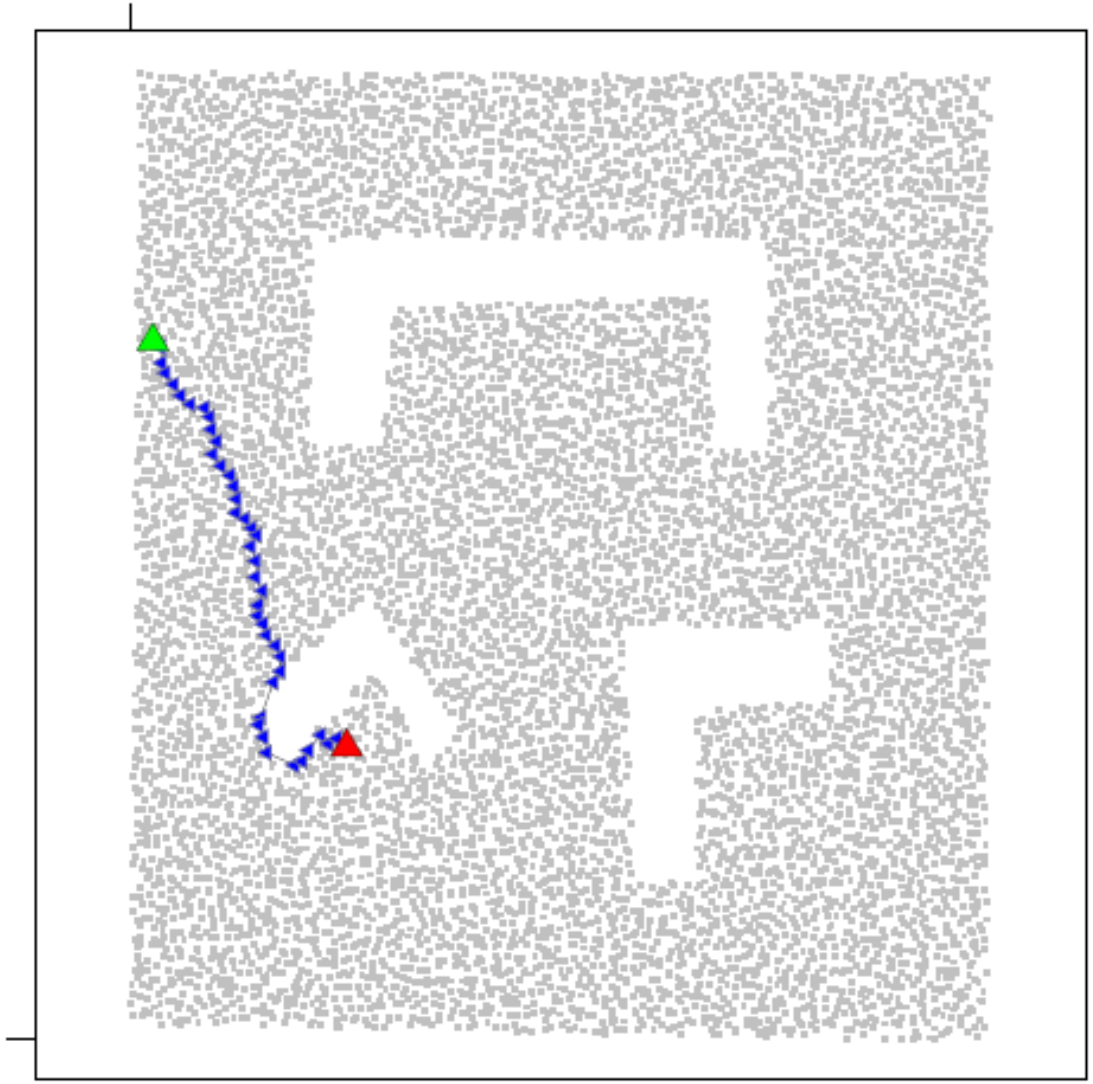
- Target: a metric with pre-specified curvature



Ricci flow on sensor networks

- Curvature flow (Ricci flow) is a natural **distributed** algorithm.
  - Each node computes its current curvature & compare with target curvature
  - Each node changes the length of its adjacent edges ~ the curvature difference
  - Iterate
- Ricci energy is strictly convex ➔ unique solution

Routing

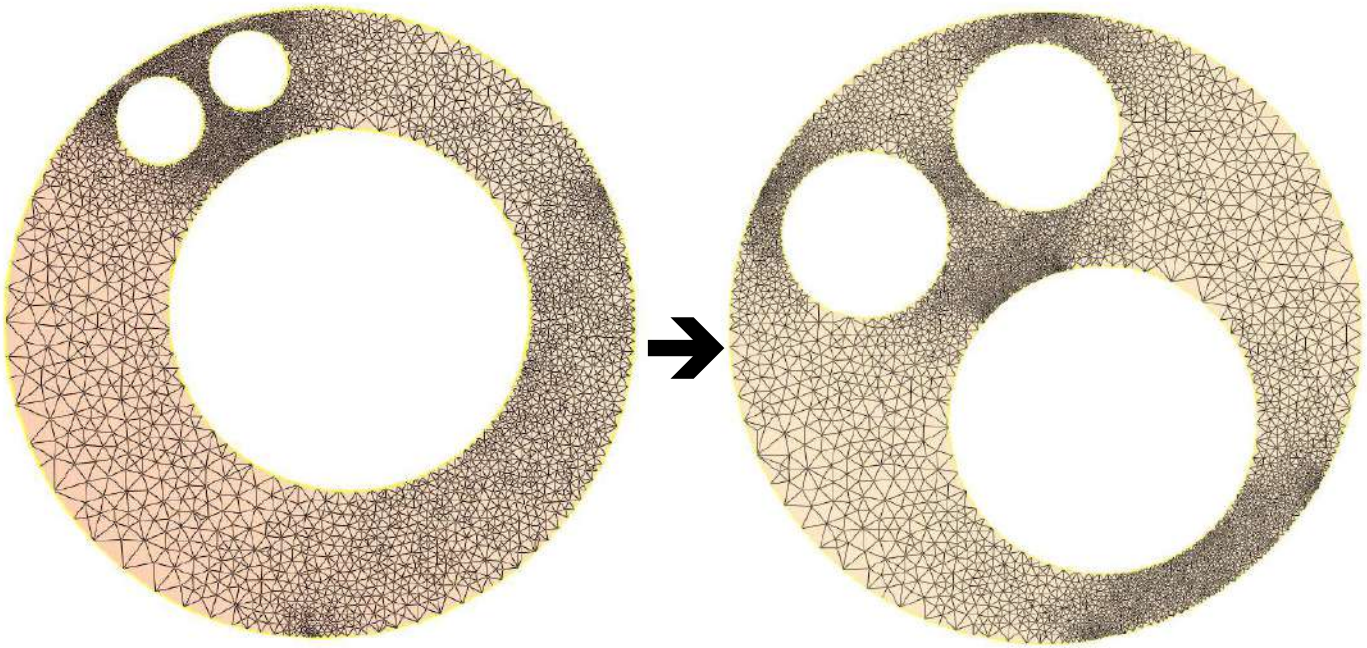


Deforming into a circular domain

- Deform a complex shape to a simple one, making it easy to explore the **space of paths**.
  - Multi-path routing
  - Recover from link failure

Circular Domains are not Unique

- Embedding into a circular domain is not unique, they differ by a Möbius transformation.

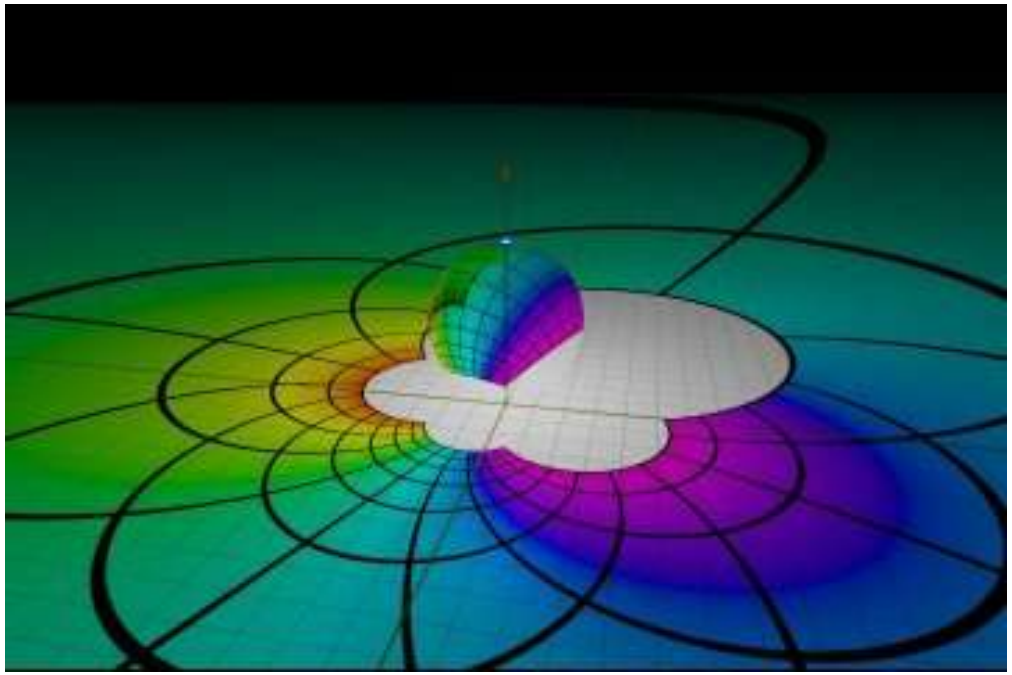


Möbius Transform

- Möbius transform
  - Conformal: maps circles to circles
  - Four basic elements: translation, dilation, rotation, inversions.

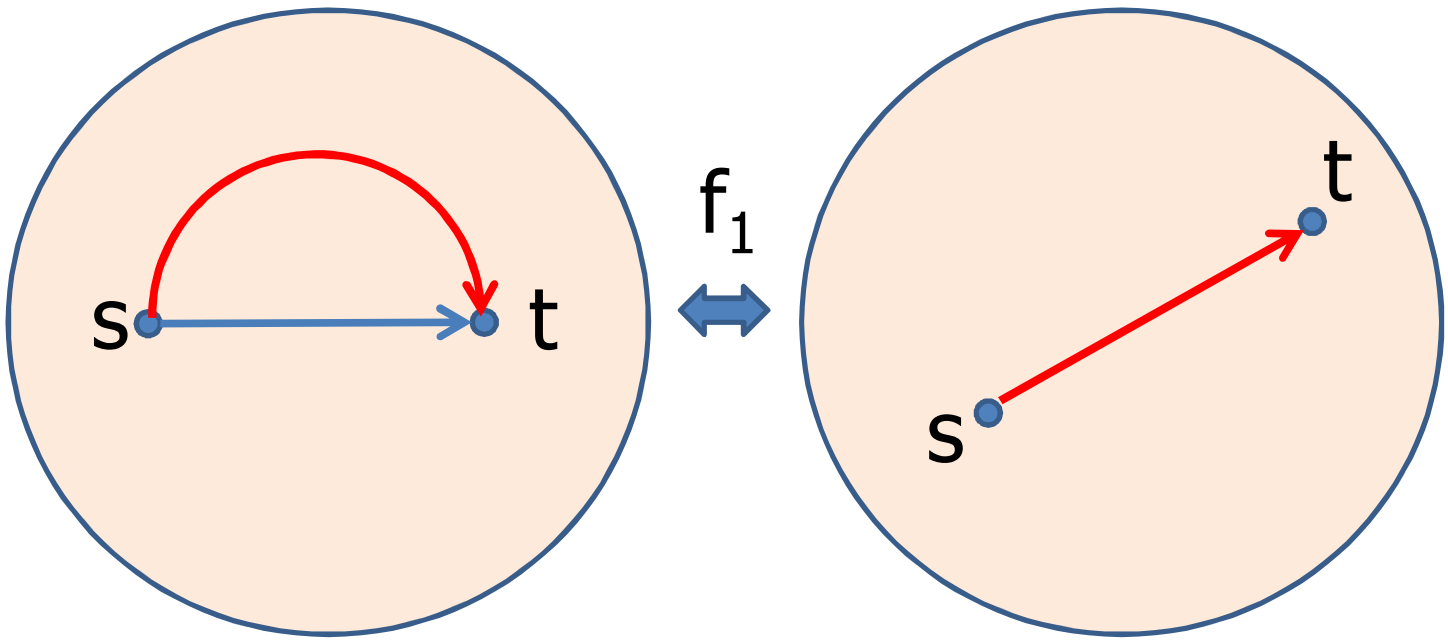
$$f(z) = \frac{az + b}{cz + d}$$

a, b, c, d are 4 complex numbers, ad ≠ bc



Greedy Paths in Different Circular Domains

- Generate disjoint paths using different transformations



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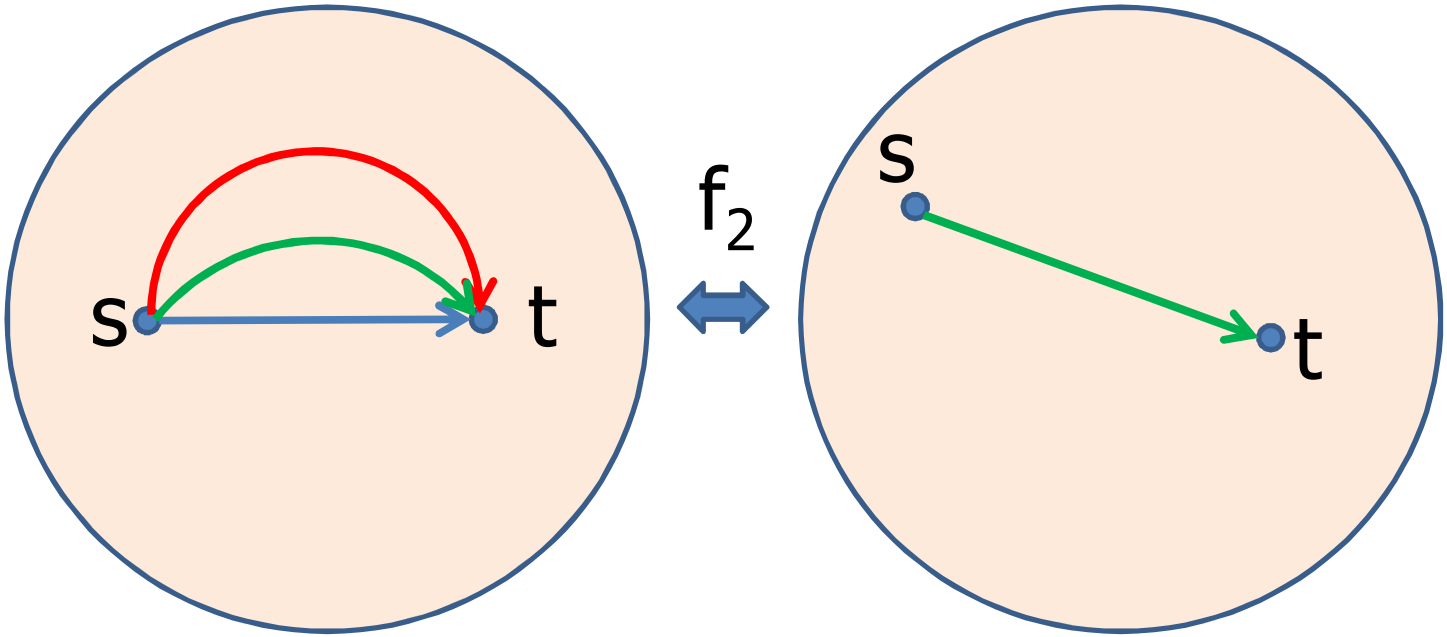
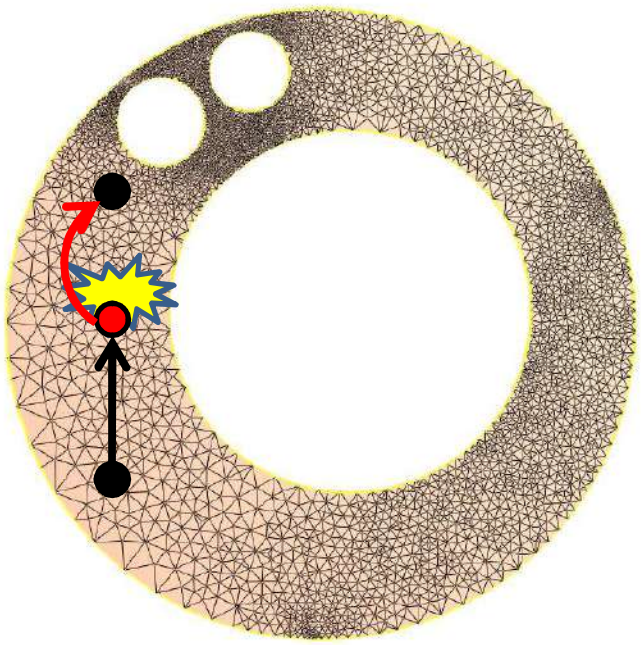
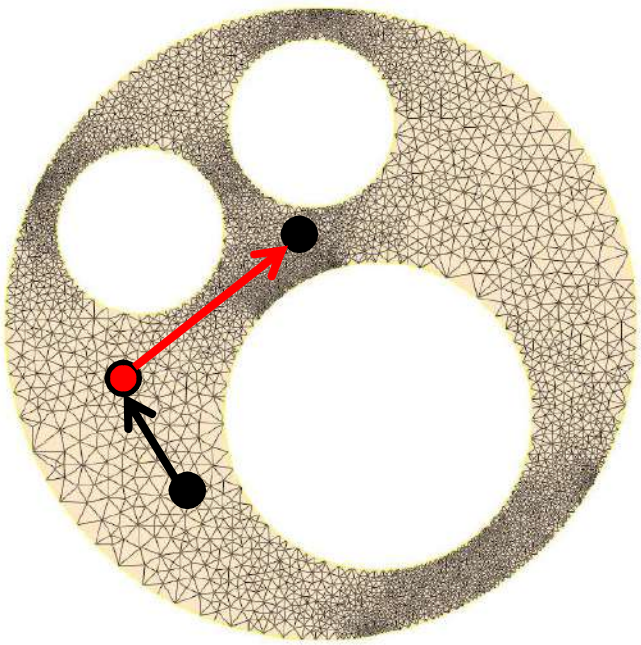


Recovery from Temporary Link Failure

- We compute a Möbius transformation
  - S.t. the broken link is **NOT** on the greedy path.
  - Möbius transform attached to packet.
  - Make big jumps





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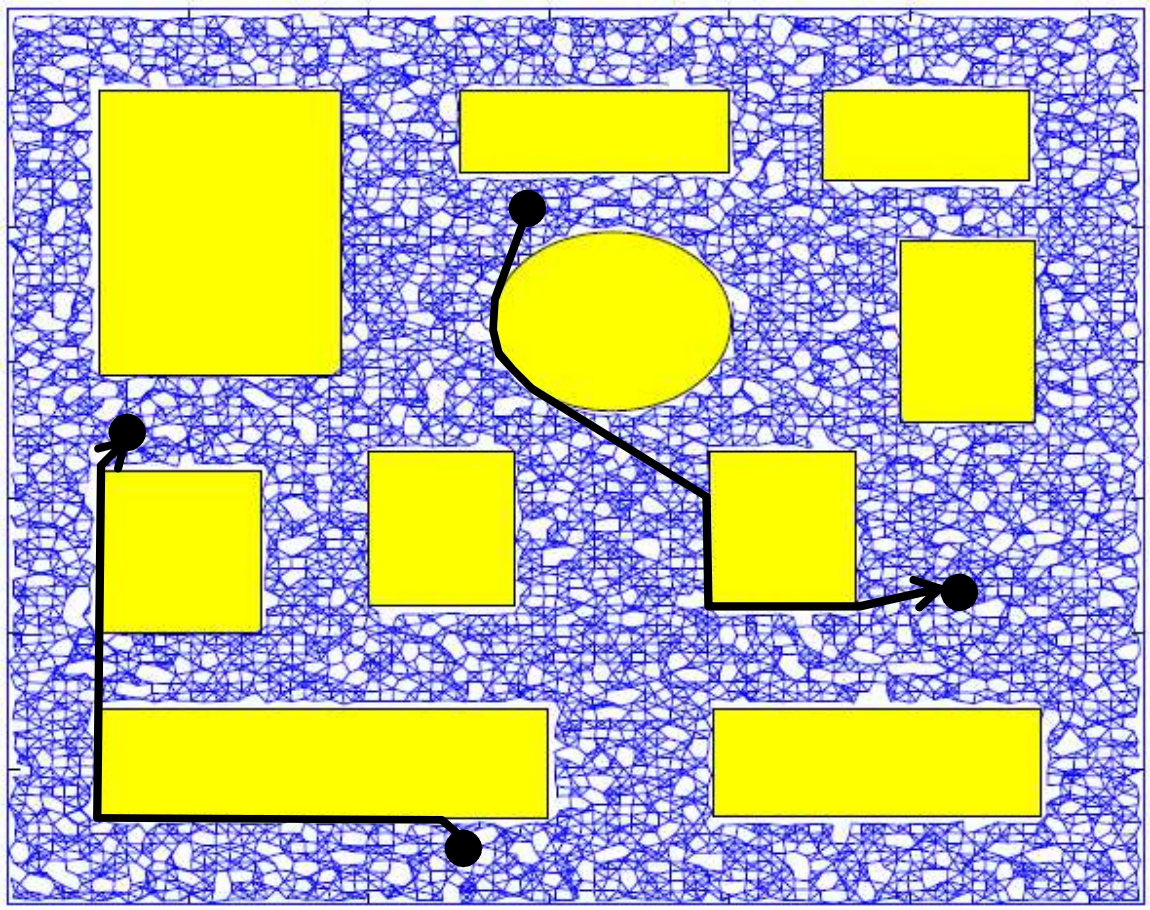
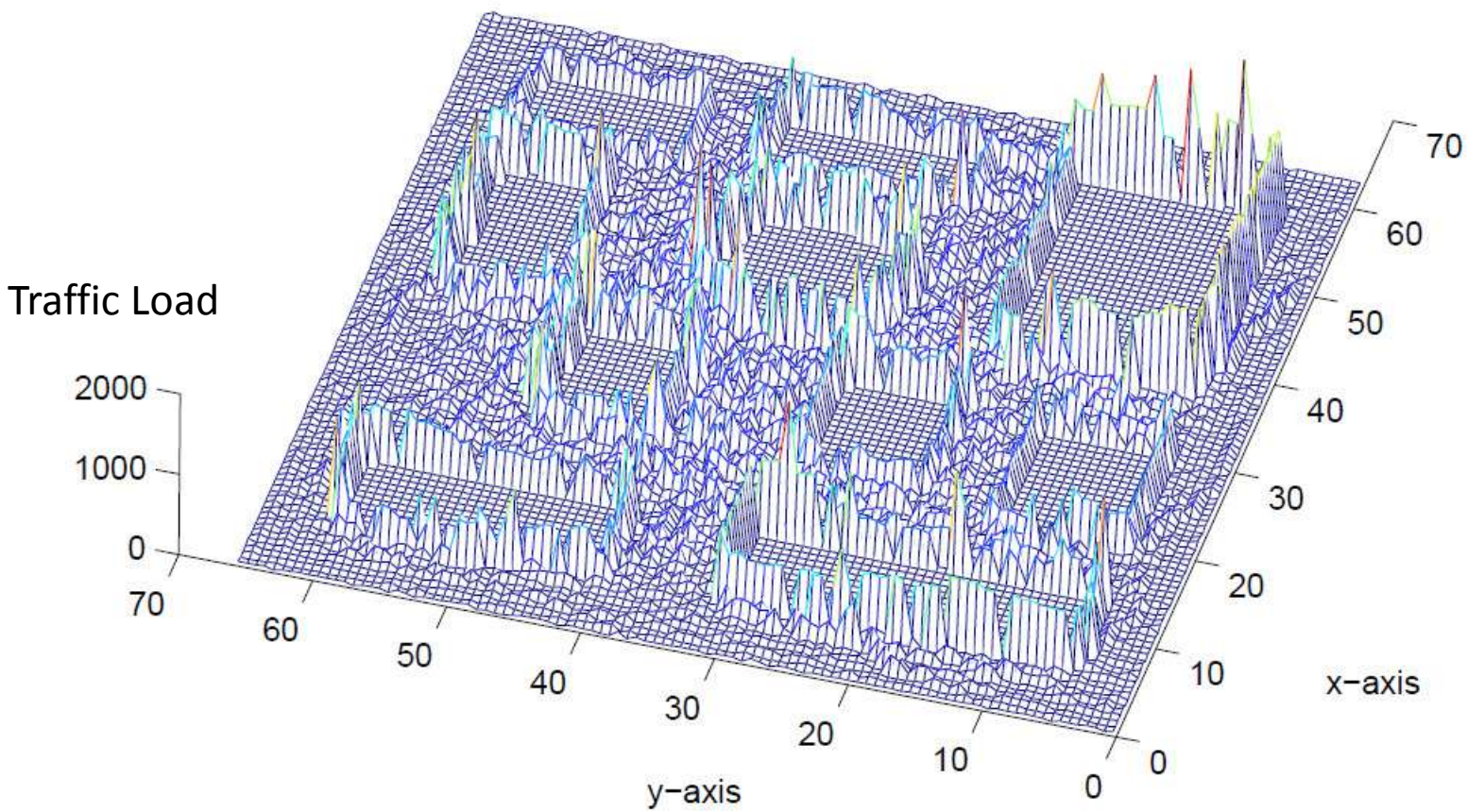
   - Node mobility

Part II: Load Balancing

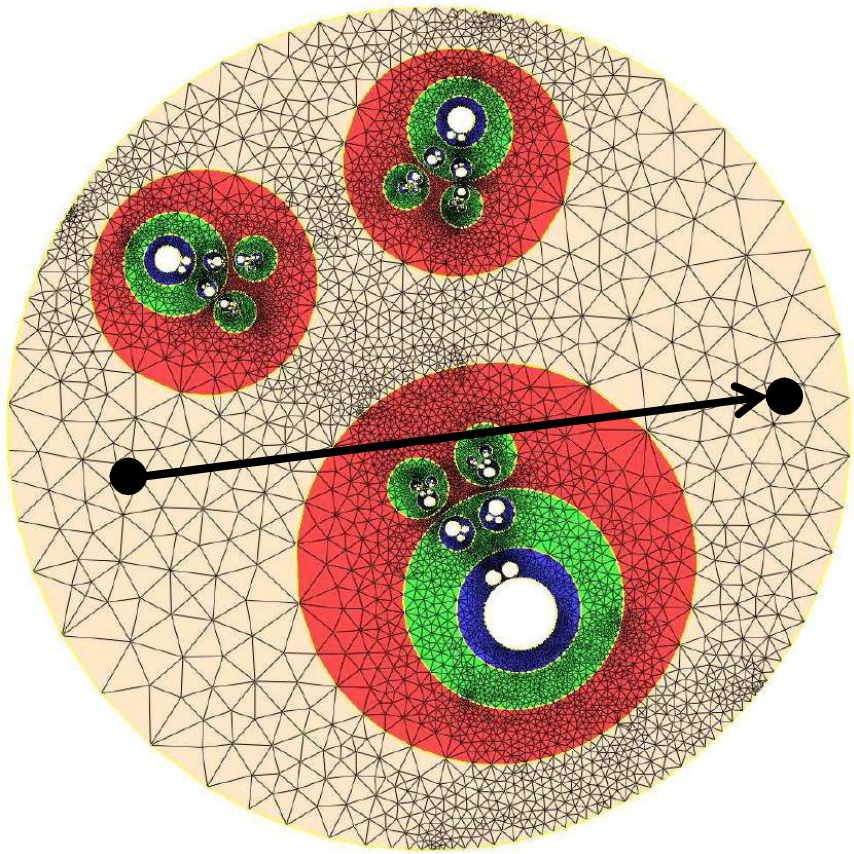


Boundary nodes get overloaded



Soln: Reduce boundary load

- Use covering space to tile up the domain;
- Boundary “disappears”.



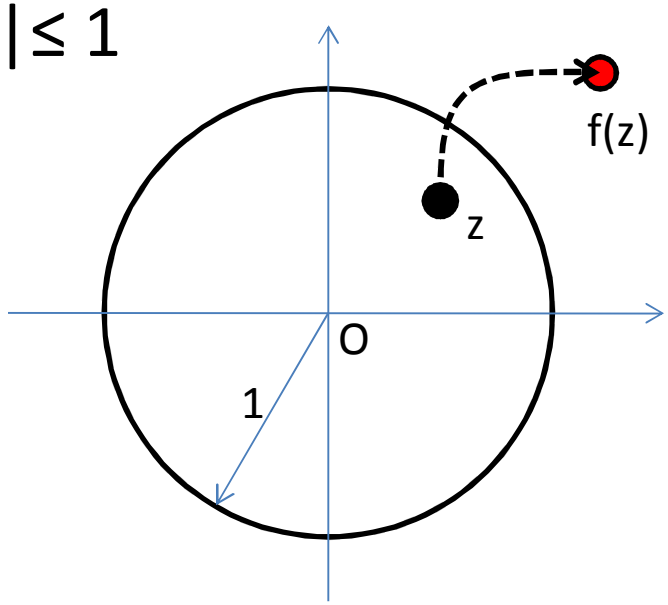
Reflections using Möbius Transform

- Map inside out

Unit circle at origin

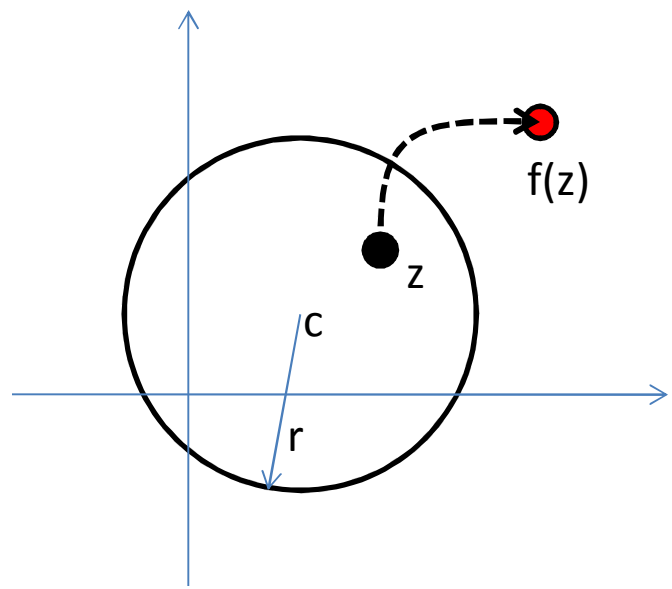
$$f(z) = \frac{1}{z}$$

$|z| \leq 1$

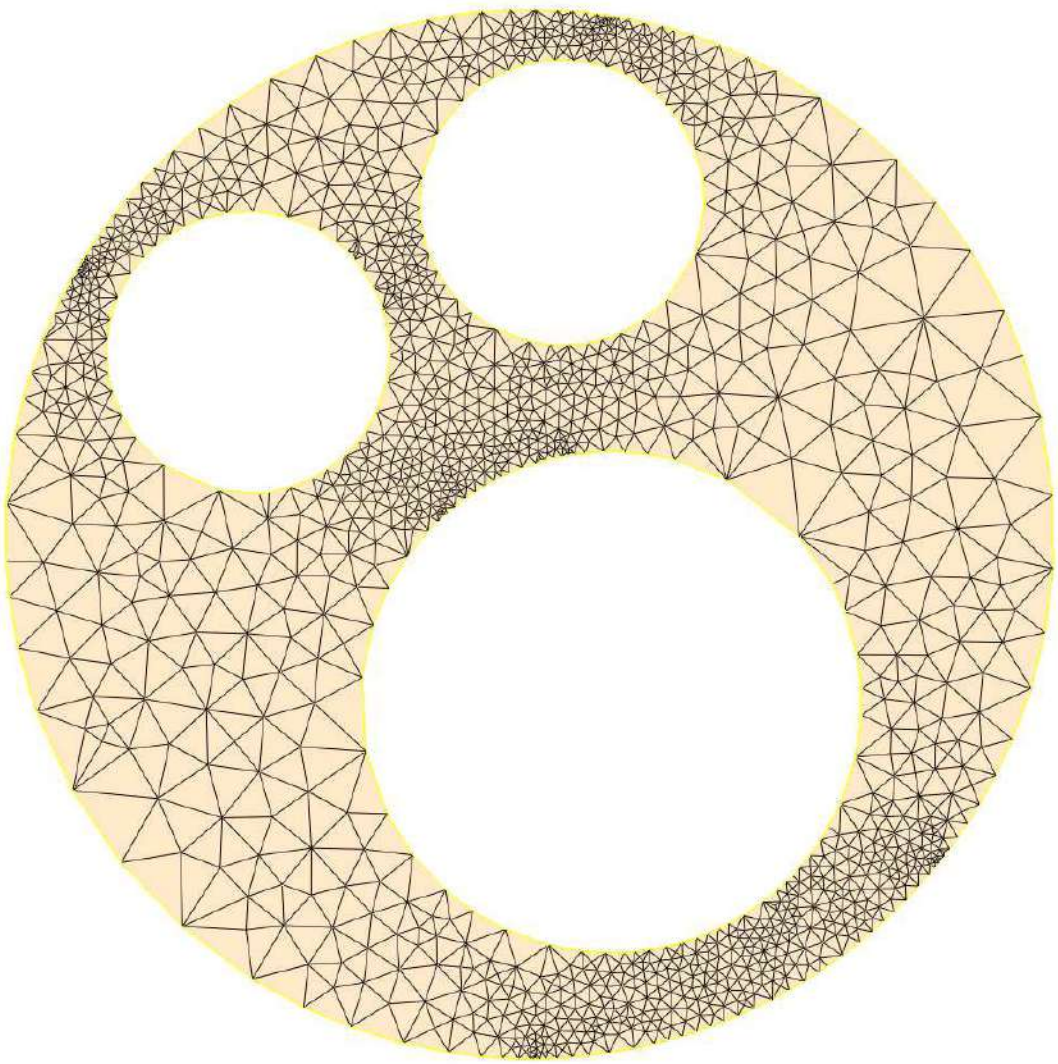


A circle at c with radius r

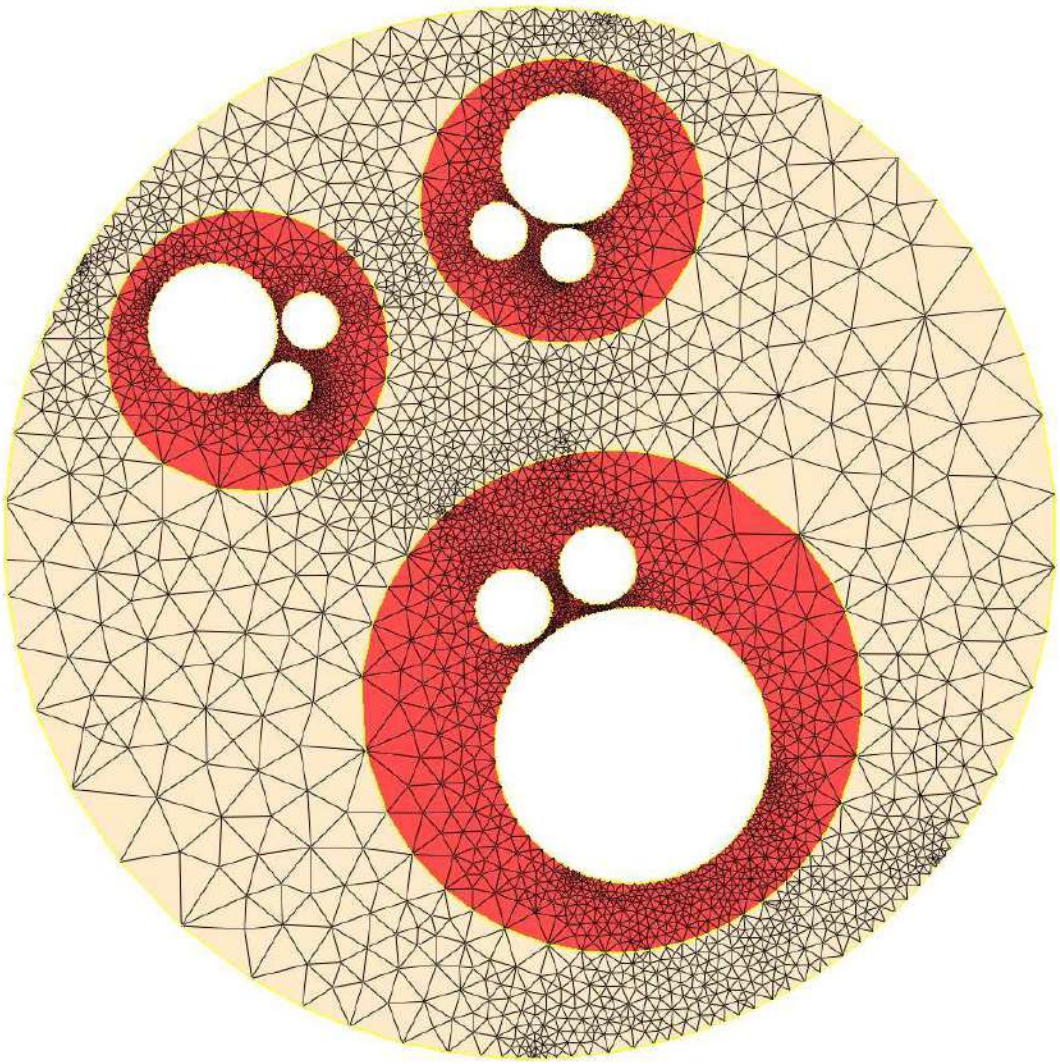
$$f(z) - c = \frac{r^2}{\bar{z} - \bar{c}}$$



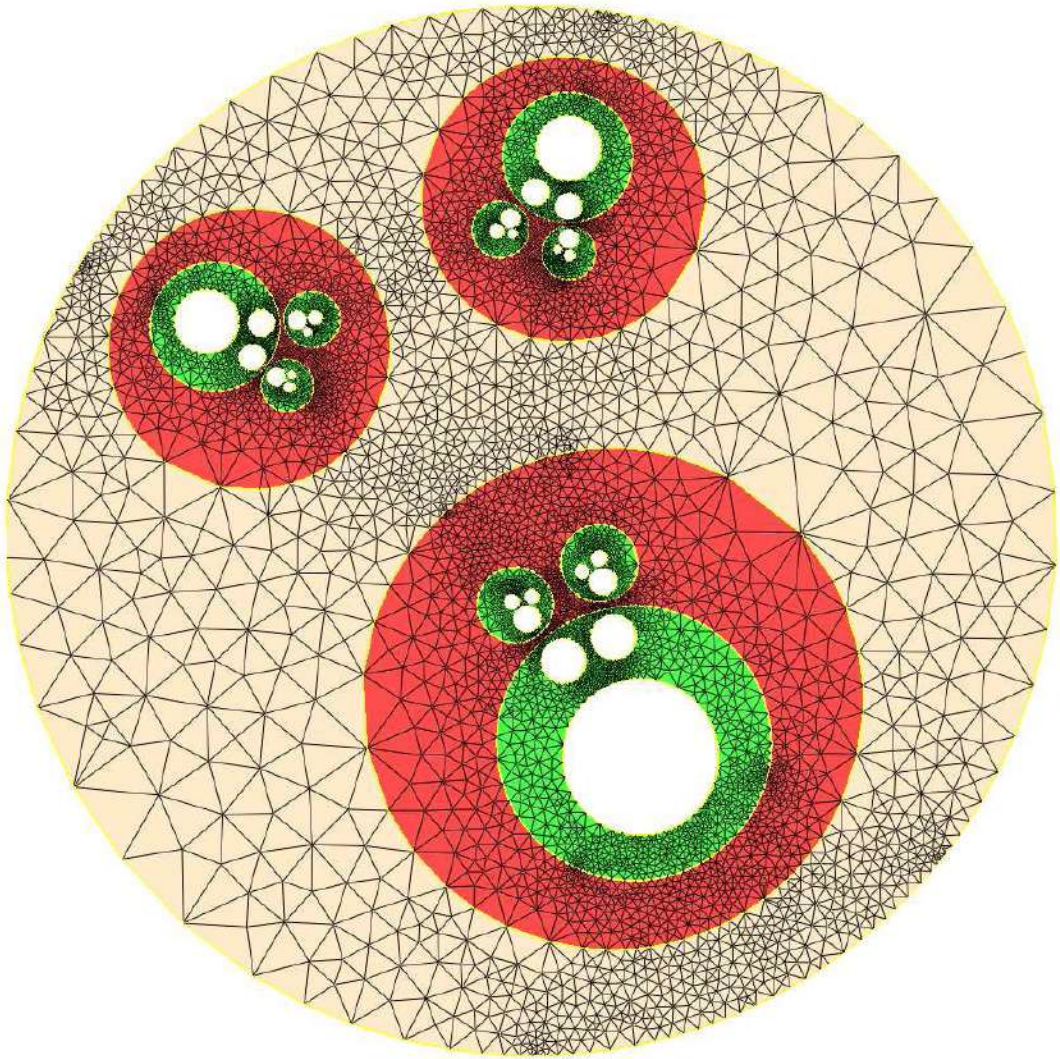
Step 1: Map All Holes Circular



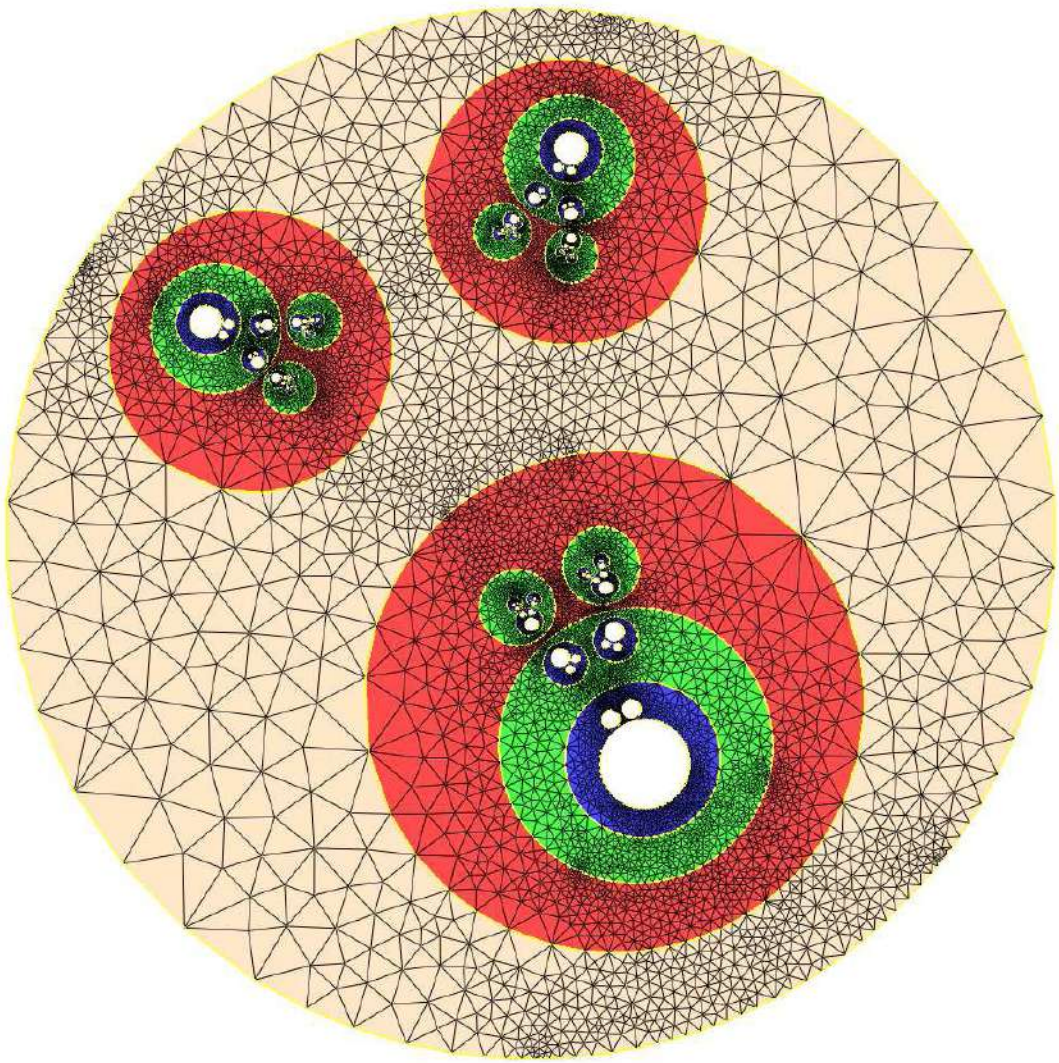
Step 2: Reflect Network for Each Hole



Step 3: Continue until all Holes are Small



Covering Space: Tile up the domain



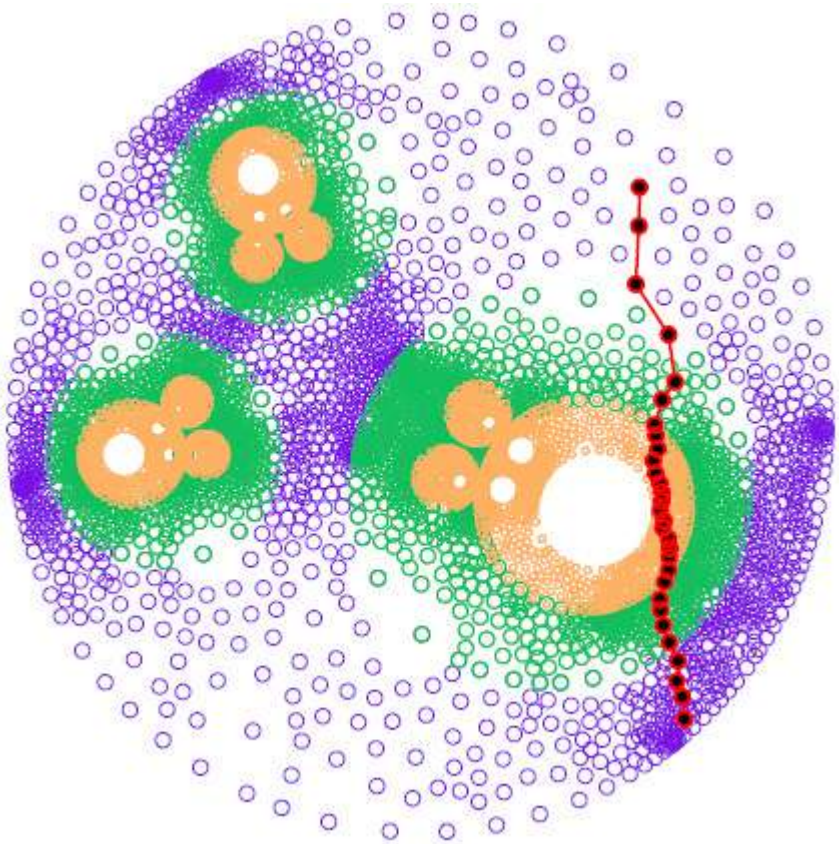
Reflections

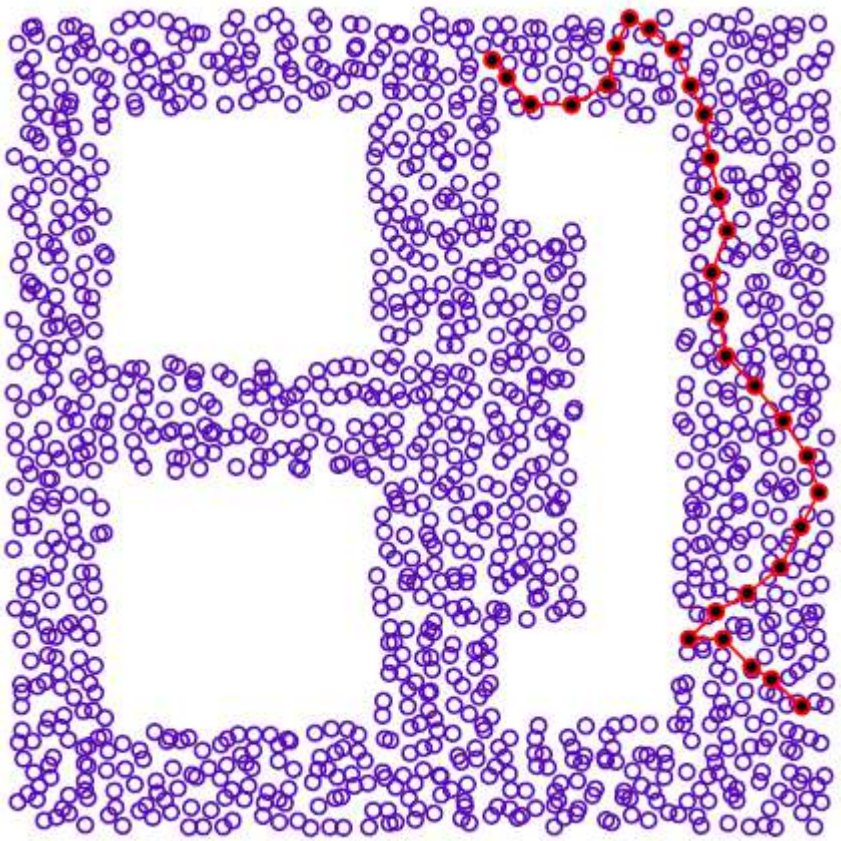
- Theorem:
  - Total area of the holes shrinks exponentially fast.
  - After $O(\log 1/\varepsilon)$ reflections, size of holes ~ ε.

- In practice
  - ≤ 5 levels of reflections
  - Reflections are computed on demand.

Routing in the covering space

- Equivalently, reflect on the hole boundary





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Circular domain embedding for mobile networks?

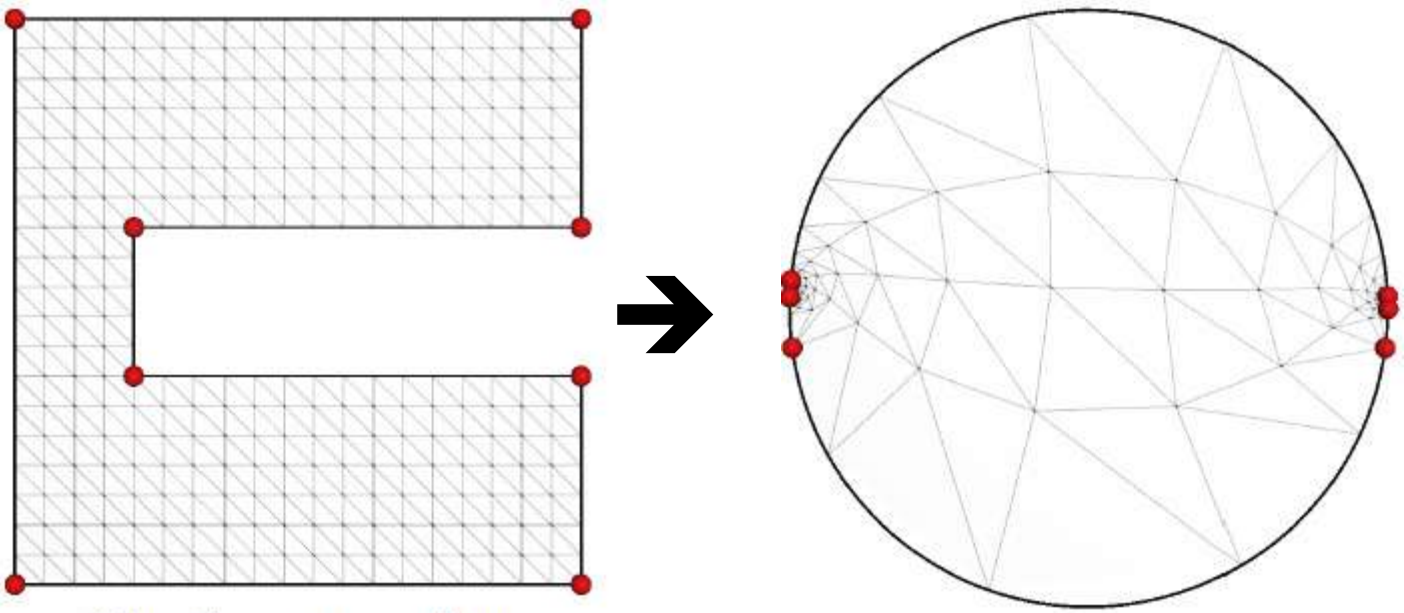
- What if nodes move around:
  - Run Ricci flow again to get the new coordinates
  - Location management scheme to update and maintain the current location information
- Our goal:
  - Avoid running Ricci flow again when nodes move
  - Compute the new virtual coordinates efficiently

Our Solution

- Assuming mobile nodes densely covering the domain
- Compute a compact map for the geometric **domain** in advance
- Preload the map to all nodes.
- Apply greedy routing based on virtual coordinates directly computed from the map

Compute the Compact Map

- Compute the continuous conformal map from the geometric domain to a circular domain
- Encode the map by the Schwarz-Christoffel Transformation



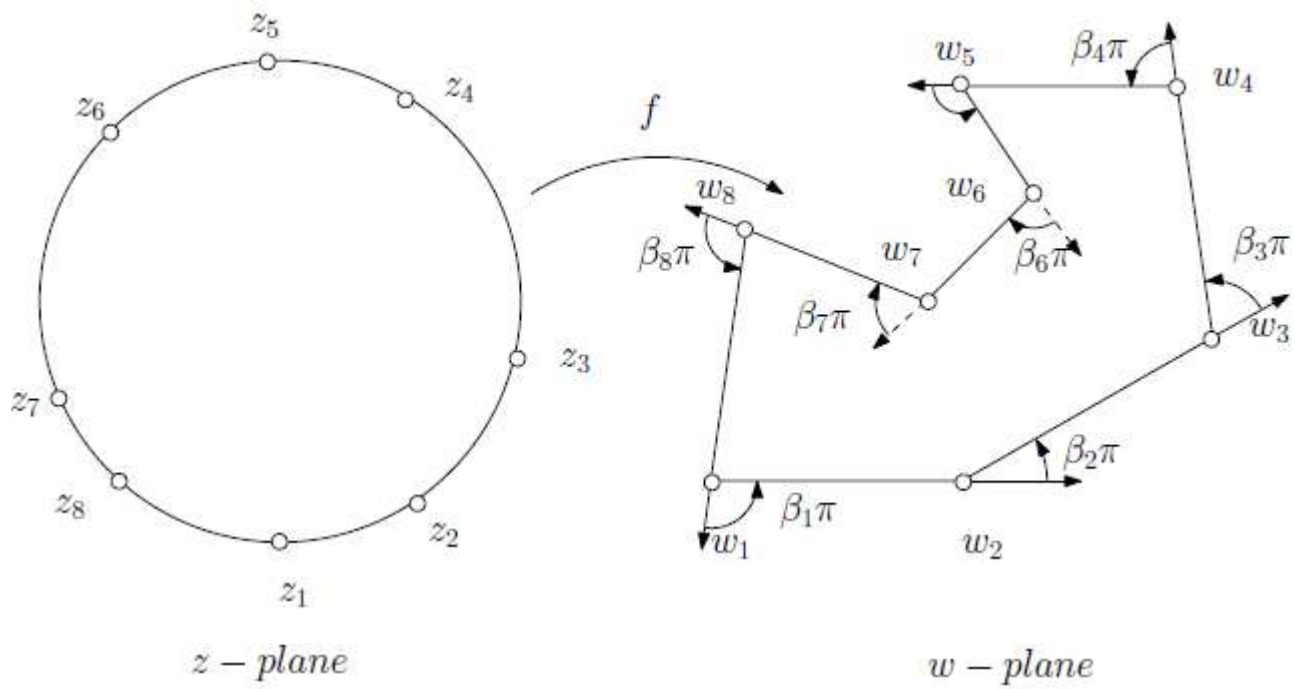
(a) vertices on $w - plane$

(b) prevertices on $z - plane$

Schwarz-Christoffel Transformation

- Describe a conformal map of a disk onto the interior of a simple polygon

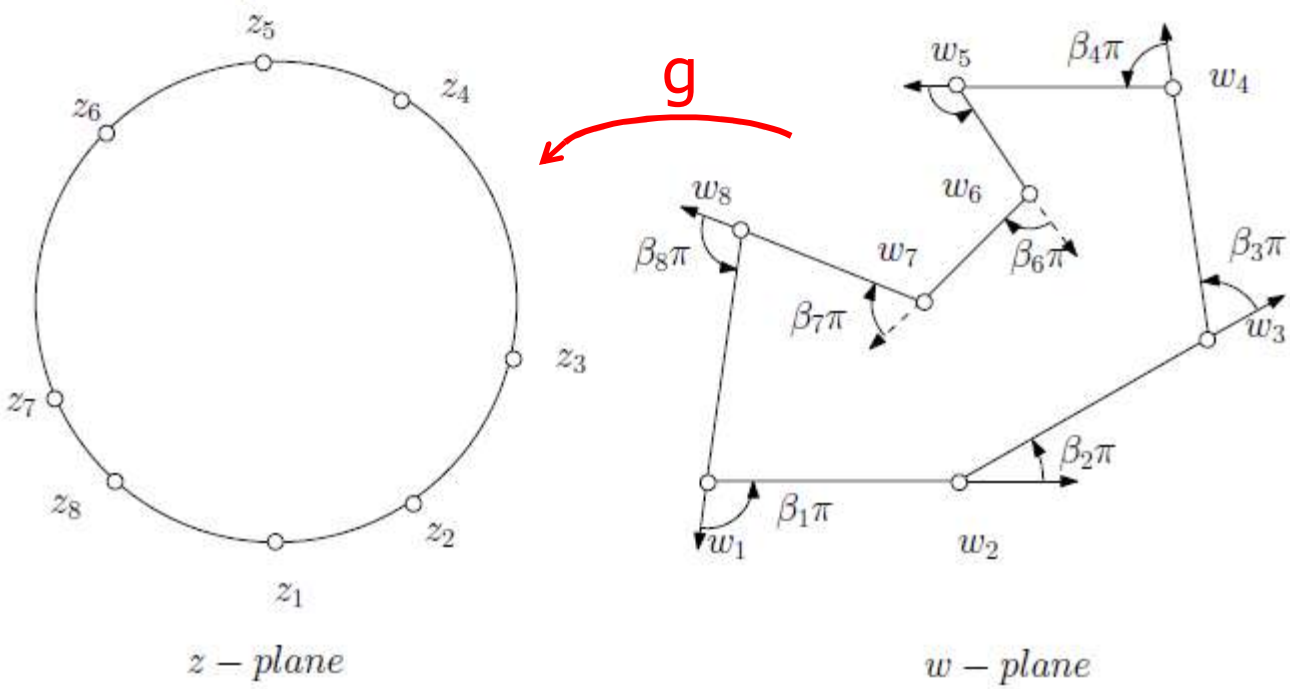
$$f(z) = A\int^{z}\prod_{k=1}^{n}(\zeta - z_k)^{-\beta_k}\,d\zeta + B$$



Schwarz-Christoffel Transformation

- We need the inverse: mapping a simple polygon onto a disk

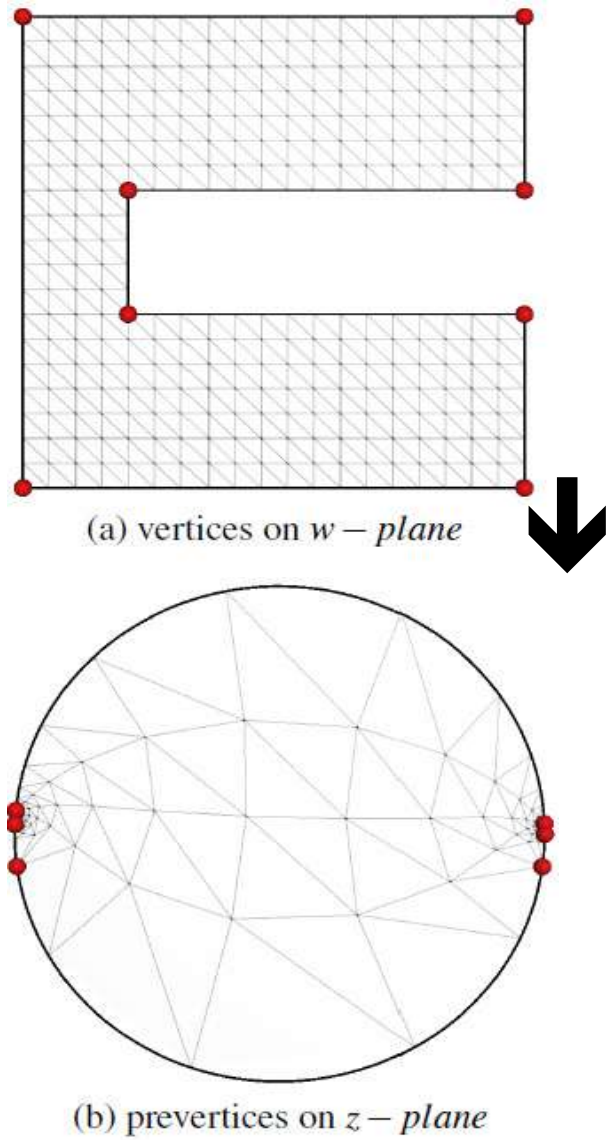
$$g(w) = f^{-1}(w) = \int_{w_0}^{w} \frac{dg(w)}{dw} dw + 0 = \int_{w_0}^{w} \prod_{k=1}^{n} (g(\tau) - z_k)^{\beta_k} d\tau + 0.$$



An Example

w	β	z
1 + 1i	0.5	-0.98993 + 0.14155i
20 + 1i	0.5	-0.99478 + 0.10202i
20 + 8i	0.5	-0.99480 + 0.10184i
5 + 8i	-0.5	-0.99978 - 0.02078i
5 + 13i	-0.5	0.97883 - 0.20466i
20 + 13i	0.5	0.99872 - 0.05063i
20 + 20i	0.5	0.99873 - 0.05041i
1 + 20i	0.5	1.00000 + 0.00000i

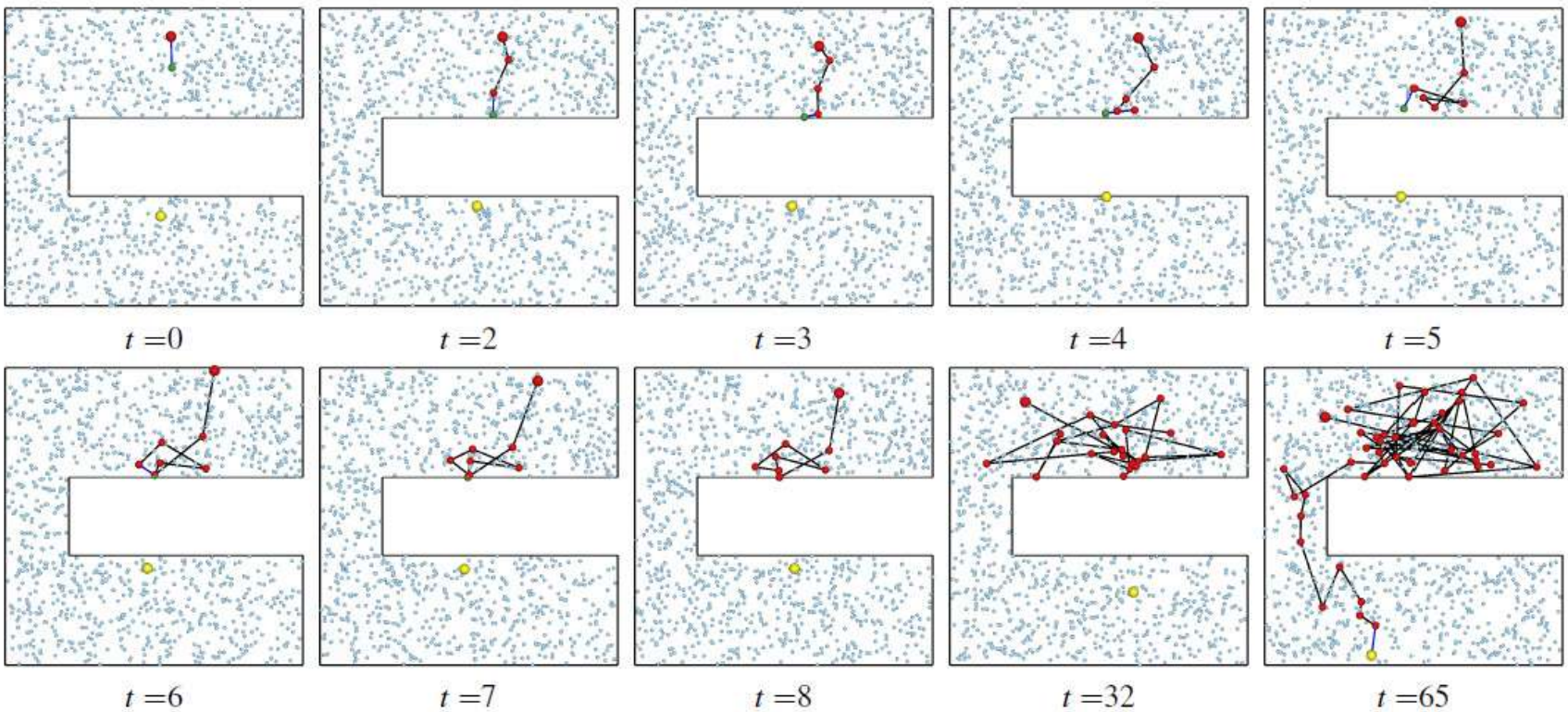
A	-0.16623648 + 2.5343952i
B	3.15 + 10.65i



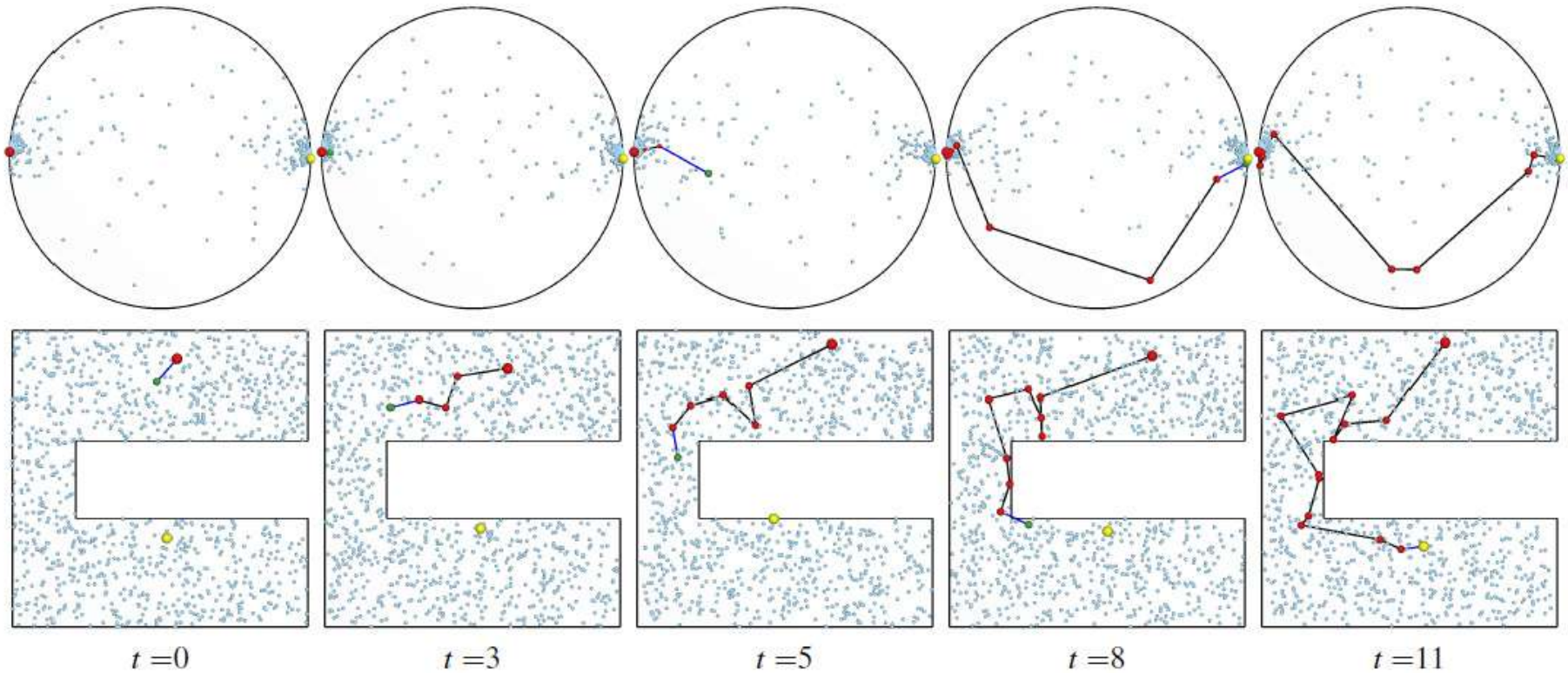
(a) vertices on $w-plane$

(b) prevertices on $z-plane$

Greedy Routing with Geographical Coordinates

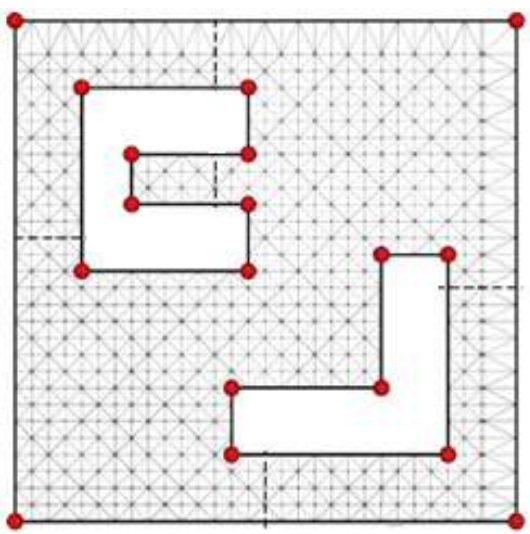


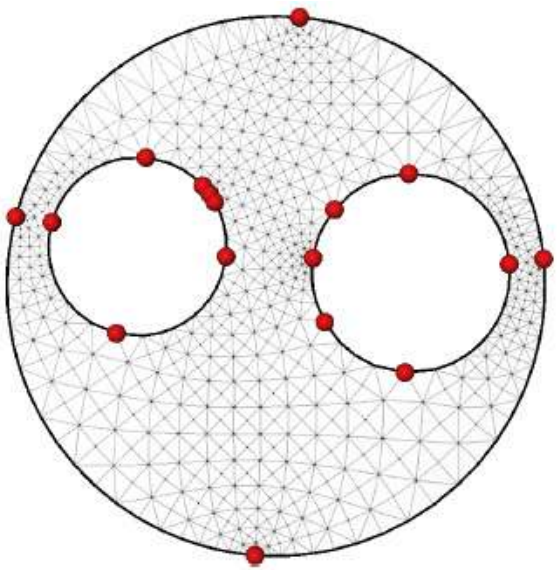
Greedy Routing with Virtual Coordinates



Extensions

- Multiply connected domain
  - Similar ideas
  - Use Laurent series





Summary

- Deformation of network metric by changing local curvatures.
- “Simplify” the network shape to benefit network management.

Acknowledgement

- Joint work with Xianfeng David Gu, Feng Luo, Rik Sarkar, Wei Zeng, Xiaomeng Ban, Ruirui Jiang, Mayank Goswami, Xiaotian Yin, Xiaokang Yu, Siming Li

References

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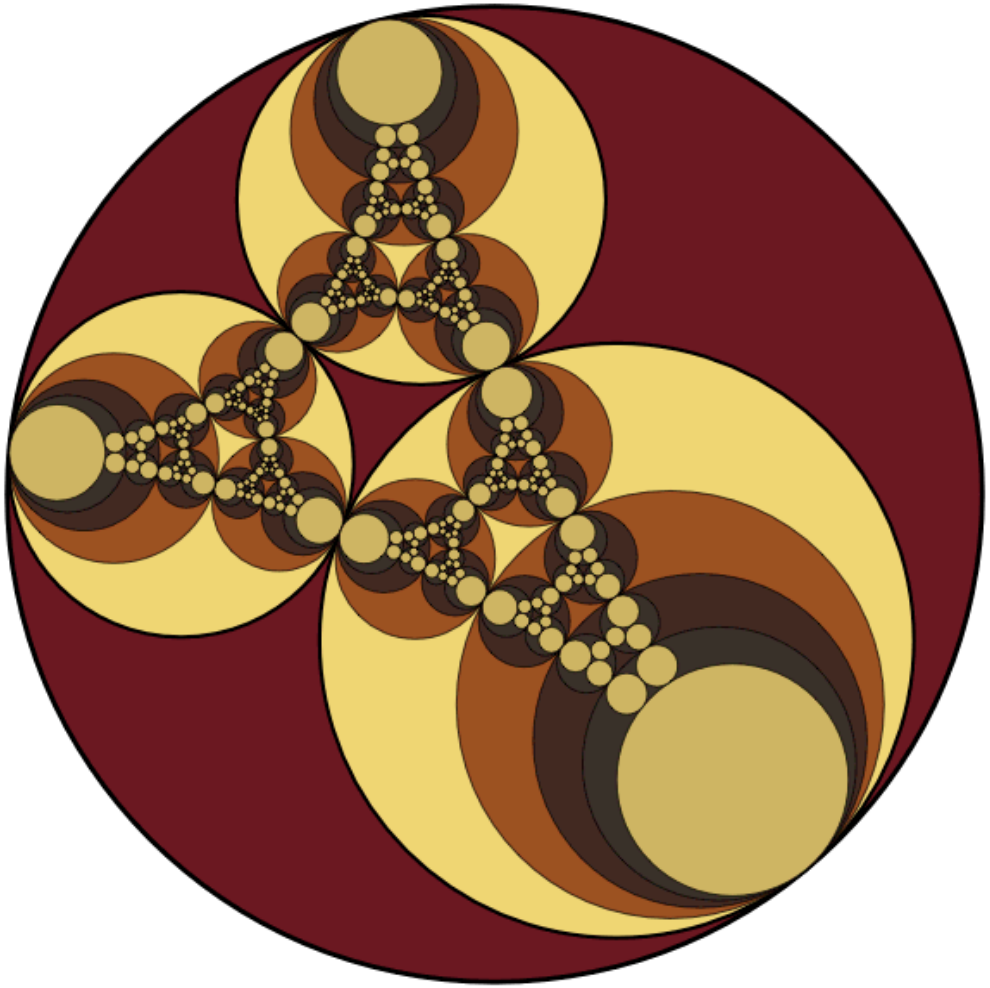
- Resilient Routing for Sensor Networks using Hyperbolic Embedding of Universal Covering Space, INFOCOM’10
- Covering Space for In-Network Sensor Data Storage, IPSN’10.
- Spherical Representation and Polyhedron Routing for Load Balancing in Wireless Sensor Networks, INFOCOM’11.
- Exploration of Path Space using Sensor Network Geometry, IPSN’11.
- Scalable Routing in 3D High Genus Sensor Networks Using Graph Embedding, INFOCOM’12.
- Topology Dependent Space Filling Curves for Sensor Networks and Applications , INFOCOM’13.
- Compact Conformal Map for Greedy Routing in Wireless Mobile Sensor Networks, INFOCOM’13.

Questions?

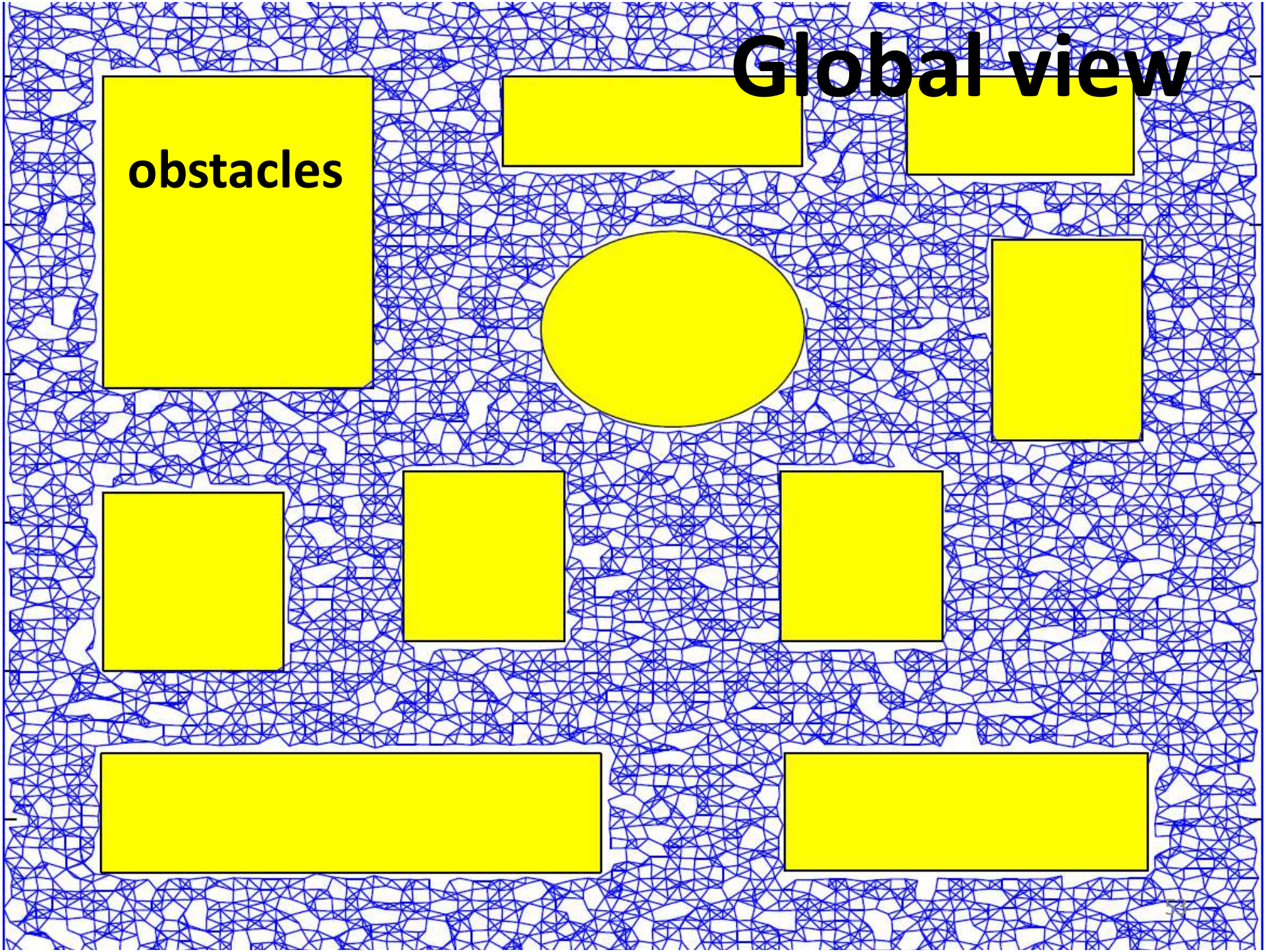


Local view

Graph abstraction of sensor networks

- The “graph” view of a sensor network depends very much on the network connectivity, which is <u>not stable</u>: links come and go, nodes fail…

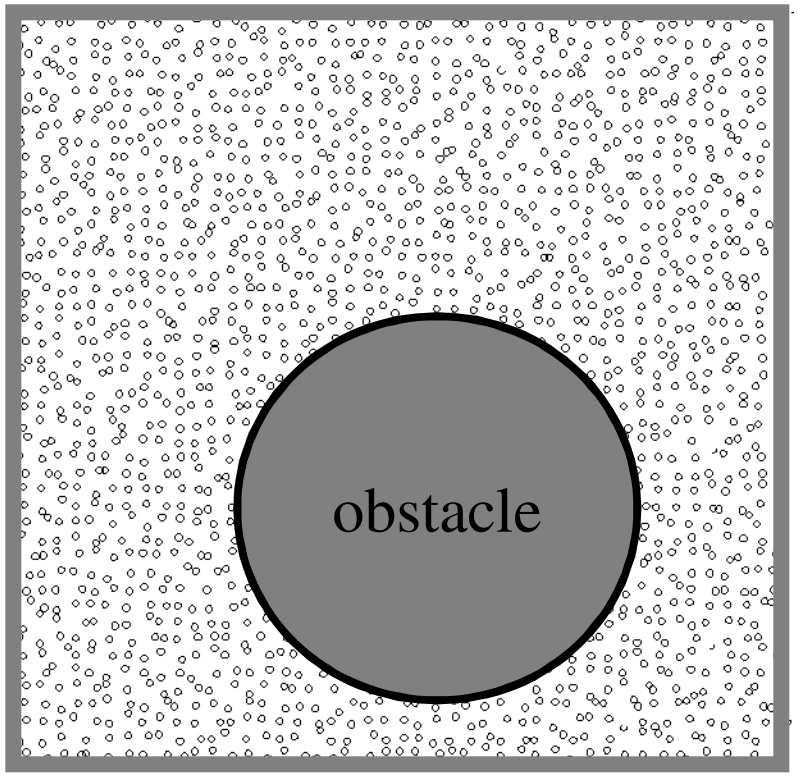
- Dynamic graph problems, in general, are difficult problems.


Global view
obstacles

Geometric View of Sensor Networks

- The **global geometry/ topology** is stable

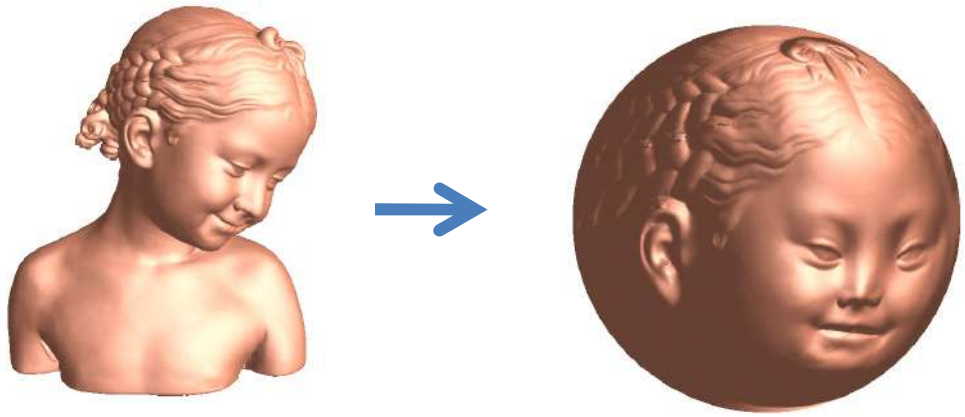
- Computational geometers know how to handle shapes



Greedy embedding

- An embedding in which greedy routing always succeeds. [Papadimitriou, Ratajczak 05]
  - Not all graphs have a greedy embedding.
  - 3-connected planar graph has a greedy embedding [Leighton, Moitra 08][Angelini etal 08]
- Very nice theory
- In practice: links go up and down; maintaining the greedy drawing is non-trivial.

Ricci Flow: diffuse curvature

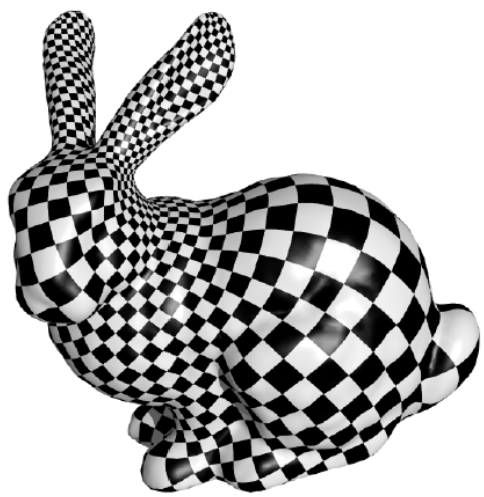


- Riemannian metric g on M
- We modify g by curvature $$\frac{dg_{ij}(t)}{dt} = -2K(t)g_{ij}(t)$$
- Curvature evolves: $$\frac{dK(t)}{dt} = \Delta_{g(t)}K(t)$$
- Same equation as heat diffusion

Δ: Laplace operator

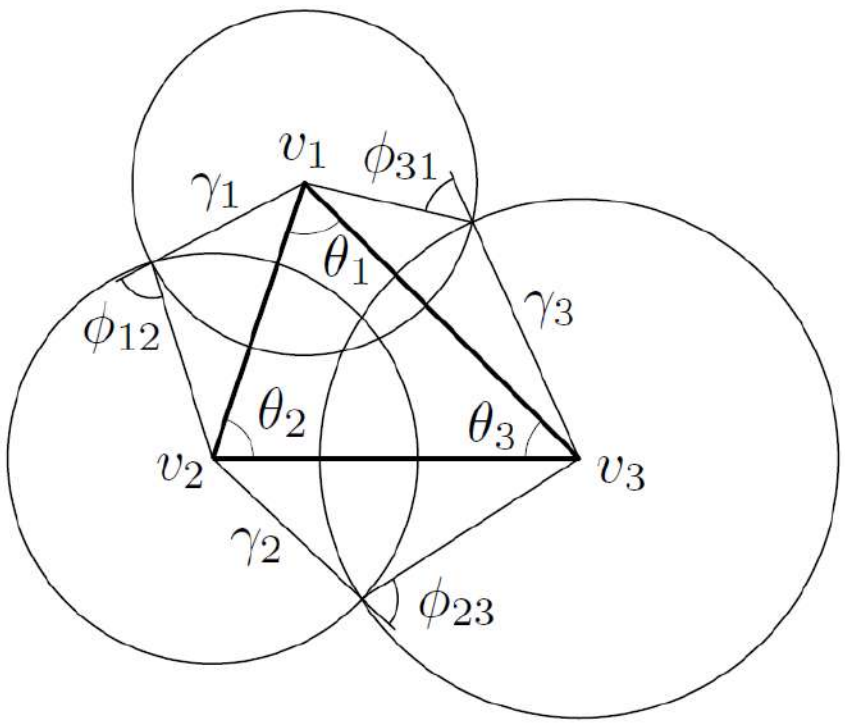
Ricci Flow: diffuse curvature

- Ricci energy is strictly convex ➔ unique solution with constant curvature everywhere and equal surface area.
- Conformal map: angle preserving



Discrete Flow : Circle Packing

- Conformal deformation : Change edge lengths of the triangulation, keep angles same



Circle packing metric: Radius at each vertex + angle at intersection

Related to edge length by cosine law

Remark: no need of an embedding (vertex location)

Background

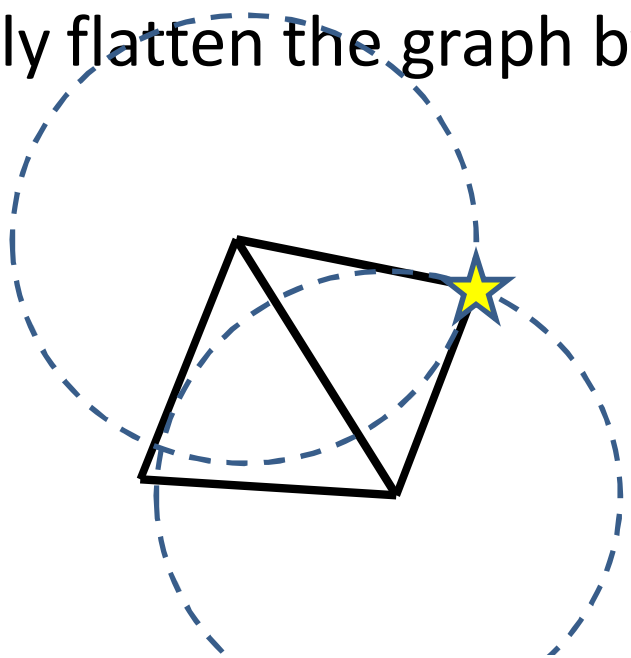
- [Hamilton 82, Chow 91] : Smooth curvature flow flattens the metric
- [Thurston 85, Sullivan and Rodin 87] : Circle packing – discrete conformal maps
- [He and Schramm 93, 96] : Discrete flow with non-uniform triangulations
- [Chow and Luo 03] : Discrete flow, existence of solutions, criteria, fast convergence

Ricci Flow in Sensor Networks

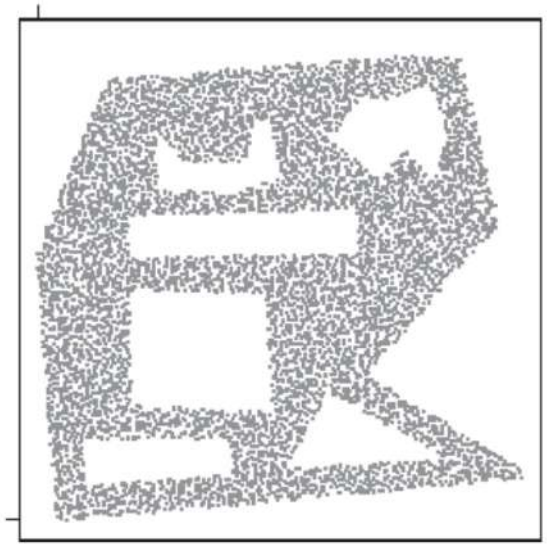
- Build a triangulation from network graph
  - Requirement: triangulation of a 2D manifold
- Compute virtual coordinates
  - Apply distributed Ricci flow algorithm to compute edge lengths d(u, v) with the target curvature (0 at interior vertices and $2\pi/|B|$ at boundary vertices B)
  - Calculate the virtual coordinates with the edge lengths d(u, v)

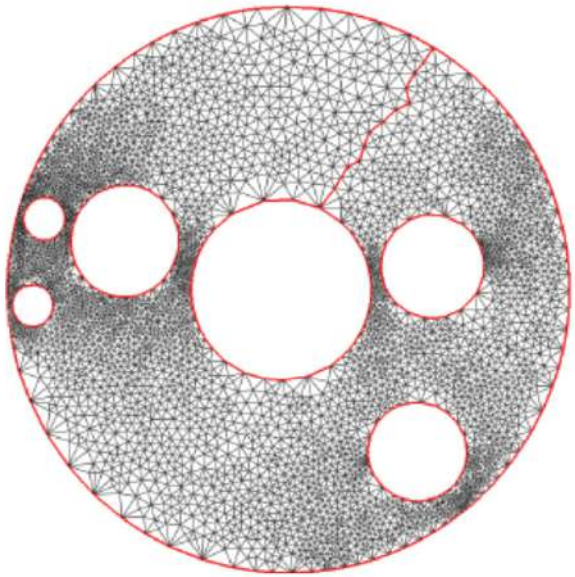
Ricci Flow in Sensor Networks

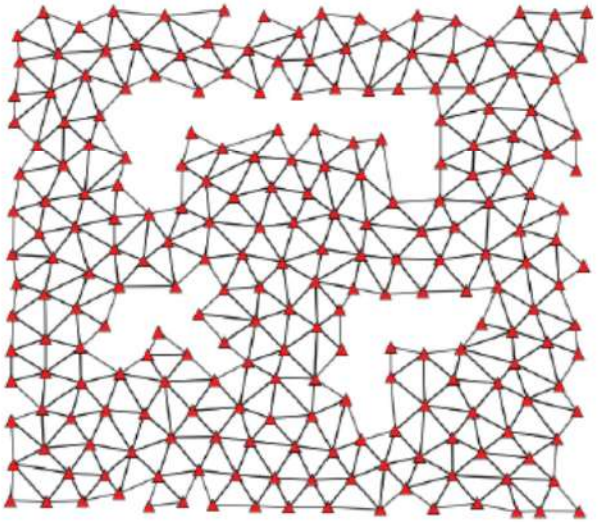
- Ricci flow is a distributed algorithm
  - Each node modifies its own circle radius.
- Compute virtual coordinates
  - Start from an arbitrary triangle
  - Iteratively flatten the graph by triangulation.

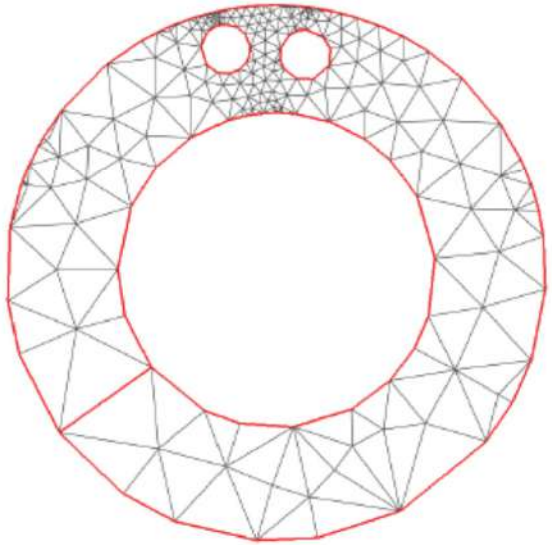


Examples







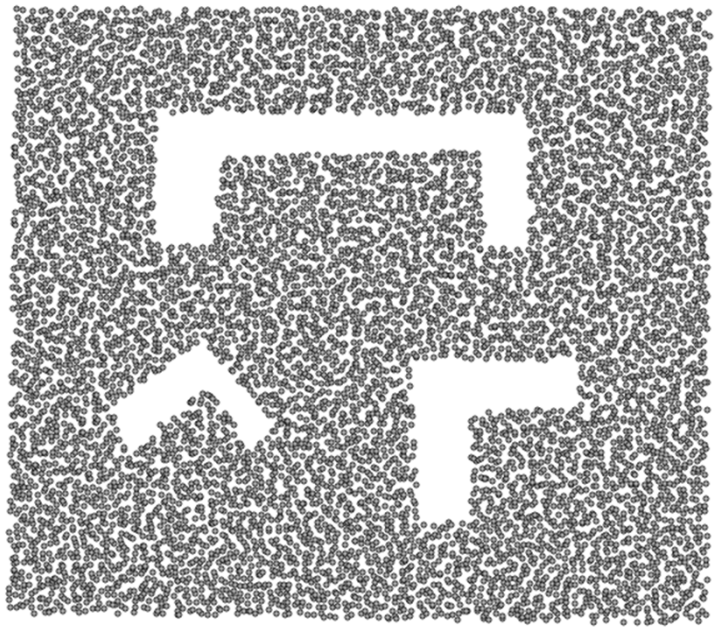


Experiments and Comparison

- NoGeo : Fix locations of boundary, replace edges by tight rubber bands : Produces convex holes

  [Rao, Papadimitriou, Shenker, Stoica – Mobicom 03]

- Does not guarantee delivery: cannot handle concave holes

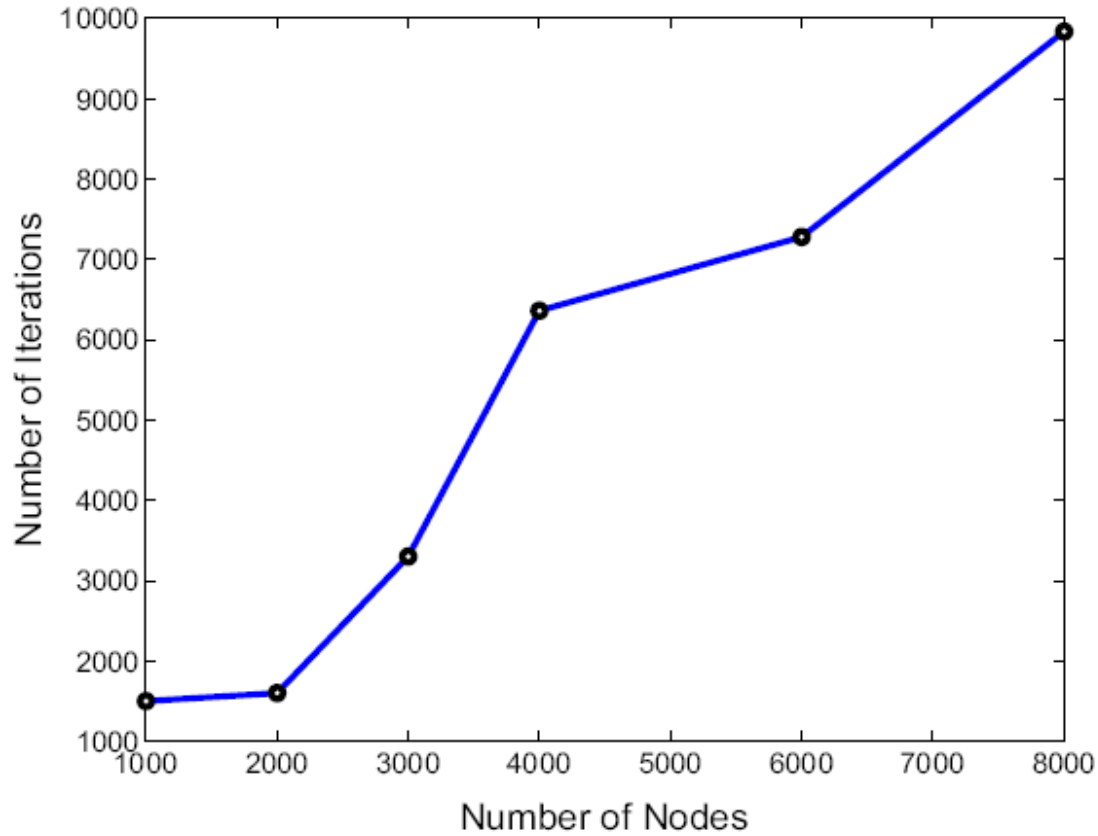


Method	Delivery	Avg Stretch	Max Stretch
Ricci Flow	100%	1.59	3.21
NoGeo	83.66%	1.17	1.54

Convergence rate

- Curvature error bound

- Ricci flow step size



Iterations Vs Number of Nodes

Theoretical guarantee of delivery

Theoretically,

- Ricci flow is a numerical algorithm.
- Triangles can be skinny.

- Route on edges/triangles.

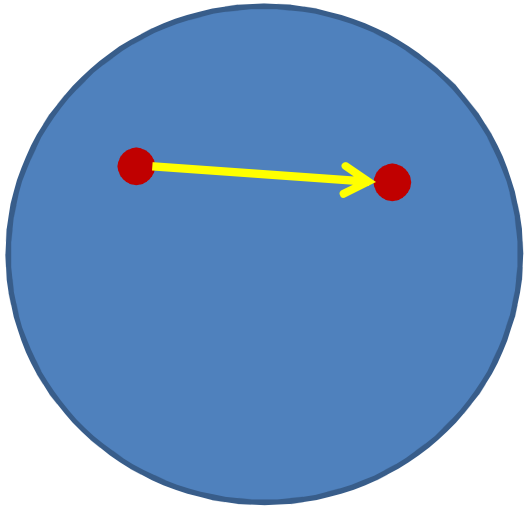
Experimentally,

- Low stretch

What about load balancing?

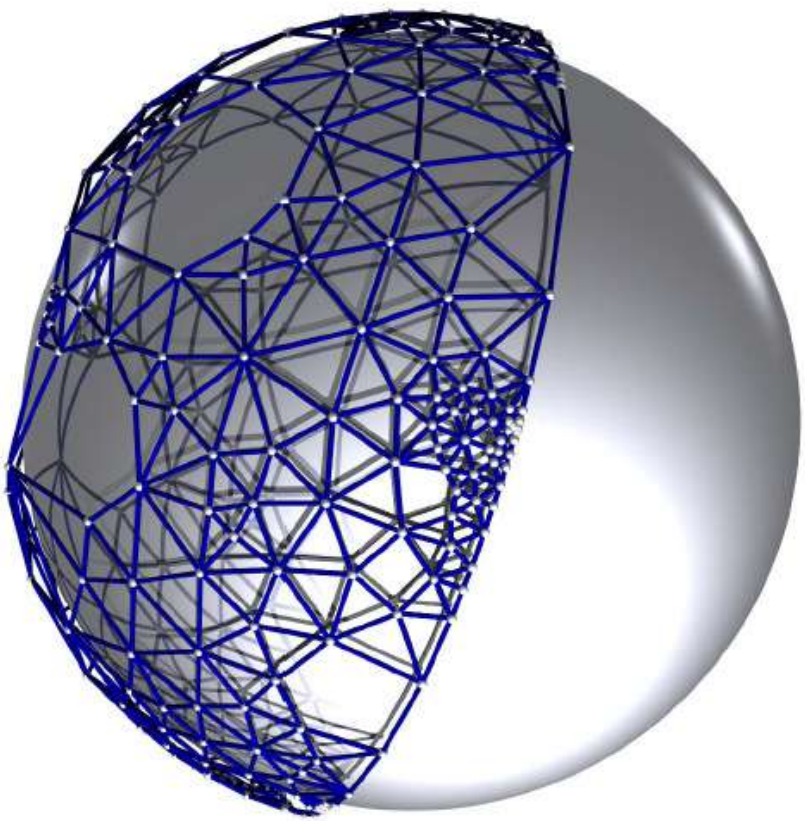
“Center” gets overloaded

- A simple case: disk shape network.
- greedy routing (send to neighbor closer to dest)
  - ≈ Shortest path routing
- Uniform traffic: All pairs of node have 1 message.
- Center load is high!



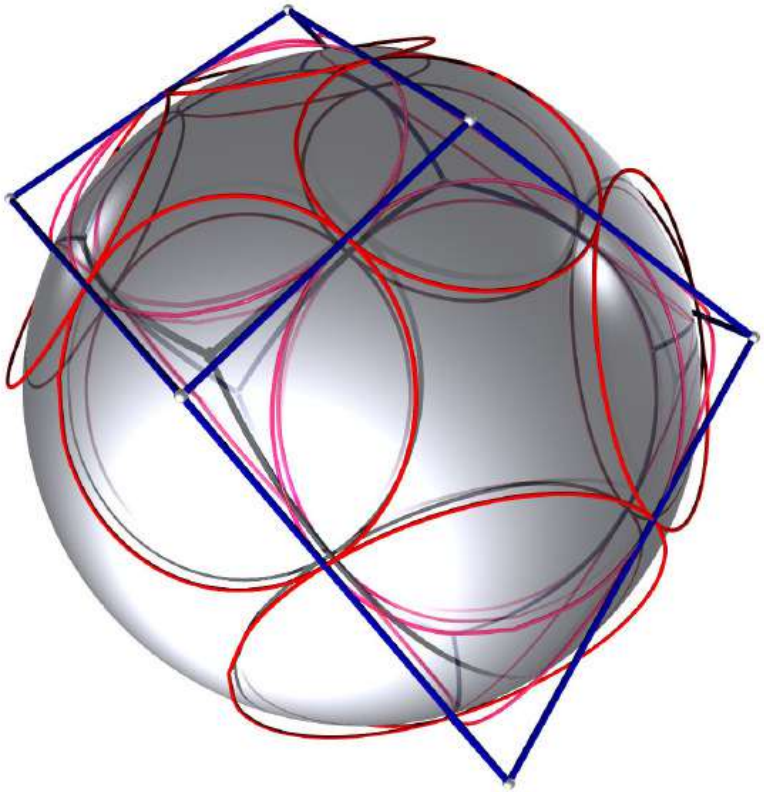
Soln #2: Reduce center load, lift to a sphere

- Greedy routing on a sphere has perfect uniform traffic load



Reduce center load: map to sphere

- **Koebe-Andreev-Thurston Theorem**: Any 3-connected graph can be embedded as a convex polyhedron
  - Circle packing with circles on vertices.
  - all edges are tangent to a unit sphere.
  - vertices are lifted from the sphere.



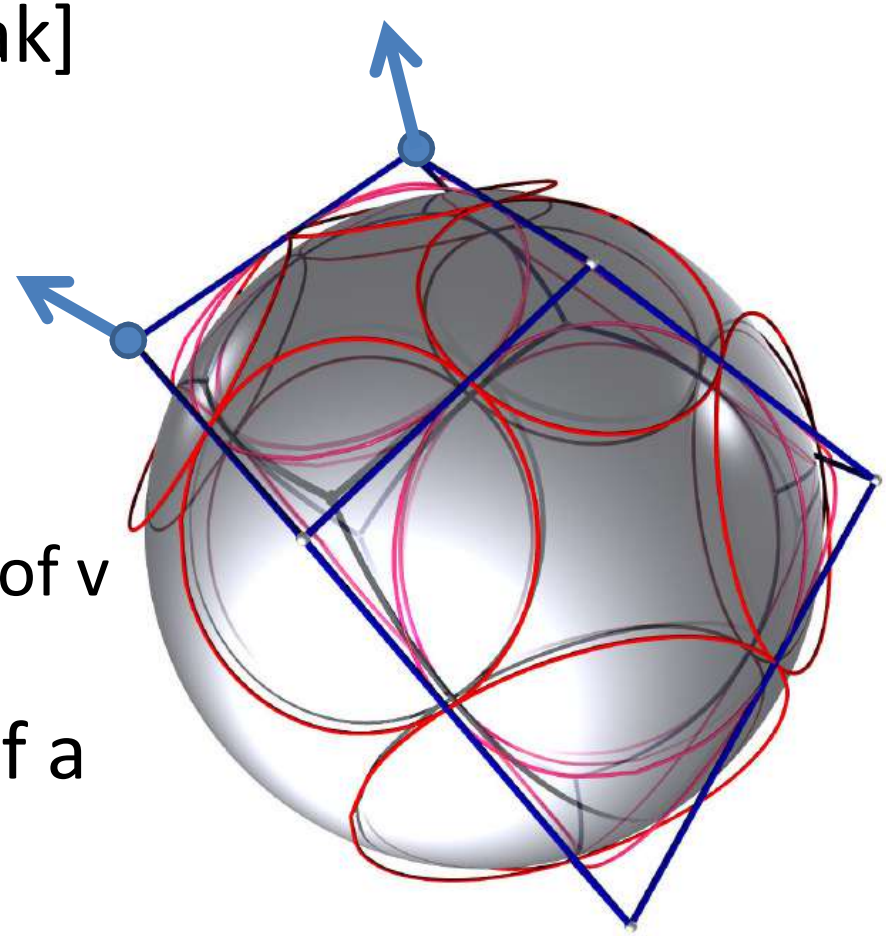
Polyhedron Routing

- [Papadimitriou & Ratajczak] Greedy routing with

  $$\mathbf{d(u, v) = -c(u) \cdot c(v)}$$

  guarantees delivery.

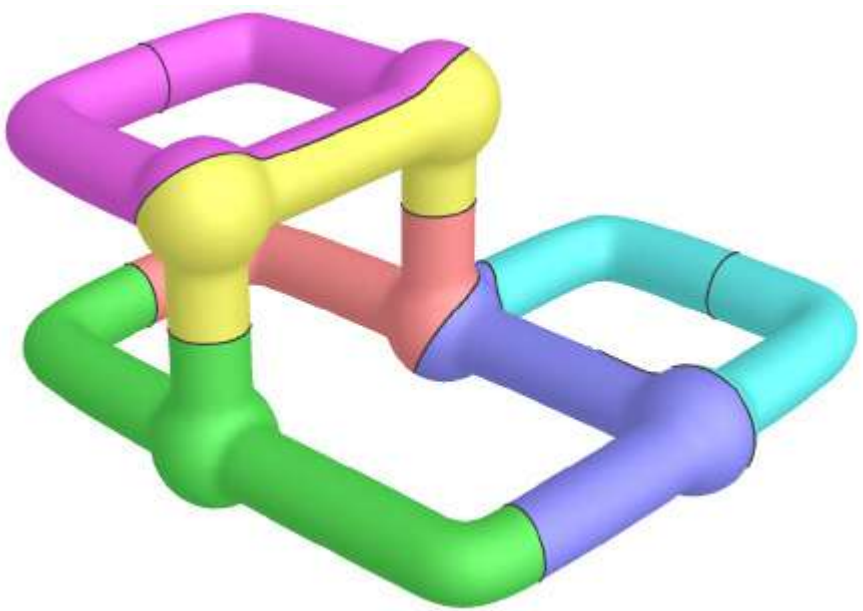
3D coordinates of v

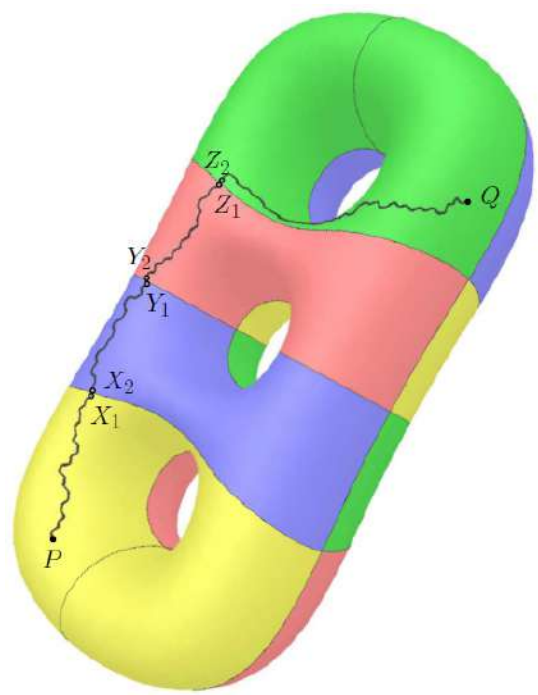


- Route along the surface of a convex polytope.

More recent work

- Extension to 3D sensor networks. [INFOCOM'12]
  - Sensors monitoring complex tunnels.
  - Embed & route on a high-genus surface





More recent work

- Space-filling curves with progressive density for non-simple domains. [Infocom'13]
  - App: data mule planning