Enabling Asymptotic Truth Learning in a Social Network

Jie Gao

Rutgers University
http://sites.rutgers.edu/jie-gao

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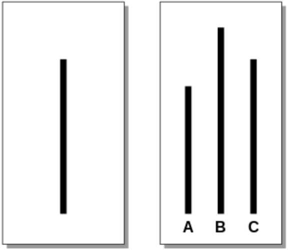
Truth Learning in a Social Setting

Asch conformity experiments [1950s]: One subject in a room with 6 actors



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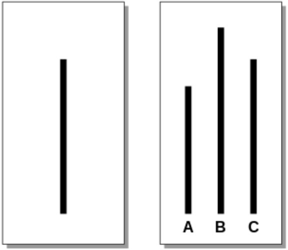

- Experiment group: 37% responses conform to the incorrect answer. 75% participants gave at least one incorrect answer out of 12 trials.

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- People line up: **in private** draw a ball, look at its color, put it back
- Make a **public** prediction whether the box is red majority or blue majority

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Third person:

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Fourth person: in the same position as the third one.

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Fourth person: in the same position as the third one.
Everyone else reports BLUE.

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- Black box: either $\color{blue}\bullet$ $\color{blue}\bullet$ $\color{red}\bullet$ balls, or $\color{blue}\bullet$ $\color{red}\bullet$ $\color{red}\bullet$ balls.
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First person: draw $\color{blue}\bullet$, report BLUE
Second person: draw $\color{blue}\bullet$, report BLUE
Third person:

- If draw $\color{blue}\bullet$, report BLUE
- If draw $\color{red}\bullet$, report BLUE

Fourth person: in the same position as the third one.
Everyone else reports BLUE.
Ground truth $\color{blue}\bullet$ $\color{red}\bullet$ $\color{red}\bullet$: w/ prob $\geq (1/3)^2$, everyone failed.

Information Cascade or Herding

Sequential Learning [Banerjee'92, BHW'92, Welch'92]

- Unknown ground-truth signal $\theta \in \{0, 1\}$.
- Rational agents take a sequential ordering σ
- Agent v: **noisy private** signal s_v, with correct prob $1/2 < q < 1$.
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Q: can we avoid herding and achieve truth learning?

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Goal: network-wide asymptotic truth learning.

$$\frac{1}{n} \cdot \sum_{v} \text{Prob}\{a_v = \theta\} \to 1$$

- Agents stay on a social network G.
- We decide an ordering of agents taking actions.

Quick Observations

Empty graph $+$ any ordering:

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- Each node make decision with only private signals.
- Agent success probability $= q$.

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What type of graph topology G + node ordering σ enables truth learning?

Our Results

Sparse graphs: average $O(1)$ degree

- Random ordering: $\times$
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A sufficient condition for truth learning

- Erdös Rényi graph
- Preferential attachment model

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Theorem

Any family of graphs of constant average degree $\Delta = O(1)$ *does not achieve asymptotic truth learning, under both Bayesian model and majority vote model.*

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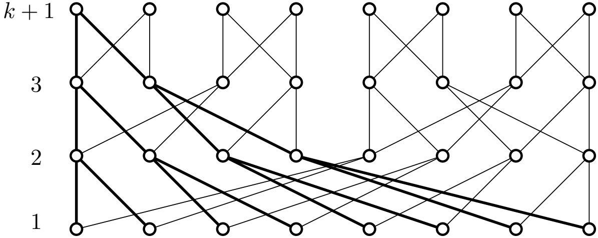
Theorem

Any family of graphs of constant average degree $\Delta = O(1)$ does not achieve asymptotic truth learning, under both Bayesian model and majority vote model.

In a random ordering, a constant fraction of nodes are independent and thus make decision with own signal.
Network-wide asymptotic learning cannot happen.

Butterfly network

2^k nodes per layer with k layers, $k = \log n$.



Truth learning is enabled with bottom up ordering

A Good Ordering

Truth learning occurs with ordering σ if

- Find a node v with a subset of neighbors $S \subseteq N(v)$ that are independent. $|S| = \omega(1)$.

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- S goes first, each making independent decisions.
- v goes next, aggregating decisions from S, achieving high probability of success.
- Complete the ordering to $n - o(n)$ nodes by finding a path where each node has at least one high quality neighbor.

Characterizing Truth Learning in an Erdös Rényi graph

$G(n, p)$: n nodes and each edge appears w/ prob $p \in [0, 1]$.

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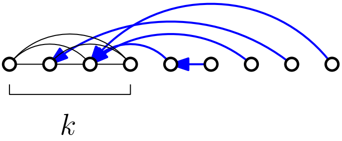
Characterizing Truth Learning in a PA graph

A preferential attachment (PA) graph, with positive integer $k = O(1)$:



- Start with a complete graph of $k + 1$ vertices.

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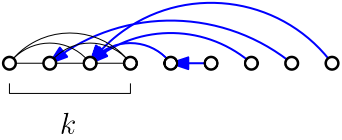

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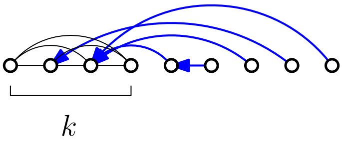

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- Models a time-evolving network with power law degree distribution.

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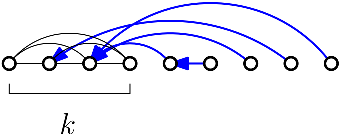

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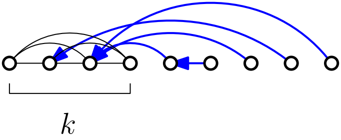

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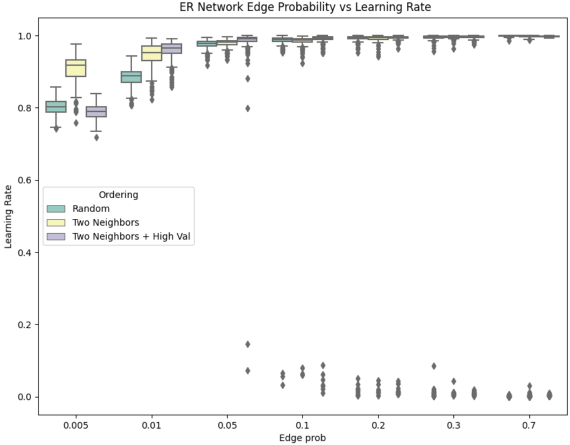
- ✓ with a good ordering.

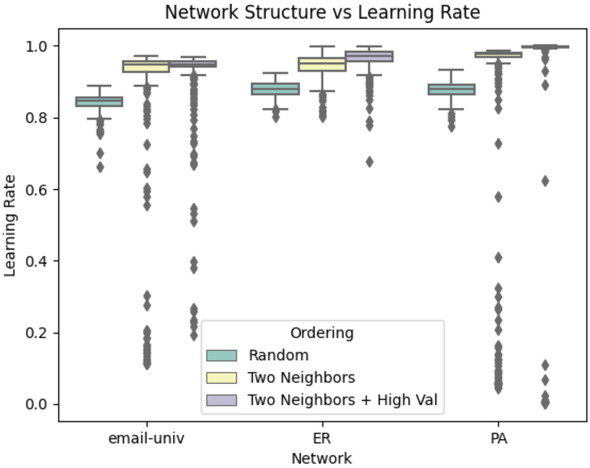
Simulations: Erdös Rényi Graph

Majority vote model. $q = 0.7$. $n = 1000$. 300 iterations.



Simulations: Real World Graphs

Majority vote model. $q = 0.7$. $n = 1133$. 300 iterations.



Summary

Interdisciplinary topic that is still largely under developed.

- Modeling: social media platforms.
- Algorithmic perspective: promote truth learning.

Acknowledgement

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