Network Dynamics and Polarization

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Modeling Social Influence

How does an agent get influenced by its neighbors?

- Diseases
- Information, ideas, innovation
- Social behaviors (e.g., smoking, obesity, voting)
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Two main factors

- **Contagion models**
- Network structures

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- Does the network converge?
- If so, what is the converged state?

Opinion Dynamics: French-DeGroot Model [1956]

Each agent i has a real-valued opinion variable $v_i \in [-1, 1]$

- -1: against; 1: supportive.
- At time 0 nodes have initial opinions.
- Edge (i, j) carries a weight $w_{ij} \geq 0$.
- At time t, every agent updates its opinion by (weighted) influence from neighbors.

$$v_i(t+1) = \sum_j w_{ij} v_j(t).$$

Research questions:

- Does the network converge? Yes.
- If so, what is the converged state? Global consensus.

Opinion Dynamics: Friedkin-Johnsen Model [1990]

Each agent i has an opinion variable $v_i \in [-1, 1]$

- -1: against; 1: supportive.
- At time 0 nodes have initial opinions u_i.
- Edge (i, j) carries a weight $w_{ij} \geq 0$.
- At time t, every agent updates its opinion by (weighted) influence from neighbors.

$$v_i(t+1) = (1 - \lambda_i) \sum_j w_{ij} v_j(t) + \lambda_i u_i.$$

Research questions:

- Does the network converge? Yes under favorable conditions on W.
- If so, what is the converged state? no consensus.

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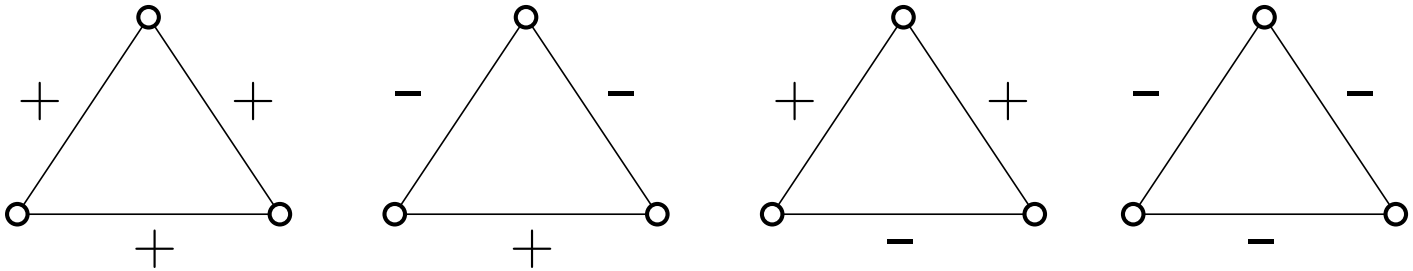
What if we have negative weights?

Structure Balance in Signed Networks [Heider46]

Poitive ties: friendship; negative ties: hostility.



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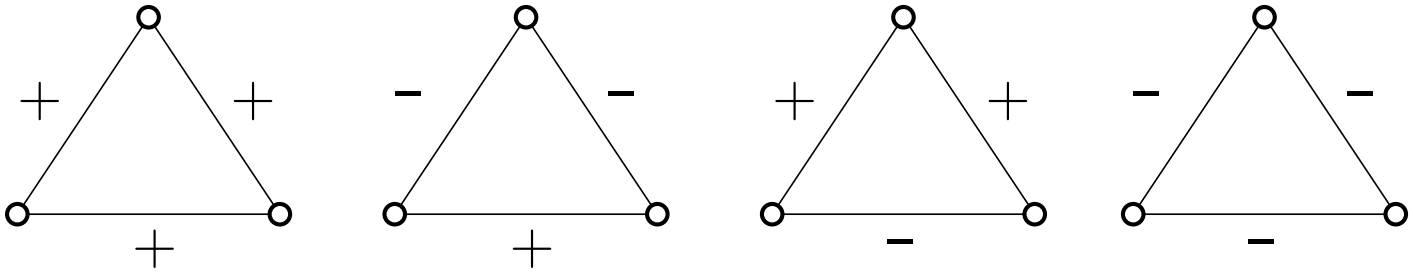

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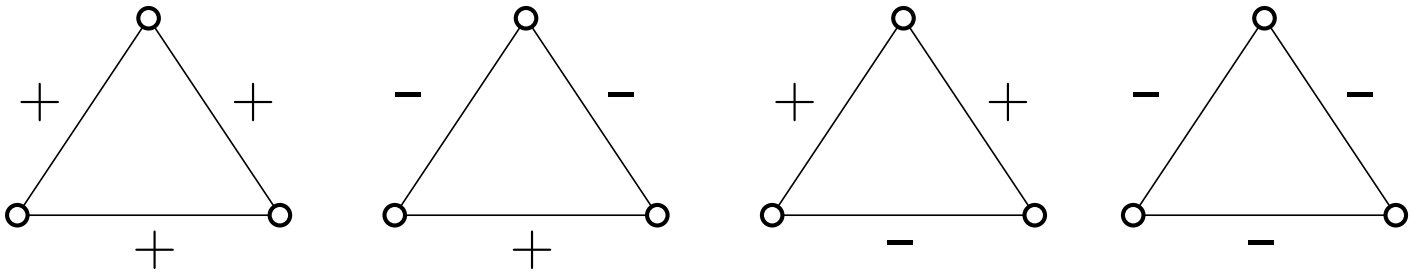

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Model on network dynamics when a graph is not balanced?

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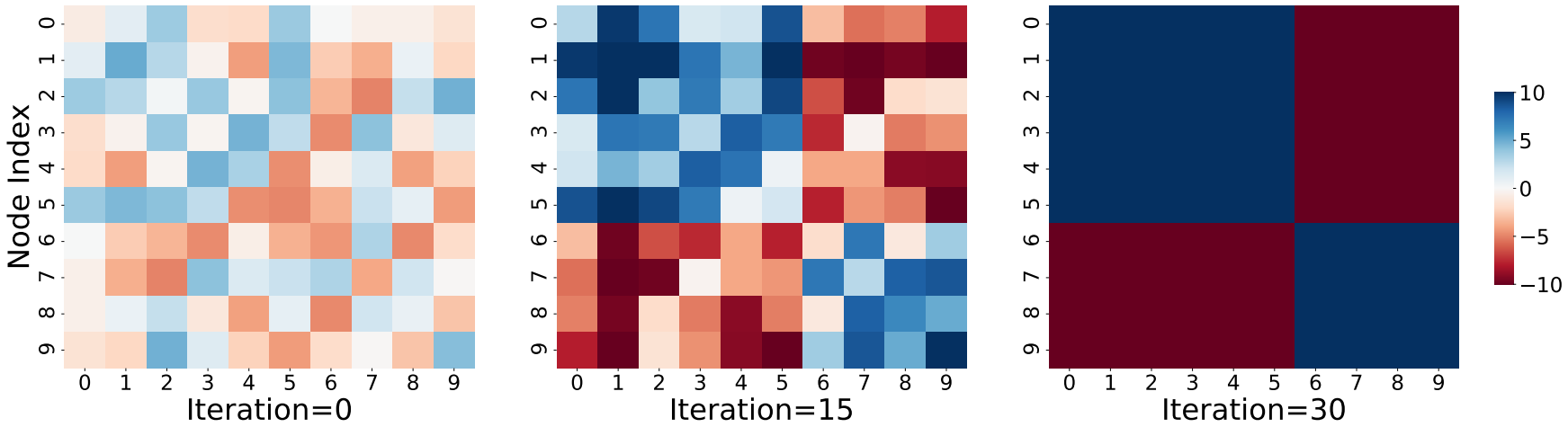
Main result:

- The network converges to structural balance, unless $|V(t)| \to 0$.
- Community membership can be solved from the initial states.

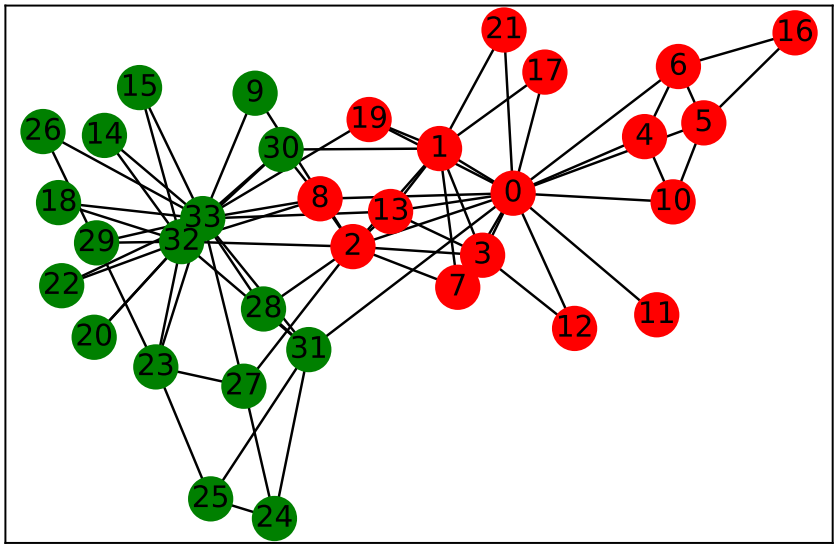
Co-Evolution Dynamics

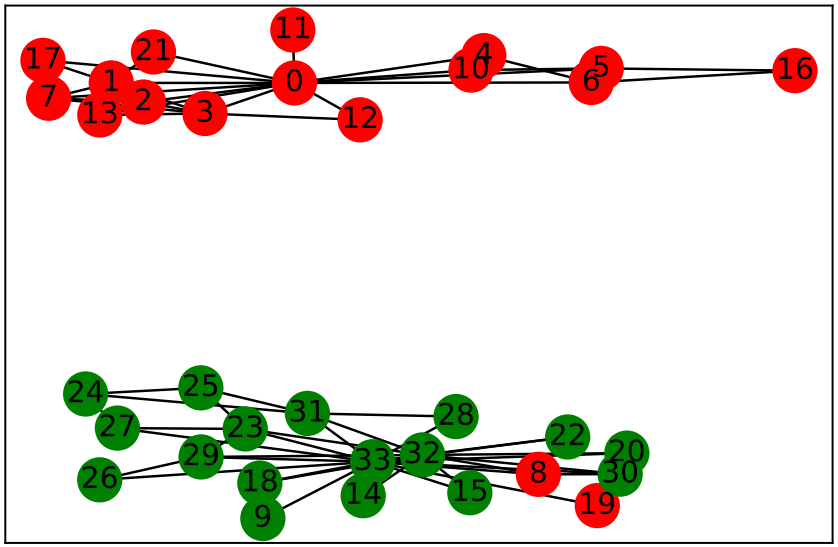
Random initial weights.



Co-Evolution Dynamics: Karate Club

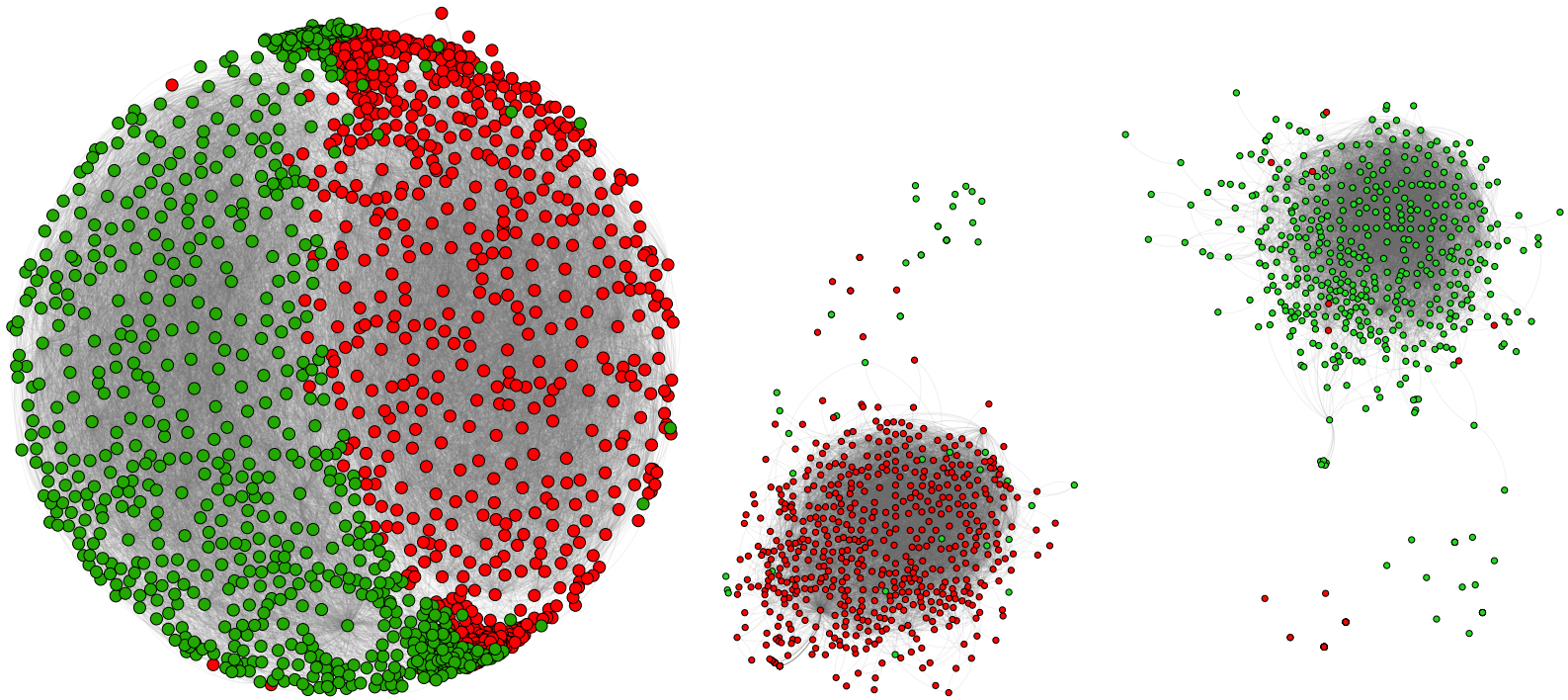
- Only two nodes: #10 and #33 have initial (opposite) opinions.
- All edges start with small positive weights.





Co-Evolution Dynamics: Political Blog Network

- 20% nodes carry ground truth opinions.
- All edges start with small positive weights.



Avg 97.21% prediction accuracy.

Summary and Acknowledgement

- Modeling: social media platforms.
- Algorithmic perspective: promote truth learning, reduce polarization

Acknowledgement

- Joint work with Professor Feng Luo (Math) and Dr. Haotian Wang
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