Exercise Manual for Math 135
Spring 2024 Edition

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Contents

1 Chapter 1: Algebra and Precalculus Review

§1.1, 1.2, 1.3, 1.4, 7.2, Appendix B

Ex. A-1 Algebra/Precalculus Fa17 Exam

Find all solutions to the following equation.

$$2\ln(x) = \ln\left(\frac{x^5}{5-x}\right) - \ln\left(\frac{x^3}{2+x}\right)$$

exf17

Ex. A-2 Algebra/Precalculus Sp19 Exam

The number N of bacteria at time t grows exponentially, so that $N(t) = N_0 e^{kt}$ for some constants N_0 and k. Suppose an initial population of 100 bacteria grows to 500 after 2 hours. How many hours does it take for an initial population of 150 bacteria to grow to 300?

exs19

Ex. A-3 Algebra/Precalculus Fa19 Exam

Solve the inequality $\dfrac{3x-6}{x+4} > 0$. Write your answer using interval notation.

exf19

Ex. A-4 Algebra/Precalculus Sp20 Exam

Solve the inequality $\dfrac{3x+6}{x-4} < 0$. *Write your answer using interval notation.*

exs20

Ex. A-5 Algebra/Precalculus Sp20 Exam

Find the domain of the function $f(x) = \dfrac{\ln(80-x)}{\sqrt{x}-5}$. *Write your answer using interval notation.*

exs20

Ex. A-6 Algebra/Precalculus Su20 Exam

Let $f(x) = 8 - \dfrac{1}{5x}$. Fully simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ with $h \neq 0$. In your work, make clear where you use the assumption $h \neq 0$.

exsu20

Ex. A-7 Algebra/Precalculus Su20 Exam

For both parts of this problem, consider the following inequality.

$$\frac{(x-3)(x-6)}{x-5} < 0$$

Your goal is to identify an error in a false solution of this inequality, and then to solve the inequality yourself.

(a) A student submits the following work for solving this equality.

> "First we multiply both sides by $(x-5)$. On the left side, this factor cancels, and on the right side we get 0. So we have $(x-3)(x-6) < 0$. The graph of $y = (x-3)(x-6)$ is a parabola that opens upward and crosses the x-axis at $x = 3$ and $x = 6$. This means that the graph is below the x-axis between these two x-values. So the solution to $(x-3)(x-6) < 0$ is the interval $(3, 6)$. But since the original inequality was undefined at $x = 5$, we also have to exclude 5. So the final answer is $(3, 5) \cup (5, 6)$."

The student's teacher does not give full credit for this solution, simply noting that $x = 4$ is included in the student's answer, but $x = 4$ does not satisfy the original inequality. So the final answer must be wrong.

What is the student's error? Be as specific as possible and explain why this is an error. ***To explain why the given solution is wrong, it is not enough to simply write the correct solution and observe that the two solutions are different.***

(b) Solve the original inequality. Write your answer using interval notation.

exsu20

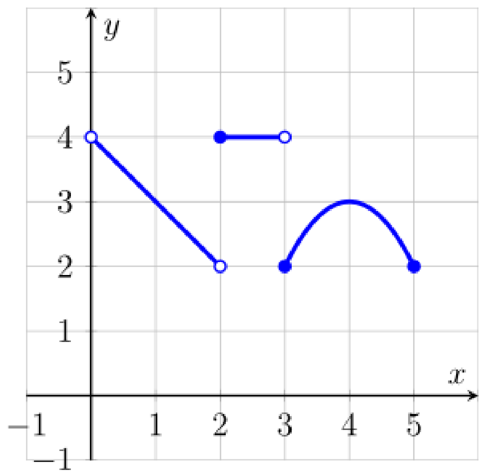
Ex. A-8 Algebra/Precalculus Fa20 Exam

Fully simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for $f(x)=\sqrt{x+2}$ and $h \neq 0$. Write your answer without square roots or fractional exponents in the numerator.

exf20

Ex. A-9 Algebra/Precalculus Fa20 Exam

For each part, use the graph of $y = g(x)$ below.



(a) Find the domain of $g(x)$. Write your answer in interval notation.
(b) Calculate $g(g(4))$.
(c) As $x \to 2$, which of the left-sided and right-sided limits of $g(x)$ exist?

exf20

Ex. A-10 Algebra/Precalculus Fa20 Exam

While solving the logarithmic equation

$$\log_2(3x+1) = 3$$

a student wrote the following steps (this work contains two distinct errors):

$$\log_2(3x) + \log_2(1) = 3 \quad (1)$$
$$\log_2(3x) + 0 = 3 \quad (2)$$
$$3x = 3^2 \quad (3)$$
$$x = 3 \quad (4)$$

(a) Identify the lines in which the two errors occur and describe each error.
(b) What is the correct solution to the original equation?

exf20

Ex. A-11 Algebra/Precalculus Fa20 Exam

Fully simplify the difference quotient $\dfrac{f(3+h)-f(3)}{h}$ for $f(x) = \dfrac{6}{9-2x}$ and $h \neq 0$. Your answer cannot contain a complex fraction (fraction within a fraction).

exf20

Ex. A-12 Algebra/Precalculus Fa20 Exam

Suppose we have all of the following:

$$\log_3(x) = A \quad , \quad \log_3(y) = B \quad , \quad \log_{b^5}(z) = C$$

Write each of the following in terms of A, B, and C. Your final answer cannot contain any "log" symbol.

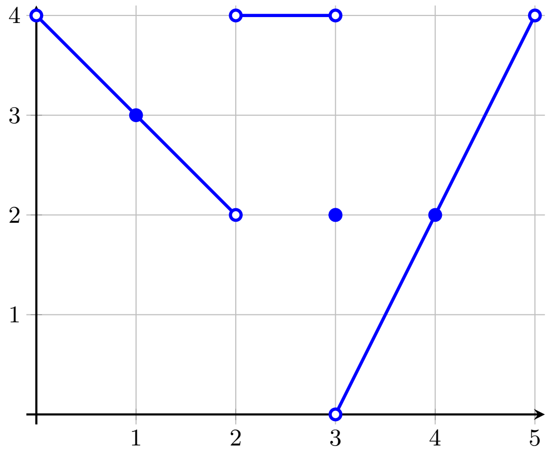
(a) $\log_3\left(\dfrac{\sqrt{x}}{9y^4}\right)$

(b) $\log_b(z)$

exf20

Ex. A-13 **Algebra/Precalculus** Fa20 **Exam**

For each part, use the graph of $y = g(x)$ below.



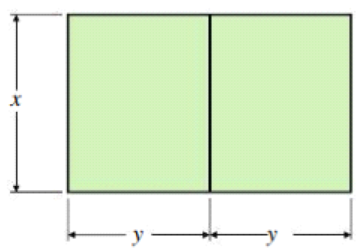
(a) Calculate $g(g(1.5))$.

(b) Find the range of $g(x)$. Write your answer in interval notation.

exf20

Ex. A-14 **Algebra/Precalculus** Sp21 **Exam**

Suppose you have exactly 840 ft of fencing that will be used to build an enclosure that consists of two identical rectangular pens that share a common fence. Let x be the (vertical) length of each pen and let y be the (horizontal) width of each pen. See the figure below.



(a) Find an expression for $F(x)$, the area of one individual pen, as a function of x.

(b) Now suppose that, for each of the two pens, the sum of the length and width must not exceed 250 ft. In the context of this problem, what is the domain of F? Write your answer in interval notation.

exs21

Ex. A-15 **Algebra/Precalculus** Sp21 **Exam**

Suppose $\log_3(x) = A$ and $\log_3(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_3\left(\frac{27\sqrt{x}}{y^4}\right)$$

exs21

Ex. A-16 **Algebra/Precalculus** Fa21 **Exam**

The graph of $y = f(x)$ is given below.

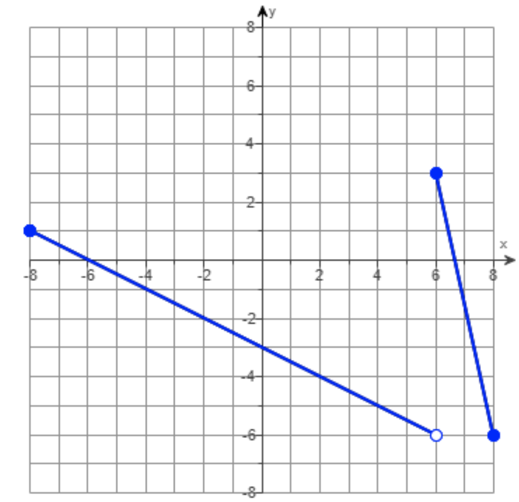
exf21

Algebra/Precalculus Fa21 **Exam**

Note that f is piecewise linear. An explicit formula for $f(x)$ can be written in the following form, where A and B are constants.

$$f(x) = \begin{cases} y_1(x) & \text{if } -8 \le x < A \\ y_2(x) & \text{if } B \le x \le 8 \end{cases}$$

Calculate each of A, B, $y_1(x)$, and $y_2(x)$.



exf21

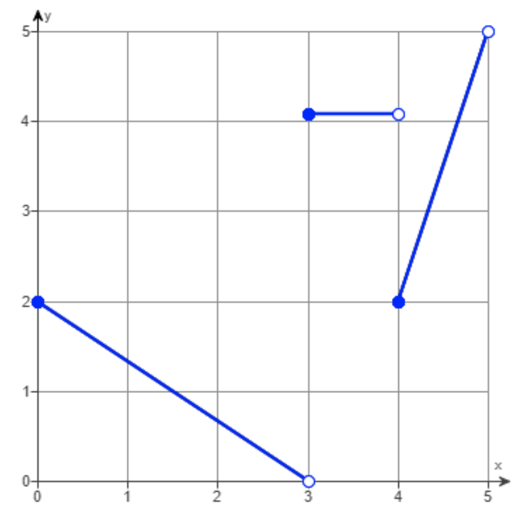
Ex. A-17 **Algebra/Precalculus** Fa21 **Exam**

For each part, use the graph of $y = f(x)$.

(a) Calculate $f(f(2))$.

(b) State the domain of f in interval notation.

(c) State the range of f in interval notation.



exf21

Ex. A-18 **Algebra/Precalculus** Fa21 **Exam**

Suppose $\log_3(x) = A$ and $\log_3(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_3\left(\frac{27\sqrt{x}}{y^4}\right)$$

exf21

Ex. A-19 **Algebra/Precalculus** Fa21 **Exam**

Rewrite the expression below as a single logarithm. Assume x and y are positive.

$$\frac{1}{2}\left(\log_5(x) - 7\log_5(y)\right) + 3\log_5(x-1)$$

exf21

Ex. A-20 **Algebra/Precalculus** Fa21 **Exam**

Suppose $\cos(\theta) = \frac{A}{7}$ with $0 < A < 7$ and $\sin(\theta) < 0$. Find $\sec(\theta)$, $\sin(\theta)$, and $\tan(\theta)$ in terms of A.

exf21

Ex. A-21 **Algebra/Precalculus** Fa21 **Exam**

A bacteria colony has an initial population of 3500. The population grows exponentially and triples every 7 hours. Recall that this means the population P at time t satisfies $P(t) = P_0 e^{kt}$ for some constants P_0 and k.

(a) Find the exact value of the growth constant k.
(b) Find the population after 25 hours.
(c) Find the time (in hours) when the population will be 12,600.

exf21

Ex. A-22 **Algebra/Precalculus** Fa21 **Exam**

A rectangular box is constructed according to the following rules.

- the length of the box is twice its width
- the height of the box is 5 feet more than three times the length

Let ℓ, w, and h denote the length, width, and height of the box, respectively, measured in feet.

(a) Write the height of the box in terms of w.
(b) Write an expression for $V(w)$, the volume of the box measured in cubic feet, as a function of its width.
(c) Suppose the rules also require that the sum of the box's width and height to be less than 26 feet. Under this condition, what is the domain of the function $V(w)$?

exf21

Ex. A-23 **Algebra/Precalculus** Fa21 **Exam**

Let $f(x) = \dfrac{2}{3x}$ and assume $h \neq 0$. Fully simplify each of the following expressions:

(a) $f(x+h)$ (b) $f(x+h) - f(x)$ (c) $\dfrac{f(x+h) - f(x)}{h}$

exf21

Ex. A-24 **Algebra/Precalculus** Fa21 **Exam**

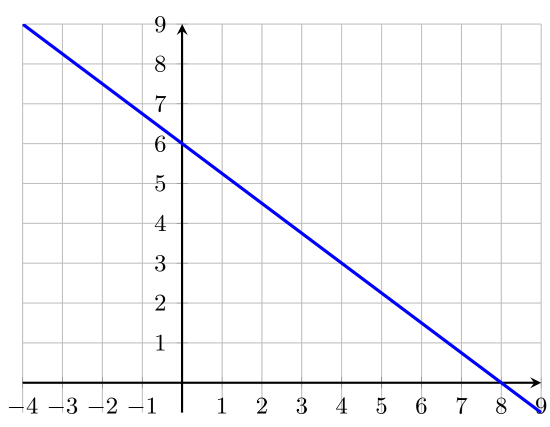
Find the domain of the function $f(x) = \sqrt{x^2 + x - 6} + \ln(10 - x)$. Write your answer using interval notation.

exf21

Ex. A-25 **Algebra/Precalculus** Sp22 **Exam**

For each part, use the graph of $y = g(x)$ given below and let $f(x) = 8x^2 - 4x + 15$.

(a) Find an expression for $g(x)$.
(b) Calculate the y-intercept of the graph of $y = f(g(x))$.
(c) Calculate $g(f(x))$.



exs22

Ex. A-26 **Algebra/Precalculus** Sp22 **Exam**

A 100-gram sample of a radioactive substance decays to 65% of its initial mass in 15 hours. Recall that the mass of the sample M at time t satisfies $M(t) = M_0 e^{kt}$ for some constants M_0 and k.

(a) Find the growth constant k.
(b) Find the mass of the sample after 22 hours.

exs22

Algebra/Precalculus Sp22 Exam

(c) Find the time in hours when the sample will have a mass of 41 grams.

exs22

Ex. A-27 Algebra/Precalculus Sp22 Exam

A rectangular box is constructed according to the following rules.

- The length of the box is 5 times its width.
- The volume of the box is 110 cubic feet.

Let L, W, and H be the length, width, and height of the box (measured in feet), respectively.

(a) Write an equation in terms of L, W, and H that expresses the first constraint.
(b) Write an equation in terms of L, W, and H that expresses the second constraint.
(c) Write an expression for $S(W)$, the total surface area of the box as a function of W.
(d) Suppose the rules also require that the sum of the box's length and width be less than 78 feet. What is the domain of $S(W)$ in this context?

exs22

Ex. A-28 Algebra/Precalculus Sp22 Exam

Suppose $\log_{16}(x) = A$ and $\log_{16}(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_{16}\left(\frac{4x^7}{\sqrt[9]{y}}\right)$$

exs22

Ex. A-29 Algebra/Precalculus Sp22 Exam

Let $f(x) = \sqrt{3x}$ and assume $h \neq 0$. Fully simplify each of the following expressions:

(a) $f(x+h)$ (b) $f(x+h) - f(x)$ (c) $\dfrac{f(x+h) - f(x)}{h}$

exs22

Ex. A-30 Algebra/Precalculus Sp22 Exam

Consider the function $f(x) = \dfrac{x-6}{x^2 - 9x + 20}$.

(a) Solve the equation $f(x) = 0$.
(b) List all numbers that are not in the domain of $f(x)$.
(c) Solve the inequality $f(x) > 0$ and write your answer using interval notation.

exs22

Ex. A-31 Algebra/Precalculus Sp22 Exam

Find all solutions to the following equation in the interval $[0, 2\pi)$.

$$2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$$

exs22

Ex. A-32 Algebra/Precalculus Sp22 Exam

Complete each of the following algebra exercises.

(a) Fully factor the polynomial $5x^4 + 25x^3 - 180x^2$.
(b) Solve the rational equation below.

$$\frac{4}{x+5} + \frac{9x}{x^2-25} = \frac{6}{x-5}$$

(c) Simplify the complex fraction below by writing it as a simple fraction.

$$\frac{\dfrac{4}{x} - \dfrac{2}{xy}}{8 + \dfrac{7}{y}}$$

exs22

Ex. A-33 Algebra/Precalculus Su22 Exam

Complete each of the following algebra exercises.

(a) Simplify $\left(\dfrac{27x^{3/5}}{x^{-3}z^{15}}\right)^{-1/3}$, leaving positive exponents and integer coefficients.

(b) Simplify $\dfrac{x^2-9}{3-\sqrt{6-x}}$ for $x \neq -3$. (All common factors must be canceled.)

(c) Factor the expression completely: $5x^9 - 14x^8 - 3x^7$.

(d) Fully simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = \dfrac{2}{x} - 3$ and $h \neq 0$.

exsu22

Ex. A-34 Algebra/Precalculus Su22 Exam

For each part, find all solutions to the given equation.

(a) $\sqrt{2x+1} + 1 = x$

(b) $(10-x^2)^{1/2} - x^2(10-x^2)^{-1/2} = 0$

(c) $2 + \sin(\theta) = 2\cos(\theta)^2$ (find solutions in $[0, 2\pi)$ only)

exsu22

Ex. A-35 Algebra/Precalculus Su22 Exam

Find the domain of the function $f(x) = \ln(x^2 - 20)$. Write your answer using interval notation.

exsu22

Ex. A-36 Algebra/Precalculus Su22 Exam

The length of a rectangular box is three times its width, and the total surface area of the box is 200 in^2. Let W be the width of the box in inches. Find the volume of the box in terms of W.

exsu22

Ex. A-37 Algebra/Precalculus Su22 Exam

For each part, write an equation for the line in the xy-plane that satisfies the given description.

(a) The line through the point $(-2, 10)$ with slope -3.

(b) The line through the points $(3, 5)$ and $(-1, 4)$.

(c) The line through the point $(5, 1)$ and perpendicular to the line $x + 3y = 10$.

(d) The horizontal line through the point $(-2, 15)$.

exsu22

Ex. A-38 Algebra/Precalculus Su22 Exam

The number of bacteria in a certain colony grows exponentially. Recall that this means the number of bacteria N at time t is $N(t) = N_0e^{kt}$, where N_0 and k are constants. Suppose there are initially 500 bacteria, and the number of bacteria triples every 2 hours. How much time must pass before the number of bacteria increases from 500 to 5000?

exsu22

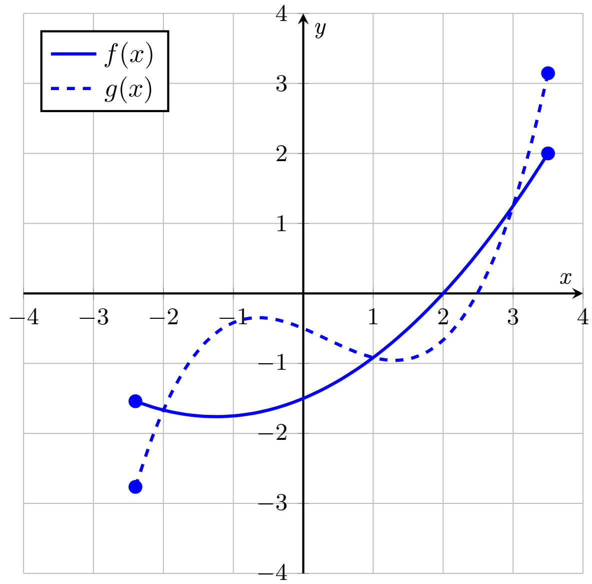
Ex. A-39 Algebra/Precalculus Su22 Exam

For each part, use the graphs of $y = f(x)$ and $y = g(x)$ below.

exsu22

Algebra/Precalculus

Su22 **Exam**



(a) Calculate $f(2)$.
(b) Estimate the value of $g(0) - f(0)$.
(c) Find all solutions to the equation $f(x) = g(x)$.
(d) Solve the inequality $g(x) > f(x)$. Write your answer using interval notation.

exsu22

Ex. A-40 **Algebra/Precalculus** Sp18 **Quiz**

Complete the following algebra exercises.

(a) Simplify the expression $\dfrac{x^3 - 4x}{x^3 - x^2 - 6x}$ as much as possible.
(b) Write the expression $\dfrac{\sqrt{xy^3}}{(x^{2/3}y^{-5/2})^6}$ in the form $x^a y^b$.
(c) Let $f(x) = 3x^2$. Simplify the expression $\dfrac{f(x+h) - f(x)}{h}$ as much as possible.
(d) Evaluate the expression $\log_6(9) + \log_6(4)$.
(e) Write the solution to the inequality $x^2 - 3x + 2 < 0$ using interval notation.
(f) Find all values of θ in the interval $[0, 2\pi)$ such that $2\sin(2\theta) = 1$.
(g) Let $f(x) = \dfrac{1}{x}$. Simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ as much as possible.

qs18

Ex. A-41 **Algebra/Precalculus** Sp18 **Quiz**

For each part, find an equation of the described line.

(a) The line whose slope is -3 and which passes through the point $(1, 4)$.
(b) The line that passes through the point $(-\pi, 1)$ with slope $\sqrt{2}$.

qs18

Ex. A-42 **Algebra/Precalculus** Sp18 **Quiz**

Find all solutions to the following equation.

$$\log_2(x) + \log_2(x - 3) = 2$$

qs18

Ex. A-43 **Algebra/Precalculus** Su22 **Quiz**

Complete the following algebra exercises.

(a) Find all solutions to the given equation.

$$2x^{5/2} + x^{3/2} + x^{1/2} = 0$$

(b) Simplify the expression; assume $x \neq -10$.

$$\frac{x^3 + 10x^2}{\sqrt{15 - x} - 5}$$

(c) Simplify the expression; assume any common factors are non-zero.

$$\frac{\frac{x-1}{x+1} + \frac{6}{x}}{\frac{2}{x^2+x} + \frac{1}{x+1}}$$

qsu22

Ex. A-44 **Algebra/Precalculus** Fa22 **Quiz**

Find all solutions to the given equation.

$$\log_2(x-3) + 2 = \log_2(x+9)$$

qf22

Ex. A-45 **Algebra/Precalculus** Fa22 **Quiz**

Fully simplify the given expression. Assume any common factors are non-zero.

$$\frac{100}{x^2 - 25} - \frac{2x}{x+5}$$

qf22

Ex. A-46 **Algebra/Precalculus** Fa22 **Quiz**

Use rationalization to simplify the expression below. All common factors must be canceled.

$$\frac{3 - \sqrt{2-x}}{x+7}$$

qf22

Ex. A-47 **Algebra/Precalculus**

For each of the following problems, zero or more of the choices are exact answers. Identify all of the exact answers, and ***explain why the other choices are wrong***. If the exact value of the correct answer does not appear as one of the choices, find the exact value of the correct answer.

(a) Find all real numbers x such that $x^2 = 2$.

A. 1.41 **B.** $\sqrt{2}$ **C.** ± 1.41 **D.** 1.41 and -1.41 **E.** $\pm\sqrt{2}$

(b) Find all real numbers t such that $t^3 + 4 = 0$.

A. -1.59 **B.** ± 1.59 **C.** $\pm\sqrt[3]{-4}$ **D.** $-2^{2/3}$ **E.** no real solution

(c) Find the circumference of a circle whose radius is 1.

A. 6.28 **B.** ± 6.283185 **C.** $\frac{44}{7}$ **D.** none of the above

(d) Find all real solutions to the equation $2^x = 3$.

A. 1.585 **B.** ± 1.585 **C.** 3^{-2} **D.** $\log_2(3)$ **E.** $\log_3(2)$ **F.** $\frac{\ln(3)}{\ln(2)}$ **G.** $\frac{1}{2}\log_2(9)$

unit1

Ex. A-48 **Algebra/Precalculus**

Simplify each of the following expressions according to the instructions.

unit1

Algebra/Precalculus

(a) Positive exponents and integer coefficients only (assume $x, y > 0$): $\left(\dfrac{x^8 y^{-4}}{16y^{4/3}}\right)^{-1/4}$

(b) Positive exponents only (assume $a, b > 0$): $\dfrac{(9ab)^{3/2}}{(27a^3b^{-4})^{2/3}} \cdot \left(\dfrac{3a^{-2}}{4b^{1/3}}\right)^{-1}$

(c) Common factors canceled (assume $h \neq 5$): $\dfrac{2h-10}{\sqrt{5}-\sqrt{h}}$

(d) Expand and fully simplify: $\left(\sqrt{9s^2+4}+2\right)\left(\sqrt{9s^2+4}-2\right)$

(e) Factor completely: $5y^2(y-3)^5 + 10y(y-3)^4$

(f) Factor completely: $3x^3 + x^2 - 12x - 4$

(g) Factor completely: $3x^{-1/2} + 4x^{1/2} + x^{3/2}$

(h) Common factors canceled, positive exponents only ($x \neq y$ and $x, y \neq 0$): $\dfrac{y^{-1}-x^{-1}}{x^{-2}-y^{-2}}$

(i) Common factors canceled ($u \neq 1$ and $u \neq -2$): $\dfrac{\dfrac{4}{u-1}-\dfrac{4}{u+2}}{\dfrac{3}{u^2+u-2}+\dfrac{3}{u+2}}$

unit1

Ex. A-49 **Algebra/Precalculus**

For each given function $f(x)$, fully simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$. Assume $h \neq 0$.

(a) $f(x) = 2x^2 - 2x$

(b) $f(x) = 9 - 5x$

(c) $f(x) = -4$

(d) $f(x) = \dfrac{1}{x}$

unit1

Ex. A-50 **Algebra/Precalculus**

Solve each equation or inequality. (Parts (b) – (d) are related!)

(a) $p^2 = p + 1$

(b) $2u^2 - 3u + 1 = 0$

(c) $2x^{5/2} - 3x^{3/2} + x^{1/2} = 0$

(d) $2\sin(\theta)^2 - 3\sin(\theta) + 1 = 0$

(e) $2x = x^2$

(f) $\dfrac{1-x}{1+x} + \dfrac{1+x}{1-x} = 6$

(g) $3\cos(x) + 2\sin(x)^2 = 3$

(h) $|2x+1| = 1$

(i) $|3x-5| = 4x$

(j) $\dfrac{x+5}{x-2} = \dfrac{5}{x+2} + \dfrac{28}{x^2-4}$

(k) $t^2 - 4t - 5 > 0$

(l) $\dfrac{x-4}{2x+1} < 0$

(m) $\dfrac{x-4}{2x+1} < 5$

unit1

Ex. A-51 **Algebra/Precalculus**

Find an equation of each described line.

(a) line through the point $(4, -6)$ with slope 3

(b) line through the points $(1, 2)$ and $(-3, 4)$

(c) line through the point $(5, 5)$ and perpendicular to the line described by $2x - 4y = 3$

(d) line through the point $(-1, -2)$ and parallel to the line described by $3x + 8y = 1$

(e) horizontal line through the point $(3, -1)$

(f) vertical line through the point $(2, -4)$

unit1

Ex. A-52 **Algebra/Precalculus**

If $f(x)$ and $g(x)$ are functions, then $f(g(x))$ is also a function, called the composition of f and g. We also write $f \circ g$ to mean $f(g(x))$. Similarly, $g \circ f$ means $g(f(x))$.

(a) Let $f(x) = \sin(3x) + 7$ and $g(x) = e^{2x} + 1$. Write expressions for both $f(g(x))$ and $g(f(x))$.

(b) Let $h(x) = \log_{10}(\sin(\sqrt{x}) + 1)$. Find four functions f_1, f_2, f_3, and f_4 such that $h(x) = f_4(f_3(f_2(f_1(x))))$. You

unit1

Algebra/Precalculus

may not use the function $f(x) = x$ for any of your choices.

unit1

Ex. A-53 **Algebra/Precalculus**

For each of the following function pairs, find a simplified formula for $f \circ g$ and $g \circ f$. Then find the domain of f, g, $f \circ g$, and $g \circ f$.

(a) $f(x) = \sin(x)$ and $g(x) = 2x + 3$

(b) $f(x) = \dfrac{2+x}{1-2x}$ and $g(x) = \dfrac{x-2}{2x+1}$

unit1

Ex. A-54 **Algebra/Precalculus**

Find the exact value of each expression. Your final answer cannot contain "log" or "ln".

(a) $\log_2(48) - \log_2(6)$

(b) $\log_2(48) - \log_4(144)$

(c) $\ln(\log_{10}(10^e))$

(d) $3^{\log_3(4e) - \log_3(e)}$

unit1

Ex. A-55 **Algebra/Precalculus**

Sketch the graph of each of the following functions.

(a) $f(x) = e^{-x}$

(b) $f(x) = \log_5(x)$

(c) $f(x) = -2^x$

(d) $f(x) = \log_{1/3}(x)$

unit1

Ex. A-56 **Algebra/Precalculus**

Find all solutions to the following equations.

(a) $3^{x^2-x} = 9$

(b) $e^{2x+3} = 1$

(c) $\log_3(x) + \log_3(2x+1) = 1$

unit1

Ex. A-57 **Algebra/Precalculus**

Suppose $\log_{b^3}(5) = \frac{1}{6}$. Find the exact value of $\sqrt{b-16}$.

unit1

Ex. A-58 **Algebra/Precalculus**

Write the exact values of the sine, cosine, and tangent of each of the following angles: $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{\pi}{2}$, $\frac{2\pi}{3}$, π, $-\frac{\pi}{6}$, and $-\frac{3\pi}{4}$. *(You should do this without any reference or calculator.)*

unit1

Ex. A-59 **Algebra/Precalculus**

Graph each of the following curves.

(a) $y = \sin(\theta)$

(b) $y = 3\cos(\pi\theta)$

unit1

Ex. A-60 **Algebra/Precalculus**

A bank pays 6% annual interest compounded continuously. How long will it take for \$835 to triple?

unit1

Ex. A-61 **Algebra/Precalculus**

The number of bacteria in a certian petri dish obeys a law of exponential growth. Suppose there are initially 1000 bacteria and the number of bacteria doubles every 20 minutes. When will the number of bacteria reach 5000?

unit1

Ex. A-62 **Algebra/Precalculus**

A rectangular box is constructed according to the following rules.

- the length of the box is twice its width
- the height of the box is 5 feet more than three times the length

(a) If x is the width of the box in feet, write an expression for $V(x)$, the volume of the box in cubic feet as a function of its width.

(b) Suppose the rules also require that the sum of the box's width and height to be no more than 26 feet. Under

unit1

Algebra/Precalculus

this condition, what is the domain of the function $V(x)$?

unit1

Ex. A-63 Algebra/Precalculus

The total cost (in \$) of producing q units of some product is $C(q) = 30q^2 + 400q + 500$.

(a) Compute the cost of making 20 units.
(b) Compute the cost of making the 20th unit.
(c) What is the initial setup cost?

unit1

Ex. A-64 Algebra/Precalculus

The speed of blood that is a distance r from the central axis in an artery of radius R is $v(r) = C(R^2 - r^2)$, where C is some constant.

(a) What is the speed of the blood on the central axis?
(b) What is the speed halfway between the central axis and the artery wall?

unit1

Ex. A-65 Algebra/Precalculus

An account in a certain bank pays 5% annual interest, compounded continuously. An initial deposit of \$200 is made into the account. How many years does it take for the \$200 to double?

unit1

Ex. A-66 Algebra/Precalculus

A radioactive frog hops out of a pond full of nuclear waste. If its level of radioactivity declines to $\frac{1}{3}$ of its original value in 30 days, when will its level of radioactivity reach $\frac{1}{100}$ of its original value?
Hint: Use the exponential growth formula $P(t) = P_0 e^{rt}$.

unit1

Ex. A-67 Algebra/Precalculus

Complete each of the following exercises from various topics in algebra and precalculus.

(a) Simplify the expression $\dfrac{|2-x|}{x-2}$ for $x > 2$.
(b) Find all solutions to the equation $2^{x^2-2x} = 8$.
(c) Simplify the expression $2^{\log_2(3)-\log_2(5)}$.
(d) Find an equation of the line through the point $(-1, 4)$ with slope 2.
(e) Find the domain of $f(x) = \dfrac{\ln(x)}{x-2}$. Write your answer in interval notation.
(f) Solve the inequality $\dfrac{3x+6}{x(x-4)} \leq 0$. Write your answer in interval notation.

unit1

Ex. A-68 Algebra/Precalculus ⋆Challenge

Let $f(x) = \dfrac{2}{3-\sqrt{x}}$. Fully simplify the difference quotient $\dfrac{f(4+h)-f(4)}{h}$ for $h \neq 0$ (i.e., simplify the expression all common factors of h have been canceled.)

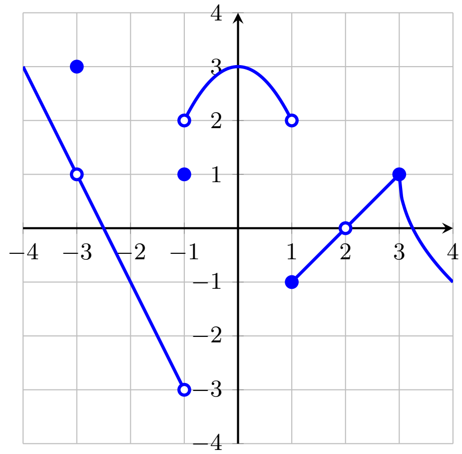
challenge

2 Chapter 2: Limits

§2.1, 2.2: Introduction to Limits

Ex. B-1 2.1/2.2 Sp19 Exam

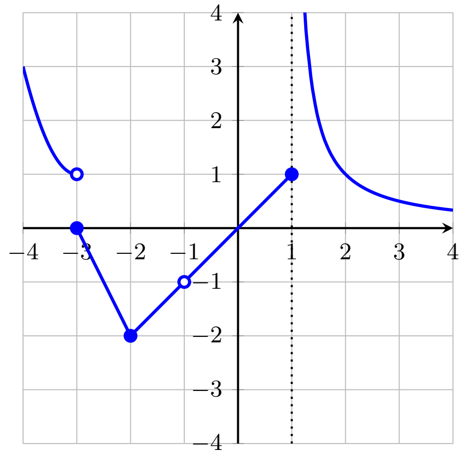
The graph of $y = f(x)$ is given below. Find all values of a in the interval $(-4, 4)$ for which $\lim_{x\to a} f(x)$ does not exist, or determine that no such values of a exist.



exs19

Ex. B-2 2.1/2.2 Sp20 Exam

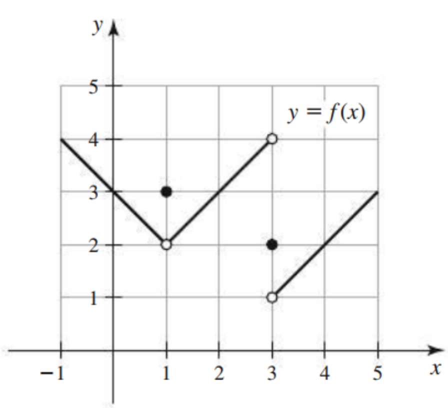
The graph of $y = f(x)$ is given below. Find all values of a in $(-4, 4)$ such that $\lim_{x\to a} f(x)$ does not exist.



exs20

Ex. B-3 2.1/2.2 Su20 Exam

For each part, use the graph of $f(x)$ below.



exsu20

2.1/2.2 Su20 Exam

(a) Calculate $\lim_{x\to 3} f(x)$ or determine that the limit does not exist.

(b) Find all values of a such that both $\lim_{x\to a} f(x)$ exists and this limit is not equal to $f(a)$.

exsu20

Ex. B-4 2.1/2.2 Su20 Exam

Consider the function below.

$$f(x) = \begin{cases} x^2+4x-1 & x<2 \\ 11 & x=2 \\ 19-x^3 & x>2 \end{cases}$$

A student correctly calculates that $\lim_{x\to 2} f(x) = 11$ and enters this as their final answer on an online exam, initially getting full credit. However, after inspecting the student's work, the teacher overrides this score and gives no credit. The teacher writes the comment "you have not correctly justified your answer." The student wrote the following:

"Since $f(x)$ is defined for all x and $f(2) = 11$, the answer is $\lim_{x\to 2} f(x) = 11$."

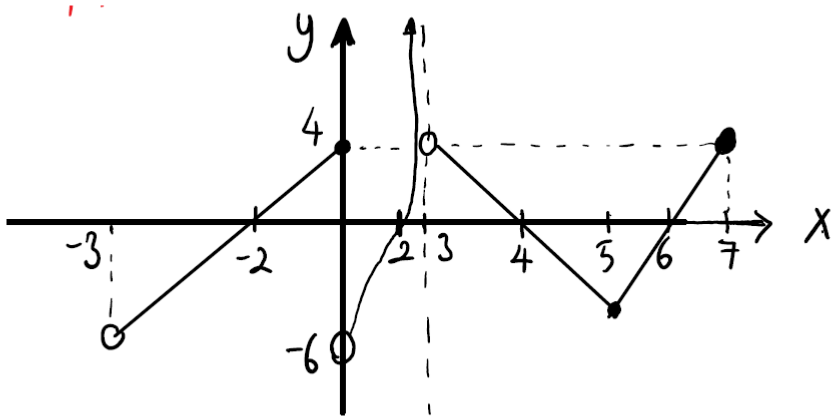
(a) Why is the student's justification incorrect?

(b) Write a complete and correct justification for the statement $\lim_{x\to 2} f(x) = 11$.

exsu20

Ex. B-5 2.1/2.2 Sp21 Exam

For each part, use the graph of $y = f(x)$.



(a) Calculate $f(f(2))$.

(b) Find where $f(x) = 0$.

(c) State the domain of f in interval notation.

(d) State the range of f in interval notation.

(e) For each part below, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(i) $\lim_{x\to 0^-} f(x)$ (ii) $\lim_{x\to 0^+} f(x)$ (iii) $\lim_{x\to 0} f(x)$ (iv) $\lim_{x\to 3^-} f(x)$ (v) $\lim_{x\to 3^+} f(x)$

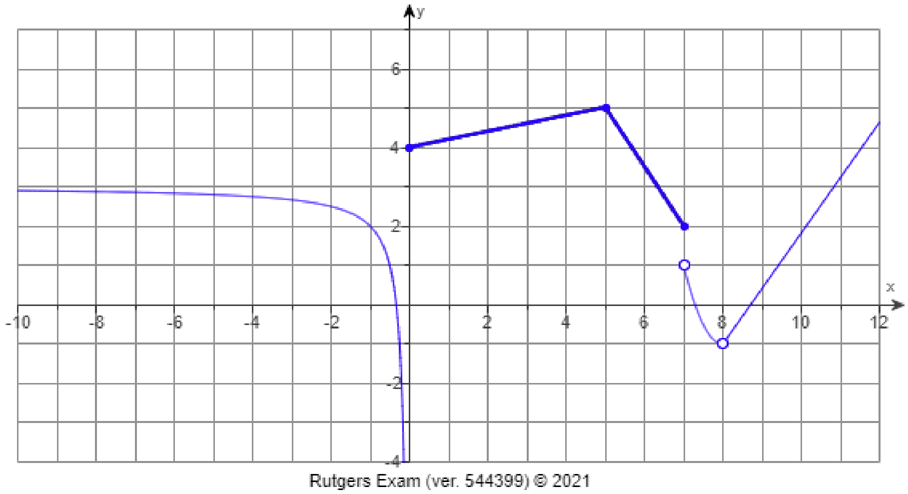
exs21

Ex. B-6 2.1/2.2, 2.3, 2.4, 2.5 Fa21 Exam

For each part, use the graph of $y = f(x)$.

exf21

2.1/2.2, 2.3, 2.4, 2.5 Fa21 **Exam**



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

exf21

Ex. B-7 **2.1/2.2** Fa21 **Exam**

The position of a particle (measured in feet) after t seconds is modeled by the following function.

$$h(t) = -16t^2 + 96t + 100$$

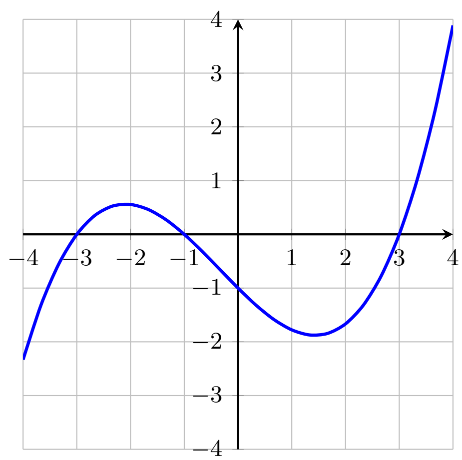
(a) Calculate the average velocity of the particle (in feet per second) between $t = 4$ and $t = 5$.

(b) Find an equation of the secant line between $(4, h(4))$ and $(5, h(5))$.

exf21

Ex. B-8 **2.1/2.2, 3.7, 4.3/4.4** Fa21 **Exam**

For each part, use the graph of $y = g(x)$.



(a) How many solutions does the equation $g'(x) = 0$ have?

(b) Order the following quantities from least to greatest: $g'(-2.5)$, $g'(-2)$, $g'(0)$, and $g'(4)$. In your answer, write these quantities symbolically; do not give a numerical estimate.

(c) What is the sign of $g''(-3)$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

exf21

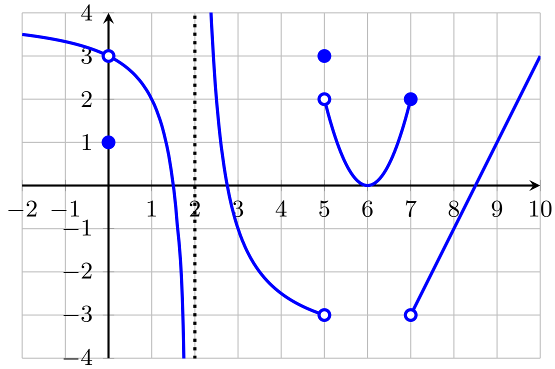
2.1/2.2, 3.7, 4.3/4.4 Fa21 Exam

(d) Let $h(x) = g(x)^2$. What is the sign of $h'(-4)$(negative, positive, or zero)? If there is not enough information to determine the value, explain why.

exf21

Ex. B-9 2.1/2.2 Sp22 Exam

For each part, use the graph of $y = g(x)$ below to calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

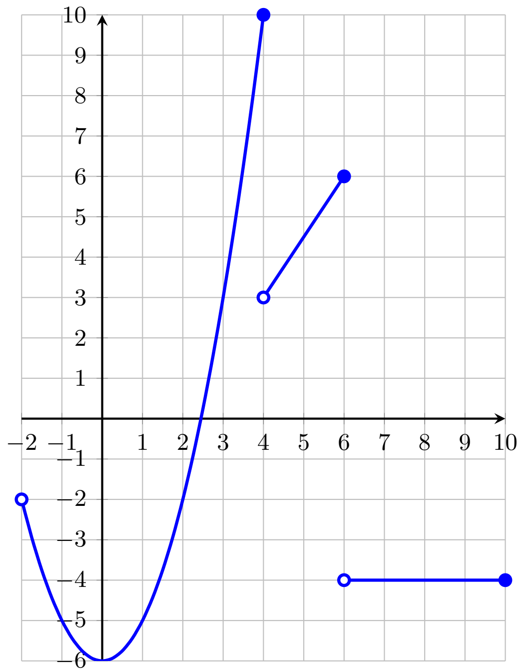


(a) $\lim_{x\to 0} g(x)$ (b) $\lim_{x\to 2^-} g(x)$ (c) $\lim_{x\to 5^-} g(x)$ (d) $\lim_{x\to 5^+} g(x)$ (e) $\lim_{x\to 7} g(x)$

exs22

Ex. B-10 2.1/2.2 Sp18 Quiz

Evaluate the limits using the given graph.



(a) $\lim_{x\to -2^+} f(x)$ (b) $\lim_{x\to 4^-} f(x)$ (c) $\lim_{x\to 4^+} f(x)$ (d) $\lim_{x\to 6} f(x)$

qs18

Ex. B-11 2.1/2.2 Fa22 Quiz

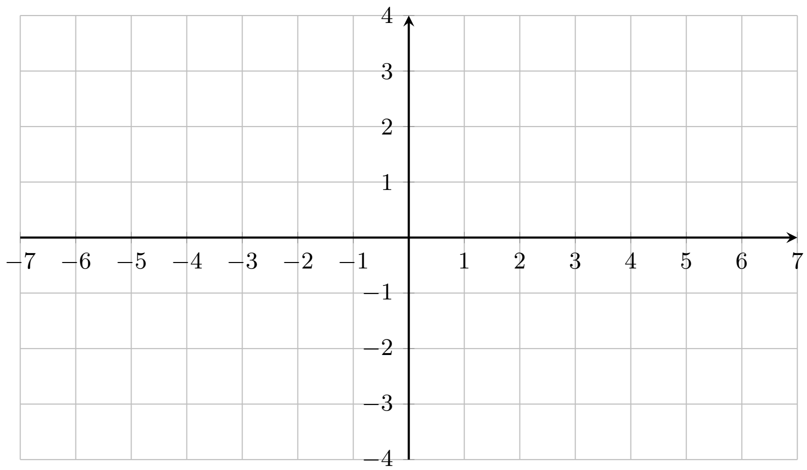
On the axes below, sketch the graph of a function $f(x)$ that satisfies the following properties:

- the domain of $f(x)$ is $[-7, 4) \cup (4, 7]$
- $\lim_{x\to -5} f(x) \neq f(-5)$

qf22

2.1/2.2 Fa22 **Quiz**

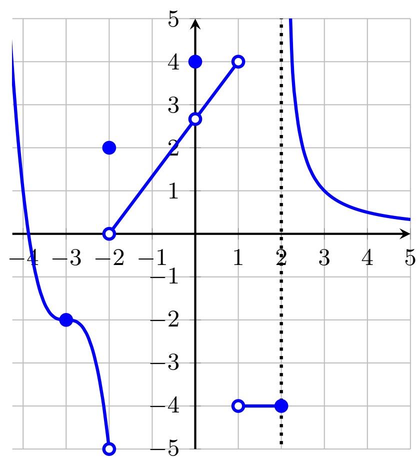
- $\lim_{x\to -3^-} f(x) = f(-3)$ but $\lim_{x\to -3} f(x)$ does not exist
- $\lim_{x\to 2} f(x) = f(2) = 4$
- $\lim_{x\to 4^+} f(x) = 2$ but $\lim_{x\to 4} f(x)$ does not exist



qf22

Ex. B-12 **2.1/2.2**

For each part, use the graph of $y = f(x)$ below to calculate the limit or determine the limit does not exist.



(a) $\lim_{x\to -3^-} f(x)$	(e) $\lim_{x\to -2^-} f(x)$	(i) $\lim_{x\to 0^-} f(x)$	(m) $\lim_{x\to 1^-} f(x)$	(q) $\lim_{x\to 2^-} f(x)$
(b) $\lim_{x\to -3^+} f(x)$	(f) $\lim_{x\to -2^+} f(x)$	(j) $\lim_{x\to 0^+} f(x)$	(n) $\lim_{x\to 1^+} f(x)$	(r) $\lim_{x\to 2^+} f(x)$
(c) $\lim_{x\to -3} f(x)$	(g) $\lim_{x\to -2} f(x)$	(k) $\lim_{x\to 0} f(x)$	(o) $\lim_{x\to 1} f(x)$	(s) $\lim_{x\to 2} f(x)$
(d) $f(-3)$	(h) $f(-2)$	(l) $f(0)$	(p) $f(1)$	(t) $f(2)$

c2s1

Ex. B-13 **2.1/2.2**

Suppose $\lim_{x\to 2}\left(\dfrac{f(x)-3}{x-2}\right) = 5$ and $\lim_{x\to 2} f(x)$ exists (and is equal to $f(2)$). What is the value of $f(2)$? Explain your answer.

c2s1

Ex. B-14 **2.1/2.2** **⋆Challenge**

Suppose $\lim_{x\to 0}(f(x)+g(x))$ exists. Is it true that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ also exist? Explain your answer.

challenge

§2.3: Techniques for Computing Limits

Ex. C-1 2.3 Fa17 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 7}\left(\dfrac{\frac{1}{7}-\frac{1}{x}}{x-7}\right)$ (b) $\lim\limits_{x\to 0}\left(\dfrac{\sin(7x)}{\tan(2x)}\right)$ (c) $\lim\limits_{x\to -1}\left(\dfrac{|x+1|}{x+1}\right)$

exf17

Ex. C-2 2.3 Sp18 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 0}\left(\dfrac{(2x+9)^2-81}{x}\right)$ (b) $\lim\limits_{x\to 3^-}\left(\dfrac{|x-3|}{x-3}\right)$ (c) $\lim\limits_{x\to 1}\left(\dfrac{5-\sqrt{32-7x}}{x-1}\right)$

exs18

Ex. C-3 2.3, 3.1 Fa18 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{u\to 4}\left(\dfrac{(u+6)^2-25u}{u-4}\right)$

(b) $\lim\limits_{s\to 1} g(s)$ where $g(s)=\begin{cases}\sqrt{1-s} & s\le 1\\ \dfrac{s^2-s}{s-1} & s>1\end{cases}$

(c) $\lim\limits_{h\to 0}\left(\dfrac{\sin(7+h)-\sin(7)}{h}\right)$

Hint: Use the definition of the derivative.

(d) $\lim\limits_{x\to 6}\left(\dfrac{\frac{1}{36}-x^{-2}}{x^2-36}\right)$

exf18

Ex. C-4 2.3 Sp19 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 5}\left(\dfrac{x-5}{x^2-2x-15}\right)$ (b) $\lim\limits_{x\to 0}\left(\dfrac{\sin(9x)}{\sin(16x)}\right)$

exs19

Ex. C-5 2.3 Fa19 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 5}\left(\dfrac{x^2-3x-10}{x^2-x-20}\right)$ (b) $\lim\limits_{x\to 0}\left(\dfrac{\sin^2(4x)}{x^2}\right)$ (c) $\lim\limits_{x\to 4}\left(\dfrac{3-\sqrt{2x+1}}{x-4}\right)$

exf19

Ex. C-6 2.3 Sp20 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 0}\left(\dfrac{(4x+1)^2-1}{x}\right)$

(b) $\lim\limits_{x\to 0}\left(\dfrac{9x\cos(2x)}{\sin(4x)}\right)$

(c) $\lim\limits_{x\to -1}\left(\dfrac{4-\sqrt{16x+32}}{x+1}\right)$

(d) $\lim\limits_{x\to 4^-}\left(\dfrac{|x^2-16|}{4-x}\right)$

(e) $\lim\limits_{x\to 3} g(x)$, where $g(x)=\begin{cases}\dfrac{x-3}{x^3-9x} & x<3\\ 18 & x=3\\ \dfrac{x-2}{x^2+9} & x>3\end{cases}$

exs20

Ex. C-7 2.3 Su20 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 3}\left(\dfrac{x^3+2x^2-15x}{x^2-9}\right)$ (b) $\lim_{x\to 0}\left(\dfrac{\sin(6x)^2}{x^2\cos(2x)}\right)$

exsu20

Ex. C-8 2.3 Su20 Exam

The parts of this problem are related.

(a) Suppose $x<3$. Write an algebraic expression that is equivalent to $|x-3|$ but without absolute value symbol.

(b) Calculate $\lim_{x\to 2}\left(\dfrac{|x-3|-1}{x-2}\right)$. Explain why your work to part (a) is relevant here and precisely where you use it.

exsu20

Ex. C-9 2.3 Su20 Exam

The parts of this problem are related.

(a) Consider the function below.

$$f(x)=\begin{cases}\dfrac{x-1}{3-\sqrt{10-x}} & x\neq 1\\ -6 & x=1\end{cases}$$

Show that $\lim_{x\to 1} f(x)\neq f(1)$.

(b) Now consider the similar function below.

$$g(x)=\begin{cases}\dfrac{x-1}{3-\sqrt{10-x}} & x\neq 1\\ b & x=1\end{cases}$$

where b is an unspecified constant. Explain how to determine whether the following statement is true: $\lim_{x\to 1} g(x)\neq g(1)$. How does your work for part (a) change, if at all, to determine the truth of the statement? Explain your answer.

exsu20

Ex. C-10 2.3 Fa20 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 5}\left(\dfrac{25-x^2}{x-5}\right)$ (b) $\lim_{x\to 4}\left(\dfrac{\frac{1}{x}-\frac{1}{4}}{4-x}\right)$

exf20

Ex. C-11 2.3 Fa20 Exam

A student is asked to solve a certain limit and determines the limit does not exist. (This may or may not be the correct answer.) They write the following for their justification:

> "I used the direct substitution property to evaluate the limit. I noticed the denominator gives me a zero, therefore the limit does not exist."

Explain why the student's justification is incorrect.

Note: To solve this problem, it is not necessary to be given the actual limit the student was asked to compute.

exf20

Ex. C-12 2.3 Fa20 Exam

Determine whether $\lim_{x\to 0} f(x)$ exists, where $f(x)=\begin{cases}3e^x-7 & x<0\\ 4+\sin(x) & x\geq 0\end{cases}$.

exf20

Ex. C-13 **2.3** Fa20 **Exam**

A student is asked to calculate the following limit:

$$L = \lim_{x\to 0}\left(\frac{x\cos x}{\sin(3x)}\right)$$

Analyze their work below, which contains two distinct errors. ***Note:*** The correct answer is $\frac{1}{3}$, not 0.

$$\begin{aligned} L &= \lim_{x\to 0}\left(\frac{x\cos(x)}{3\sin(x)}\right) && (1)\\ &= \left[\lim_{x\to 0}\left(\frac{1}{3}\right)\right]\left[\lim_{x\to 0}\left(\frac{x}{\sin(x)}\right)\right]\left[\lim_{x\to 0}(\cos(x))\right] && (2)\\ &= \left(\frac{1}{3}\right)(1)(0) && (3)\\ &= 0 && (4) \end{aligned}$$

Identify the lines in which the two errors occur and describe each error.

exf20

Ex. C-14 **2.3** Fa20 **Exam**

Consider the function $f(x)$ below, where $g(x)$ is an unspecified function with domain $[4,\infty)$.

$$f(x) = \begin{cases} 4 & x \le 0 \\ \dfrac{x-4}{\frac{1}{4}-\frac{1}{x}} & 0 < x < 4 \\ 16 & x = 4 \\ g(x) & x > 4 \end{cases}$$

(a) Show that $\lim_{x\to 4^-} f(x) = f(4)$.

(b) Suppose $g(4) = 16$. Is it necessarily true that $\lim_{x\to 4} f(x)$ exists? Justify your response.

exf20

Ex. C-15 **2.3** Fa20 **Exam**

A student is asked to solve a certain limit and determines the limit does not exist. (This may or may not be the correct answer.) They write the following for their justification:

> "I used the direct substitution property to evaluate the limit. I obtained the expression "$\frac{0}{0}$", which is undefined. Therefore the limit does not exist."

Is the student's justification correct? Explain.

Note: To solve this problem, it is not necessary to be given the actual limit the student was asked to compute.

exf20

Ex. C-16 **2.3** Fa20 **Exam**

Consider the limit $\lim_{x\to 3}\left(\frac{(5x-c)(x+4)}{x-3}\right)$, where c is an unspecified constant.

(a) For what value(s) of c does this limit exist? Explain.

(b) Suppose the limit exists. What is its value? Show all work.

exf20

Ex. C-17 **2.3** Sp21 **Exam**

Suppose $\lim_{x\to 0} f(x) = 4$. Calculate $\lim_{x\to 0}\left(\frac{xf(x)}{\sin(5x)}\right)$ or show that the limit does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

exs21

Ex. C-18 **2.3** Sp21 **Exam**

Consider the following limit, where a is an unspecified constant.

$$\lim_{x\to -3}\left(\frac{x^2-a}{x^3+x^2-6x}\right)$$

(a) Find the value of a for which this limit exists.

(b) For this value of a, calculate the value of the limit.

exs21

Ex. C-19 **2.3** Sp21 **Exam**

Consider the following function, where k is an unspecified constant.

$$g(x)=\begin{cases} xe^{x+4}-7\ln(x+5) & x<-4 \\ -4\cos(\pi x) & -4<x<5 \\ 10 & x=5 \\ \sqrt{2x-5}+k & 5<x \end{cases}$$

Note that $g(-4)$ is undefined.

(a) Does $\lim\limits_{x\to -4} g(x)$ exist? Choose the best answer below.

(i) Yes, $\lim\limits_{x\to -4} g(x)$ exists and is equal to ______.

(ii) Yes, $\lim\limits_{x\to -4} g(x)$ exists but we cannot determine its value with the given information.

(iii) No, $\lim\limits_{x\to -4} g(x)$ does not exist because the corresponding one-sided limits are not equal.

(iv) No, $\lim\limits_{x\to -4} g(x)$ does not exist because $g(-4)$ does not exist.

(v) No, $\lim\limits_{x\to -4} g(x)$ does not exist because the limit is infinite.

(b) Calculate the following limits. Your answer may contain k.

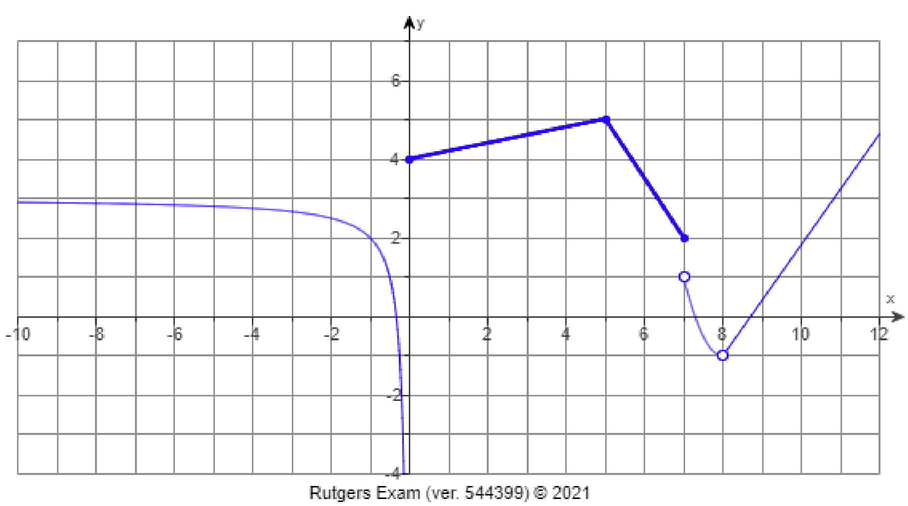
(i) $\lim\limits_{x\to 5^-} g(x)$ (ii) $\lim\limits_{x\to 5^+} g(x)$

(c) Is it possible to choose a value of k so that $\lim\limits_{x\to 5} g(x)$ exists? If so, what is that value of k?

exs21

Ex. B-6 **2.1/2.2, 2.3, 2.4, 2.5** Fa21 **Exam**

For each part, use the graph of $y=f(x)$.



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

exf21

2.1/2.2, 2.3, 2.4, 2.5

Fa21 Exam

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

exf21

Ex. C-20 2.3 Fa21 Exam

Suppose $\lim_{x\to 6} |f(x)| = 2$. Which of the following statements must be true about $\lim_{x\to 6} f(x)$?

(i) $\lim_{x\to 6} f(x)$ does not exist.

(ii) $\lim_{x\to 6} f(x) = 2$.

(iii) $\lim_{x\to 6} f(x)$ exists and is equal to either 2 or -2, but there is not enough information to determine which of these possibilities must be true.

(iv) There is not enough information about $f(x)$ to determine whether $\lim_{x\to 6} f(x)$ exists.

(v) $\lim_{x\to 6} f(x) = -2$.

exf21

Ex. C-21 2.3 Fa21 Exam

Consider the following function, where k is an unspecified constant.

$$f(x) = \frac{4x^2 - kx}{x^2 + 12x + 32}$$

(a) Find the value of k for which $\lim_{x\to -4} f(x)$ exists.

(b) For the value of k described in part (a), evaluate $\lim_{x\to -4} f(x)$.

exf21

Ex. C-22 2.3 Fa21 Exam

Suppose $\lim_{x\to 0}\left(\frac{f(x)}{x}\right) = 8$. Calculate $\lim_{x\to 0}\left(\frac{f(x)}{\sin(6x)}\right)$ or show that the limit does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

exf21

Ex. C-23 2.3, 2.4 Sp22 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 3}\left(\frac{x-3}{10-\sqrt{x+97}}\right)$

(b) $\lim_{x\to 6}\left(\frac{36-x^2}{\frac{1}{x}-\frac{1}{6}}\right)$

(c) $\lim_{x\to 0}\left(\frac{x^2\csc(3x)}{\cos(7x)\sin(4x)}\right)$

(d) $\lim_{x\to 2^-}\left(\frac{6x^2-7x}{x^2-4}\right)$

exs22

Ex. C-24 2.3 Su22 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 5}\left(\frac{6x+10}{x^2-25}-\frac{4}{x-5}\right)$

(b) $\lim_{x\to 6}\left(\frac{x-\sqrt{5x+6}}{6-x}\right)$

(c) $\lim_{x\to \infty}\left(\frac{5e^{2x}-3e^{x}}{9e^{3x}-12}\right)$

exsu22

Ex. C-25 2.3 Fa22 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

exf22

2.3 Fa22 Exam

(a) $\lim\limits_{x\to 8}\left(\dfrac{(x-2)^2-36}{x-8}\right)$ (b) $\lim\limits_{x\to 5}\left(\dfrac{40-8x}{\sqrt{19-3x}-2}\right)$ (c) $\lim\limits_{x\to 2^-}\left(\dfrac{4+x}{x^2+x-6}\right)$

exf22

Ex. C-26 2.3 Sp18 Quiz

Evaluate each of the following limits or show why it does not exist.

(a) $\lim\limits_{x\to 2}\left(\dfrac{2x^2-3x-2}{x^2+2x-8}\right)$ (b) $\lim\limits_{x\to 4}\left(\dfrac{3-\sqrt{x+5}}{x-4}\right)$

qs18

Ex. C-27 2.3 Sp20 Quiz

Evaluate each of the following limits or show why it does not exist.

(a) $\lim\limits_{x\to 1}\left(\dfrac{\sqrt{7x+9}-4}{x-1}\right)$ (b) $\lim\limits_{x\to 5}\left(\dfrac{\frac{1}{5}-\frac{1}{x}}{\frac{x}{5}-\frac{5}{x}}\right)$ (c) $\lim\limits_{x\to 3}\left(\dfrac{3-\sqrt{12-x}}{x-3}\right)$

qs20

Ex. C-28 2.3 Su22 Quiz

For each part, calculate the limit or show that it does not exist.

(a) $\lim\limits_{x\to 9}\left(\dfrac{x^3-81x}{(x-4)^2-25}\right)$ (b) $\lim\limits_{x\to 1}\left(\dfrac{\sqrt{x+3}-2}{x-1}\right)$

qsu22

Ex. C-29 2.3 Su22 Quiz

Calculate $\lim\limits_{x\to 0} f(x)$ or show the limit does not exist, where $f(x)$ is the function given below. Your work must be coherent and clearly explain your answer.

$$f(x)=\begin{cases}10e^x & x<0\\ 7 & x=0\\ \dfrac{\sin(10x)}{x} & x>0\end{cases}$$

qsu22

Ex. C-30 2.3 Fa22 Quiz

For each part, calculate the limit or determine it does not exist. You must show all work, and your work will be graded on its correctness and coherence.

(a) $\lim\limits_{x\to 6}\left(\dfrac{x^2-36}{2x^2-11x-6}\right)$ (b) $\lim\limits_{x\to 2}\left(\dfrac{\frac{3x+1}{x-1}-7}{x-2}\right)$

qf22

Ex. C-31 2.3 Fa22 Quiz

Calculate $\lim\limits_{x\to -3}\left(\dfrac{\sqrt{2x+15}-3}{x^2+8x+15}\right)$ or determine that it does not exist. If the limit is infinite, write "$+\infty$" or "$-\infty$" as your answer, as appropriate, instead of "DNE".

qf22

Ex. C-32 2.3

For each part, calculate the limit or show that it does not exist.

c2s3

2.3

(a) $\lim_{x\to 2}\left(\dfrac{x^2+3x-1}{x+\sin(\pi x)}\right)$

(b) $\lim_{x\to 1}(x^4-9x)^{1/3}$

(c) $\lim_{x\to -3}\left(\dfrac{x^2-9}{x^3+x^2-6x}\right)$

(d) $\lim_{x\to 1}\left(\dfrac{\sqrt{23-7x}-4}{x-1}\right)$

(e) $\lim_{h\to 0}\left(\dfrac{(x+h)^{-2}-x^{-2}}{h}\right)$

(f) $\lim_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{x^2+x}\right)$

(g) $\lim_{x\to 1}\left(\dfrac{\frac{1}{x}-1}{\sqrt{x}-1}\right)$

(h) $\lim_{x\to 0}|x|$

(i) $\lim_{x\to 8}\left(\dfrac{|x-8|}{x-8}\right)$

(j) $\lim_{x\to 8^-}\left(\dfrac{|x^2-64|}{x-8}\right)$

(k) $\lim_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$

(l) $\lim_{x\to 0^-}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$

(m) $\lim_{x\to 0}\left(\dfrac{\sin(\pi x)}{x}\right)$

(n) $\lim_{x\to 0}\left(\dfrac{\sec(x)-1}{x\sec(x)}\right)$

(o) $\lim_{x\to 0}\left(\dfrac{1-\cos(x)}{\sin(x)}\right)$

(p) $\lim_{x\to 2}\left(\dfrac{\sin(6-3x)}{5x-10}\right)$

(q) $\lim_{x\to \pi}\left(\dfrac{\tan(x-\pi)}{x-\pi}\right)$

(r) $\lim_{x\to 0}\left(\dfrac{\sin(2x)^2\cos(3x)}{\tan(5x)\sin(7x)}\right)$

(s) $\lim_{x\to -1}g(x)$ where

$$g(x)=\begin{cases}4x-5 & \text{if } x<-1\\ x^3+x & \text{if } x\geq -1\end{cases}$$

(t) $\lim_{x\to 2}f(x)$ where

$$f(x)=\begin{cases}\dfrac{x^2-2x}{x-2} & \text{if } x<2\\ \sqrt{x+2} & \text{if } x>2\end{cases}$$

c2s3

Ex. C-33 2.3

For each part, calculate the limit or show that it does not exist.

(a) $\lim_{x\to 0}\left(\dfrac{\sin(5x)}{3x}\cos(4x)\right)$ (b) $\lim_{x\to -2}\left(\dfrac{x^2+3x+2}{x^2+x-2}\right)$ (c) $\lim_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{x^2+x}\right)$

unit2

Ex. C-34 2.3

Let $f(x)=\begin{cases}\dfrac{3x^2-15x}{x-5} & \text{if } x<5\\ \sqrt{a+3x} & \text{if } x>5\end{cases}$, where a is an unspecified constant.

(a) For what value of a does $\lim_{x\to 5}f(x)$ exist? Explain.

(b) Given that $\lim_{x\to 5}f(x)$ exists, what is its value?

other

Ex. C-35 2.3

Consider the limit $\lim_{x\to -2}\left(\dfrac{x^4-a}{x^2-2x-8}\right)$, where a is an unspecified constant.

(a) For what value of a does this limit exist? Explain.

(b) Given that the limit does exist, what is its value?

other

Ex. C-36 2.3

For each of the following, evaluate the limit or explain why it does not exist. Show all work.

(a) $\lim_{h\to 0}\left(\dfrac{(x+h)^{-2}-x^{-2}}{h}\right)$ (b) $\lim_{x\to 3}\left(\dfrac{4}{x-3}-\dfrac{8}{x^2-4x+3}\right)$ (c) $\lim_{x\to 0}\left(\dfrac{\sin(7x)^2\cos(9x)}{\tan(3x)\sin(4x)}\right)$

other

Ex. C-37 2.3 ⋆Challenge

Calculate the following limit or determine that it does not exist.

$$\lim_{x\to a}\left(\frac{\cos\left(\frac{\pi a}{2x}\right)}{x-a}\right)$$

challenge

§2.4: Infinite Limits

Ex. D-1 2.4 Sp20 Exam

Consider the following function, where a and b are unspecified constants.

$$f(x) = \frac{x^2 + ax + b}{x - 2}$$

Is the line $x = 2$ necessarily a vertical asymptote of $f(x)$? Explain your answer. *Your answer may contain either English, mathematical symbols, or both.*

For example, let $a = 0$ and $b = -4$. Then $f(x) = x + 2$ for $x \neq 2$, and so f has no vertical asymptote at $x = 2$.

exs20

Ex. D-2 2.4, 4.7 Sp20 Exam

Which of the following limits are equal to $+\infty$? Select all that apply.

(a) $\displaystyle\lim_{x\to 5^-} \left(\frac{x^2+25}{5-x}\right)$

(b) $\displaystyle\lim_{x\to 5^+} \left(\frac{x^2+25}{5-x}\right)$

(c) $\displaystyle\lim_{x\to -3^-} \left(\frac{x^3}{|x+3|}\right)$

(d) $\displaystyle\lim_{x\to 0^-} \left(\frac{x^4-2x-5}{\sin(x)}\right)$

(e) $\displaystyle\lim_{x\to 1^+} \left(\frac{x^6-x^2}{x-1}\right)$

exs20

Ex. D-3 2.4 Sp20 Exam

Consider the function below.

$$f(x) = \frac{x^3 + 2x^2 - 13x + 10}{x^2 - 1}$$

Show that $x = -1$ is a vertical asymptote of f, but $x = 1$ is *not* a vertical asymptote of f.

exs20

Ex. D-4 2.4 Sp20 Exam

Determine which of the following limits are equal to $-\infty$. Select all that apply.

(a) $\displaystyle\lim_{x\to 6^-} \left(\frac{x^2-5x-6}{x-6}\right)$

(b) $\displaystyle\lim_{x\to 6^-} \left(\frac{x^2-5x-6}{x^2-12x+36}\right)$

(c) $\displaystyle\lim_{x\to \infty} \left(\frac{x^2-5x-6}{x-6}\right)$

(d) $\displaystyle\lim_{x\to \infty} \left(\frac{x^2-5x-6}{x^2-12x+36}\right)$

exs20

Ex. D-5 2.4 Su20 Exam

Let $h(x) = \dfrac{f(x)}{g(x)}$, where f and g are continuous and $\displaystyle\lim_{x\to a} g(x) = 0$. Is the following true or false?

> "The line $x = a$ is necessarily a vertical asymptote of $h(x)$."

You must justify your answer. This means that if your answer is "true", you should explain why the above statement is always true. If your answer is "false", you should give an example to show that the above statement is sometimes false.

exsu20

Ex. D-6 2.4 Su20 Exam

Suppose that as x increases to 1, the values of $f(x)$ get larger and larger, and the values stay positive. Is the following true or false?

> "Therefore, $\displaystyle\lim_{x\to 1^-} f(x) = +\infty$."

You must justify your answer. This means that if your answer is "true", you should explain why the above statement is always true. If your answer is "false", you should give an example to show that the above statement is sometimes false.

exsu20

Ex. D-7 **2.4, 2.6** Su20 **Exam**

Let $f(x) = \dfrac{9x - x^3}{x^2 + x - 6}$.

(a) Calculate all vertical asymptotes of f. Justify your answer.

(b) Where is f discontinuous?

(c) For each point at which f is discontinuous, determine what value should be reassigned to f, if possible, to guarantee that f will be continuous there.

exsu20

Ex. D-8 **2.4, 2.5** Su20 **Exam**

Let $f(x) = \dfrac{3 + 7e^{2x}}{1 - e^x}$. Calculate each of the following limits.

(a) $\lim_{x\to-\infty} f(x)$ (b) $\lim_{x\to+\infty} f(x)$ (c) $\lim_{x\to 0^-} f(x)$

exsu20

Ex. D-9 **2.4** Fa20 **Exam**

Consider the function $f(x) = \dfrac{(ax-6)(x+1)}{x-2}$, where a is an unspecified constant.

(a) For which value(s) of a does f have a vertical asymptote? What is the equation of this vertical asymptote?

(b) For which value(s) of a does f have a horizontal asymptote? What is the equation of this horizontal asymptote?

exf20

Ex. D-10 **2.4** Fa20 **Exam**

For which value(s) of n, if any, is the following statement true: $\lim_{x\to 2^-} (2-x)^n = +\infty$? Explain your answer.

exf20

Ex. D-11 **2.4** Sp21 **Exam**

Determine whether the following statement is true or false. Explain your answer in 1 or 2 sentences. Your answer should contain English with few mathematical symbols.

> "Suppose f and g are functions with $g(3) = 1$. Put $H(x) = \dfrac{f(x)}{g(x)-1}$. Then H must have a vertical asymptote at $x = 3$."

exs21

Ex. D-12 **2.4** Sp21 **Exam**

Let $f(x) = \dfrac{(x+a)(x-3)}{(x-2)(x+1)}$, where a is an unspecified, **positive** constant. For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0} f(x)$ (b) $\lim_{x\to 2^-} f(x)$ (c) $\lim_{x\to 2^+} f(x)$ (d) $\lim_{x\to 2} f(x)$

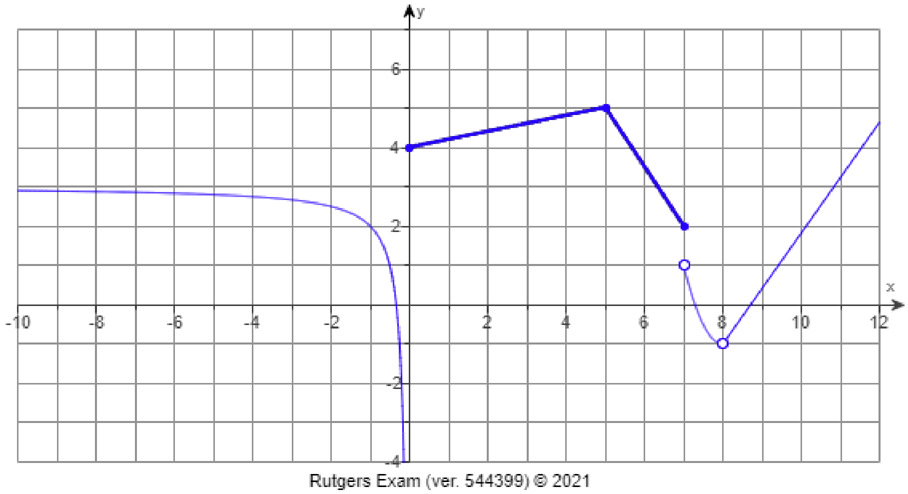
exs21

Ex. B-6 **2.1/2.2, 2.3, 2.4, 2.5** Fa21 **Exam**

For each part, use the graph of $y = f(x)$.

exf21

2.1/2.2, 2.3, 2.4, 2.5 Fa21 **Exam**



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

exf21

Ex. D-13 **2.4, 2.5, 4.7** Fa21 **Exam**

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 1}\left(\frac{x^4 - x}{\ln(77x-76)}\right)$

(b) $\lim_{x\to -\infty}\left(\frac{\sqrt{36x^2+63}}{31x}\right)$

(c) $\lim_{x\to 2^+} f(x)$, with $f(x) = \begin{cases} 1+4x & x \le 2 \\ \frac{x^2-4}{x-2} & x > 2 \end{cases}$

(d) $\lim_{x\to 5^-}\left(\frac{\cos(\pi x)}{x^2-25}\right)$

exf21

Ex. D-14 **2.4, 4.7** Fa21 **Exam**

For each part, find all vertical asymptotes of the given function.

(a) $f(x) = \frac{x^2-8x+15}{x^2-9}$

(b) $g(x) = \frac{e^{x+3}-1}{x^2-9}$

exf21

Ex. C-23 **2.3, 2.4** Sp22 **Exam**

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 3}\left(\frac{x-3}{10-\sqrt{x+97}}\right)$

(b) $\lim_{x\to 6}\left(\frac{36-x^2}{\frac{1}{x}-\frac{1}{6}}\right)$

(c) $\lim_{x\to 0}\left(\frac{x^2\csc(3x)}{\cos(7x)\sin(4x)}\right)$

(d) $\lim_{x\to 2^-}\left(\frac{6x^2-7x}{x^2-4}\right)$

exs22

Ex. D-15 **2.4, 2.5** Sp22 **Exam**

For the function f below, find its domain and all vertical and horizontal asymptotes.

$$f(x) = \frac{x^2-8x+12}{3x^2-8x+4}$$

exs22

Ex. D-16 **2.4, 2.5** Su22 **Exam**

Consider the function $f(x) = \dfrac{x^3 - 3x + 1}{x^2 - 2x + 1}$.

(a) Find all horizontal asymptotes of f, if any.

(b) Find all vertical asymptotes of f. Then calculate $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$, where $x = a$ is the rightmost vertical asymptote of f.

exsu22

Ex. D-17 **2.4** Fa22 **Exam**

Find all vertical asymptotes of the function $f(x) = \dfrac{x^3 - 36x}{x^3 - 12x^2 + 36x}$.

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exf22

Ex. D-18 **2.4, 2.5** Su22 **Quiz**

Calculate all of the vertical and horizontal asymptotes of $f(x) = \dfrac{x^2 - 100}{10x - x^2}$.

Then find the two one-sided at $x = a$, where $x = a$ is the leftmost vertical asymptote of f.

qsu22

Ex. D-19 **2.4** Fa22 **Quiz**

Find all vertical asymptotes of $f(x)$. You must justify your answers precisely.

$$f(x) = \frac{\sin(2x)}{x^2 - 10x}$$

qf22

Ex. D-20 **2.4**

For each part, calculate the limit or show that it does not exist.

(a) $\displaystyle\lim_{x \to 0^+} \left(\frac{x^2 - x + 4}{2x + \sin(x)} \right)$ (b) $\displaystyle\lim_{x \to 3^-} \left(\frac{2x^2 + 8}{x^2 - 9} \right)$ (c) $\displaystyle\lim_{x \to 4^+} \left(\frac{|16 - x^2|}{x - 4} \right)$

c2s4

Ex. D-21 **2.4**

For each part, find the vertical asymptotes of $f(x)$. Then find both corresponding one-sided limits at each vertical asymptote.

(a) $f(x) = \dfrac{(x-1)(2x+5)}{(x+1)(3x-6)}$

(b) $f(x) = \dfrac{x^2 - 18x + 81}{x^2 - 81}$

(c) $f(x) = \dfrac{(x-4)\sin(x)}{x^3 - 8x^2 + 16x}$

(d) $f(x) = \ln(x)$

(e) $f(x) = \dfrac{2e^x + 3}{1 - e^x}$

(f) $f(x) = e^{-1/x}$

c2s4

Ex. D-22 **2.4**

Find all vertical asymptotes of $f(x) = \dfrac{x^2 + x - 2}{x^2 - 4x + 3}$. Then at each vertical asymptote, calculate the corresponding one-sided limits of $f(x)$.

unit2

Ex. D-23 **2.4, 2.5** ⋆**Challenge**

For each function, find all horizontal asymptotes and vertical asymptotes. Then, at each vertical asymptote, calculate both one-sided limits.

(a) $f(x) = \dfrac{4x^3 + 4x^2 - 8x}{x^3 + 3x^2 - 4}$ (b) $f(x) = \dfrac{4x^3 - \sqrt{x^6 + 17}}{5x^3 - 40}$

challenge

§2.5: Limits at Infinity

Ex. E-1 **2.5** Sp19 **Exam**

Find the equation of each horizontal asymptote, if any, of $f(x) = \dfrac{4x^3 - 3x^2}{2x^3 + 9x + 1}$.

exs19

Ex. E-2 **2.5, 4.7** Sp19 **Exam**

The parts of this problem *are* related!

(a) Show that $\displaystyle\lim_{x\to\infty} \left(\frac{x}{x-3}\right) = 1$.

(b) Calculate the following limit or show it does not exist.

$$\lim_{x\to\infty} \left(\frac{x}{x-3}\right)^x$$

Hint: First use part (a) to identify the appropriate indeterminate form.

exs19

Ex. E-3 **2.5** Sp20 **Exam**

Find all horizontal asymptotes of

$$f(x) = \frac{12x + 5}{\sqrt{16x^2 + x + 1}}$$

or determine that there are no horizontal asymptotes.

exs20

Ex. E-4 **2.5** Sp20 **Exam**

Suppose the function f has domain $(-\infty, \infty)$. Give a brief explanation of how you would find all horizontal asymptotes of f. Note that for this problem, f is unspecified; you should not assume it has any particular form. *Your answer may contain either English, mathematical symbols, or both.*

exs20

Ex. E-5 **2.5** Su20 **Exam**

Let $f(x) = \dfrac{(x-3)(2x+1)}{(5x+2)(3x-10)}$. Calculate all horizontal asymptotes of f.

exsu20

Ex. D-8 **2.4, 2.5** Su20 **Exam**

Let $f(x) = \dfrac{3 + 7e^{2x}}{1 - e^x}$. Calculate each of the following limits.

(a) $\displaystyle\lim_{x\to-\infty} f(x)$ (b) $\displaystyle\lim_{x\to+\infty} f(x)$ (c) $\displaystyle\lim_{x\to 0^-} f(x)$

exsu20

Ex. E-6 **2.5** Fa20 **Exam**

Calculate all horizontal asymptotes of the function $h(x) = \dfrac{\sqrt{3x^2 + x + 10}}{2 - 5x}$.

exf20

Ex. E-7 **2.5** Fa20 **Exam**

Suppose the line $y = 3$ is a horizontal asymptote for f. Which of the following statements MUST be true? Select all that apply.

(a) $f(x) \neq 3$ for all x in the domain of f

(b) $f(3)$ is undefined

(c) $\displaystyle\lim_{x\to 3} f(x) = \infty$

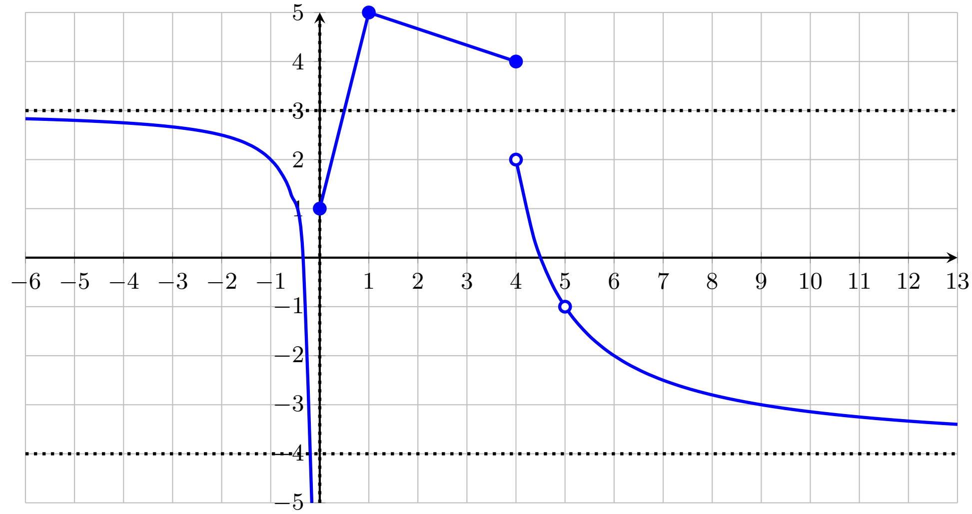
(d) $\displaystyle\lim_{x\to\infty} f(x) = 3$

(e) none of the above

exf20

Ex. E-8 **2.5, 2.6, 3.1/3.2** Sp21 **Exam**

Use the graph of f below to answer the following questions. Dashed lines indicate the location of asymptotes.



(a) Calculate $\lim_{x\to\infty} f(x)$.

(b) Calculate $\lim_{x\to-\infty} f(x)$.

(c) List the values of x where f is not continuous.

(d) List the values of x where f is not differentiable.

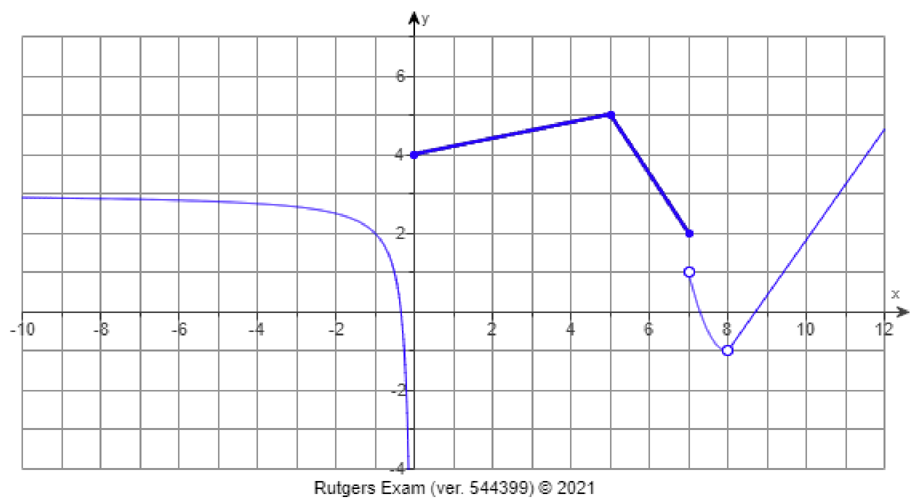
(e) What is the sign of $f'(-1)$? (choices: positive, negative, zero, does not exist)

(f) What is the sign of $f'(0.5)$? (choices: positive, negative, zero, does not exist)

exs21

Ex. B-6 **2.1/2.2, 2.3, 2.4, 2.5** Fa21 **Exam**

For each part, use the graph of $y = f(x)$.



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

exf21

Ex. E-9 **2.5** Fa21 **Exam**

Let $f(x) = \dfrac{8+6e^x}{9e^x - \pi^6}$.

(a) Evaluate $\lim\limits_{x\to\infty} f(x)$. (b) Evaluate $\lim\limits_{x\to-\infty} f(x)$. (c) List all vertical asymptotes of f.

exf21

Ex. D-13 **2.4, 2.5, 4.7** Fa21 **Exam**

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 1}\left(\dfrac{x^4 - x}{\ln(77x-76)}\right)$

(b) $\lim\limits_{x\to-\infty}\left(\dfrac{\sqrt{36x^2+63}}{31x}\right)$

(c) $\lim\limits_{x\to 2^+} f(x)$, with $f(x) = \begin{cases} 1+4x & x \le 2 \\ \dfrac{x^2-4}{x-2} & x > 2 \end{cases}$

(d) $\lim\limits_{x\to 5^-}\left(\dfrac{\cos(\pi x)}{x^2-25}\right)$

exf21

Ex. E-10 **2.5** Sp22 **Exam**

Find all horizontal asymptotes of the function $g(x) = \dfrac{2e^x - 15}{5e^{3x}+8}$.

exs22

Ex. D-15 **2.4, 2.5** Sp22 **Exam**

For the function f below, find its domain and all vertical and horizontal asymptotes.

$$f(x) = \frac{x^2 - 8x + 12}{3x^2 - 8x + 4}$$

exs22

Ex. D-16 **2.4, 2.5** Su22 **Exam**

Consider the function $f(x) = \dfrac{x^3 - 3x + 1}{x^2 - 2x + 1}$.

(a) Find all horizontal asymptotes of f, if any.

(b) Find all vertical asymptotes of f. Then calculate $\lim\limits_{x\to a^-} f(x)$ and $\lim\limits_{x\to a^+} f(x)$, where $x = a$ is the rightmost vertical asymptote of f.

exsu22

Ex. E-11 **2.5** Fa22 **Exam**

Find all horizontal asymptotes of the function $h(x) = \dfrac{6x+5}{\sqrt{4x^2-9}}$.

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exf22

Ex. D-18 **2.4, 2.5** Su22 **Quiz**

Calculate all of the vertical and horizontal asymptotes of $f(x) = \dfrac{x^2-100}{10x - x^2}$.

Then find the two one-sided at $x = a$, where $x = a$ is the leftmost vertical asymptote of f.

qsu22

Ex. E-12 **2.5** Su22 **Quiz**

Calculate the limit below.

$$\lim_{x\to-\infty}\left(\frac{2 - 3e^x + 4e^{-x}}{5 + 7e^x - 15e^{-x}}\right)$$

qsu22

Ex. E-13 2.5 Fa22 Quiz

Find all horizontal asymptotes of $g(x)$. You must justify your answers precisely.

$$g(x) = \frac{3e^{-2x} + 4e^{5x} - 10}{6e^{-9x} - 7e^{8x} + 1}$$

qf22

Ex. E-14 2.5

For each part, calculate the limit or show that it does not exist.

(a) $\displaystyle\lim_{x\to\infty} \left(\frac{3x-5}{x+1}\right)$

(b) $\displaystyle\lim_{x\to-\infty} \left(\frac{3x}{\sqrt{4x^2+9}}\right)$

(c) $\displaystyle\lim_{x\to\infty} \left(\frac{(x-3)(2x+4)(x-5)}{(3x+1)(4x-7)(x+2)}\right)$

(d) $\displaystyle\lim_{x\to-\infty} \left(\frac{(x-3)(2x+4)(x-5)}{(3x+1)(4x-7)(x+2)}\right)$

(e) $\displaystyle\lim_{x\to\infty} \cos\left(\frac{1}{x}\right)$

(f) $\displaystyle\lim_{x\to\infty} e^{-x^3}$

c2s5

Ex. E-15 2.5

For each function, find all horizontal asymptotes.

(a) $f(x) = \dfrac{(x-1)(2x+5)}{(x+1)(3x-6)}$

(b) $f(x) = \ln(x)$

(c) $f(x) = \dfrac{2e^x+3}{1-e^x}$

(d) $f(x) = e^{-1/x}$

c2s5

Ex. E-16 2.5

Find all horizontal asymptotes of $f(x) = \dfrac{\sqrt[4]{16x^4+7x+5}}{3x-8}$.

other

Ex. D-23 2.4, 2.5 ⋆Challenge

For each function, find all horizontal asymptotes and vertical asymptotes. Then, at each vertical asymptote, calculate both one-sided limits.

(a) $f(x) = \dfrac{4x^3+4x^2-8x}{x^3+3x^2-4}$

(b) $f(x) = \dfrac{4x^3-\sqrt{x^6+17}}{5x^3-40}$

challenge

Ex. E-17 2.5 ⋆Challenge

Find all horizontal asymptotes of $f(x) = \dfrac{2x}{x-\sqrt{x^2+10}}$.

challenge

§2.6: Continuity

Ex. F-1 **2.6** Fa17 **Exam**

Find the values of the constants a and b so that the following function is continuous for all x. If this is not possible, explain why.

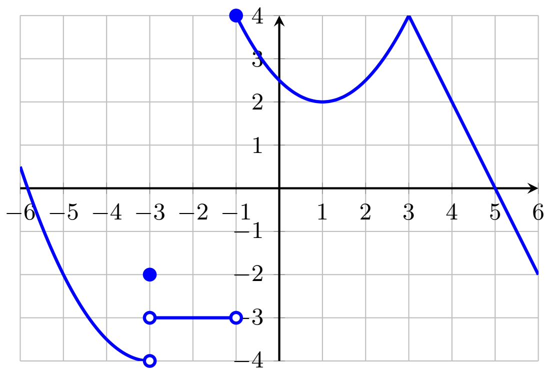
$$f(x) = \begin{cases} ax+b & \text{if } x<1 \\ -2 & \text{if } x=1 \\ 3\sqrt{x}+b & \text{if } x>1 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exf17

Ex. F-2 **2.6, 3.1/3.2** Sp18 **Exam**

For each part, use the graph of $y = f(x)$ below.



(a) Find where $f(x)$ is *not* continuous in the interval $(-5, 5)$.
(b) Find where $f(x)$ is *not* differentiable in the interval $(-5, 5)$.
(c) Find where $f'(x) = 0$ in the interval $(-5, 5)$.
(d) Find where $f'(x) < 0$ in the interval $(-5, 5)$.

exs18

Ex. F-3 **2.6** Sp18 **Exam**

Each part of this question refers to the function $f(x)$ below, where a and b are unspecified constants.

$$f(x) = \begin{cases} \dfrac{\sin(ax)}{x} & \text{if } x<0 \\ 2x+3 & \text{if } 0 \le x < 1 \\ b & \text{if } x=1 \\ \dfrac{x^2-1}{x-1} & \text{if } 1<x \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

(a) Find the value of a so that f is continuous at $x = 0$. If this is not possible, explain why.
(b) Find the value of b so that f is continuous at $x = 1$. If this is not possible, explain why.

exs18

Ex. F-4 **2.6** Fa18 **Exam**

Find the values of the constants a and b so that the following function is continuous at $x = 0$. If this is not possible,

exf18

2.6 Fa18 Exam

explain why.

$$f(x) = \begin{cases} \dfrac{4-\sqrt{16+49x^2}}{ax^2} & \text{if } x<0 \\ -23 & x=0 \\ \dfrac{\tan(2bx)}{x} & \text{if } x>0 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exf18

Ex. F-5 2.6 Sp19 Exam

Find the value of k that makes $f(x)$ continuous at $x=1$. If no such value of k exists, write "does not exist".

$$f(x) = \begin{cases} k\cos(\pi x) - 3x^2 & \text{if } x \le 1 \\ 8e^x - k\ln(x) & \text{if } x>1 \end{cases}$$

exs19

Ex. F-6 2.6 Sp19 Exam

Consider the function $f(x)$ below.

$$f(x) = \begin{cases} \dfrac{4-\sqrt{2x+10}}{x-3} & \text{if } x \ne 3 \\ 1 & \text{if } x=3 \end{cases}$$

Is $f(x)$ continuous at $x=3$? Explain your answer. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

exs19

Ex. F-7 2.6 Fa19 Exam

Find the values of a and b that make f continuous at $x=1$ or determine that no such values exist.

$$f(x) = \begin{cases} -3x+ax^2 & x<1 \\ b & x=1 \\ 4ax-1 & x>1 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exf19

Ex. F-8 2.6 Sp20 Exam

Determine where f is continuous. *Write your answer using interval notation.*

$$f(x) = \begin{cases} 9-16x & x<0 \\ 3x^2-x^3 & 0 \le x \le 3 \\ 1-e^{x-3} & x>3 \end{cases}$$

exs20

Ex. F-9 2.6 Sp20 Exam

Find the value of k that makes f continuous at $x=-2$ or determine that no such value of k exists.

$$f(x) = \begin{cases} 3x^2+k & x<-2 \\ -10 & x=-2 \\ kx^3-6 & x>-2 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exs20

Ex. F-10 **2.6** Sp20 **Exam**

Consider the function $f(x)$, where k is an unspecified constant. Find the value of k for which f continuous for all x, or show that no such value of k exists.

$$f(x) = \begin{cases} 38 + kx & x < 3 \\ kx^2 + x - k & x \geq 3 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exs20

Ex. F-11 **2.6** Su20 **Exam**

In a certain parking garage, the cost of parking is \$20 per hour or any fraction thereof. For example, if you are in the garage for two hours and fifteen minutes, you pay \$60 (\$20 for the first hour, \$20 for the second hour, and \$20 for the fifteen-minute portion of the third hour). Let $P(t)$ be the cost of parking for t hours, where t is any non-negative real number. For example, $P(2.25) = 60$. Is the following true or false?

> "$P(t)$ is a continuous function of t."

You must justify your answer.

exsu20

Ex. F-12 **2.6** Su20 **Exam**

Consider the following function, where a and b are unspecified constants.

$$f(x) = \begin{cases} 3 & x \leq -1 \\ ax^2 + 2x + b & -1 < x \leq 2 \\ 14 - ax & x > 2 \end{cases}$$

Find the values of a and b for which f is continuous for all x, or determine that no such values exist. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

exsu20

Ex. D-7 **2.4, 2.6** Su20 **Exam**

Let $f(x) = \dfrac{9x - x^3}{x^2 + x - 6}$.

(a) Calculate all vertical asymptotes of f. Justify your answer.

(b) Where is f discontinuous?

(c) For each point at which f is discontinuous, determine what value should be reassigned to f, if possible, to guarantee that f will be continuous there.

exsu20

Ex. F-13 **2.6** Fa20 **Exam**

Determine where the following function is continuous. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & x < 3 \\ 0 & x = 3 \\ 5x - 9 & 3 < x < 4 \\ 11 & x = 4 \\ 27 - x^2 & x > 4 \end{cases}$$

exf20

Ex. F-14 **2.6** Fa20 **Exam**

Consider the function f below, where A, B, and C are unspecified constants.

exf20

2.6 Fa20 Exam

$$f(x) = \begin{cases} 2x^3 + Ax & x < -1 \\ C & x = -1 \\ Bx^2 + 4 & x > -1 \end{cases}$$

(a) Calculate $\lim_{x \to -1^-} f(x)$.

(b) Calculate $\lim_{x \to -1^+} f(x)$.

(c) How must A and B be related if $\lim_{x \to -1} f(x)$ exists?

(d) Suppose $C = 10$ and f is continuous for all x. Find the values of A and B.

exf20

Ex. F-15 2.6 Fa20 Exam

Which of the following equations expresses the fact that $f(x)$ is continuous at $x = 6$. (There is only one correct choice.)

(a) $\lim_{x \to 6} f(6) = f(6)$

(b) $\lim_{x \to 6} f(6) = 6$

(c) $\lim_{x \to 6} f(x) = f(6)$

(d) $\lim_{x \to 6} f(x) = 6$

(e) $\lim_{x \to 6} f(x) = 0$

(f) $\lim_{x \to 6} f(x) = \infty$

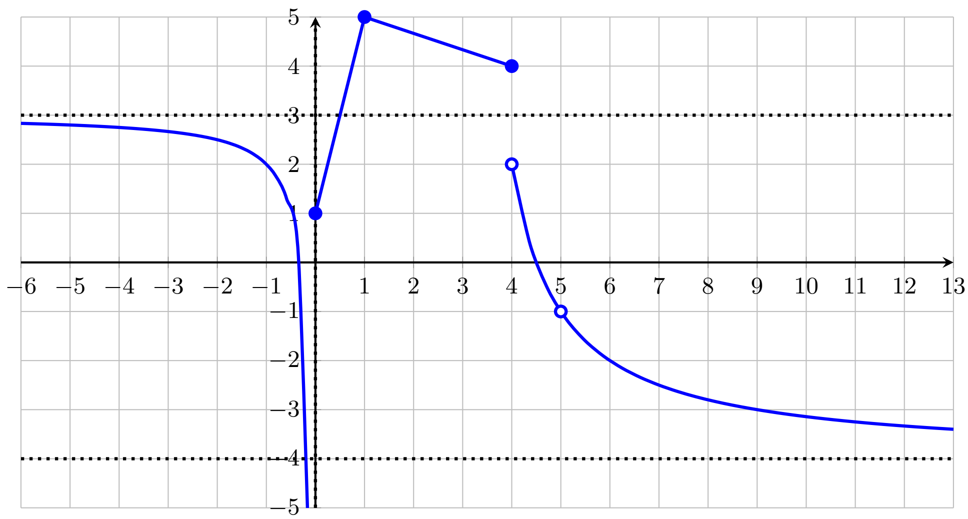
(g) $\lim_{x \to \infty} f(x) = f(6)$

(h) $\lim_{x \to \infty} f(x) = \infty$

exf20

Ex. E-8 2.5, 2.6, 3.1/3.2 Sp21 Exam

Use the graph of f below to answer the following questions. Dashed lines indicate the location of asymptotes.



(a) Calculate $\lim_{x \to \infty} f(x)$.

(b) Calculate $\lim_{x \to -\infty} f(x)$.

(c) List the values of x where f is not continuous.

(d) List the values of x where f is not differentiable.

(e) What is the sign of $f'(-1)$? (choices: positive, negative, zero, does not exist)

(f) What is the sign of $f'(0.5)$? (choices: positive, negative, zero, does not exist)

exs21

Ex. F-16 2.6 Sp21 Exam

Consider the function g below, where a and b are unspecified constants. Assume that g is continuous for all x.

$$g(x) = \begin{cases} be^x + a + 1 & x \le 0 \\ ax^2 + b(x+3) & 0 < x \le 1 \\ a\cos(\pi x) + 7bx & 1 < x \end{cases}$$

exs21

2.6 Sp21 Exam

(a) What relation must hold between a and b for g to be continuous at $x = 0$? Your answer should be an equation involving a and b.

(b) What relation must hold between a and b for g to be continuous at $x = 1$? Your answer should be an equation involving a and b.

(c) Calculate the values of a and b.

exs21

Ex. F-17 2.6 Fa21 Exam

Consider the piecewise-defined function $f(x)$ below; A and B are unspecified constants and $g(x)$ is an unspecified function with domain $[94, \infty)$.

$$f(x) = \begin{cases} Ax^2 + 8 & x < 75 \\ \ln(B) + 6 & x = 75 \\ \dfrac{x - 75}{\sqrt{x+6} - 9} & 75 < x < 94 \\ 19 & x = 94 \\ g(x) & x > 94 \end{cases}$$

(a) Find $\lim\limits_{x \to 75^-} f(x)$ in terms of A and B.

(b) Find $\lim\limits_{x \to 75^+} f(x)$ in terms of A and B.

(c) Find the exact values of A and B for which f is continuous at $x = 75$.

(d) Suppose $g(94) = 19$. What does this imply about $\lim\limits_{x \to 94} f(x)$? Select the best answer.

(i) $\lim\limits_{x \to 94} f(x)$ exists.

(ii) $\lim\limits_{x \to 94} f(x)$ does not exist.

(iii) It gives no information about $\lim\limits_{x \to 94} f(x)$.

exf21

Ex. F-18 2.6 Fa21 Exam

Consider the following function.

$$f(x) = \frac{x^2 - x - 6}{x^3 - 2x^2 - 3x}$$

(a) Where is f discontinuous?

(b) At the leftmost x-value where f is discontinuous, what type of discontinuity does f have (removable, jump, infinite (vertical asymptote), or other)?

(c) At the rightmost x-value where f is discontinuous, what type of discontinuity does f have (removable, jump, infinite (vertical asymptote), or other)?

exf21

Ex. F-19 2.6 Fa21 Exam

Let $f(x)$ be the following function, where k is an unspecified constant. Find the value of k that makes f continuous at $x = 2$ or determine that no such value of k exists.

$$f(x) = \begin{cases} 27x - kx^2 & x < 2 \\ -6 & x = 2 \\ 3x^3 + k & x > 2 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exf21

Ex. F-20 2.6 Sp22 Exam

Determine where $f(x)$ is continuous. *In your work, you must use limit-based methods to solve this problem. Solutions*

exs22

2.6 Sp22 Exam

that have work that is not based on limits will not receive full credit.

$$f(x) = \begin{cases} \dfrac{(x+1)^2-16}{2x-6} & \text{if } x < 3 \\ \\ 3 - \ln(x-2) & \text{if } x \geq 3 \end{cases}$$

exs22

Ex. F-21 2.6 Sp22 Exam

Consider the function $f(x)$ defined below, where A and B are unspecified constants. Find the values of A and B for which f is continuous at $x = 2$, or determine that no such values exist.

$$f(x) = \begin{cases} Ax + B - 4 & \text{if } x < 2 \\ 9 & \text{if } x = 2 \\ Ax^2 - 5 & \text{if } x > 2 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exs22

Ex. F-22 2.6 Su22 Exam

Consider the function $f(x) = \dfrac{\sin(7x)}{x^2 - 5x}$.

(a) Find the domain of f. Write your answer using interval notation.

(b) Find the x-values where f is discontinuous.

(c) For each value of x where f is discontinuous, classify the type of discontinuity as "removable", "jump", "infinite", or "essential". Clearly label your work and justify your answers.

exsu22

Ex. F-23 2.6 Su22 Exam

Consider the limit $\displaystyle\lim_{x\to 3} \left(\frac{x^3 - 4x^2 + ax}{x^2 - 9}\right)$, where a is an unspecified constant.

(a) For what values of a does this limit exist? Explain your answer.

(b) Given that the limit does exist, what is its value?

exsu22

Ex. F-24 2.6 Su22 Exam

Consider the function below, where a and b are unspecified constants. Find the values of a and b for which f is continuous for all x, or determine that no such values exist.

$$f(x) = \begin{cases} ax^2 + 3x + b & x < -1 \\ 2 + ax + \sin\left(\dfrac{\pi x}{2}\right) & -1 \leq x < 4 \\ b(x-3)^2 + 1 & x \geq 4 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

exsu22

Ex. F-25 2.6 Fa22 Exam

On the axes provided, sketch the graph of a function $f(x)$ that satisfies all of the following properties. ***Note:*** Make sure to read these properties carefully!

- the domain of $f(x)$ is $[-10, 7) \cup (7, 10]$
- $\displaystyle\lim_{x\to -8} f(x)$ exists but f is discontinuous at $x = -8$

exf22

2.6 Fa22 Exam

- $\lim_{x \to -5^+} f(x) = f(-5)$ but $\lim_{x \to -5} f(x)$ does not exist
- $\lim_{x \to 2^-} f(x) = 4$ and f is continuous at $x = 2$
- the line $x = 5$ is a vertical asymptote for f (***Note:*** $x = 5$ is in the domain of f.)
- $\lim_{x \to 7} f(x) = +\infty$ (***Note:*** $x = 7$ is not in the domain of f.)

exf22

Ex. F-26 2.6 Fa22 Exam

Consider the function below, where a and b are unspecified constants.

$$f(x) = \begin{cases} \dfrac{\sin(4x)\sin(6x)}{x^2} & x < 0 \\ ax + b & 0 \le x \le 1 \\ \dfrac{5x+2}{x-1} - \dfrac{2x+5}{x^2-x} & x > 1 \end{cases}$$

(a) Calculate $\lim_{x \to 0^-} f(x)$.

(b) Calculate $\lim_{x \to 1^+} f(x)$.

(c) Find the values of a and b for which f is continuous for all x, or determine that no such values exist. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

exf22

Ex. F-27 2.6 Sp18 Quiz

Consider the following function.

$$f(x) = \begin{cases} x^3 + 27 & \text{if } x \le -3 \\ \dfrac{x+3}{2 - \sqrt{1-x}} & \text{if } -3 < x < 1 \\ 4 & \text{if } x = 1 \\ x^2 + 2x - 1 & \text{if } 1 < x \end{cases}$$

(a) Find all points where f is discontinuous. *Be sure to give a full justification here.*

(b) For each x-value you found in part (a), determine what value should be assigned to f, if any, to guarantee that f will be continuous there. Justify your answer.

(For example, if you claim f is discontinuous at $x = a$, then you should determine the value that should be assigned to $f(a)$, if any, to guarantee that f will be continuous at $x = a$.)

qs18

Ex. F-28 2.6 Sp20 Quiz

Find the values of a and b for which f is continuous for all x, or show that no such values of a and b exist. You must use proper calculus methods and clearly explain your work using limits.

$$f(x) = \begin{cases} ax^2 - bx - 6 & \text{if } x < 3 \\ b & \text{if } x = 3 \\ 10x - x^3 & \text{if } x > 3 \end{cases}$$

qs20

Ex. F-29 2.6 Sp20 Quiz

Determine where $f(x)$ is continuous. Write your answer using interval notation.

$$f(x) = \begin{cases} 4x^2 - 10 & \text{if } x < -1 \\ 6\sin\left(\frac{\pi x}{2}\right) & \text{if } -1 \leq x \leq 4 \\ x - 4^{x-3} & \text{if } x > 4 \end{cases}$$

qs20

Ex. F-30 2.6 Su22 Quiz

Consider the function $f(x)$ below, where a and b are unspecified constants.

$$f(x) = \begin{cases} ax^2 - 7x + b & x < 2 \\ 10 & x = 2 \\ ae^{x-2} + b\ln(x-1) & x > 2 \end{cases}$$

Find the values of a and b for which f is continuous for all x, or determine that no such values exist. Write "NONE" in the answer boxes if no such values exist.

In your work, you must use proper notation and limit-based methods to solve this problem. Solutions that have work that does not have proper notation or which is not based on limits will not receive full credit.

qsu22

Ex. F-31 2.6 Su22 Quiz

Let $f(x) = \dfrac{x^3 - 7x^2 + 10x}{x^2 - 6x}$.

(a) Find the domain of f. Write your answer using interval notation.

(b) Find all values of x where f is discontinuous.

(c) For each value of x where f is discontinuous, classify the type of discontinuity as "removable", "jump", "infinite", or "essential". Clearly label your work and justify your answers.

qsu22

Ex. F-32 2.6 Fa22 Quiz

Find the value of A that makes $f(x)$ continuous for all x, or determine that no such value exists. Write "DNE" if no such value of A exists. Your solution must be based on limits to receive full credit.

$$f(x) = \begin{cases} \dfrac{\sin(Ax)}{x} - 2 & \text{if } x < 0 \\ 9 & \text{if } x = 0 \\ 3x^3 - A\cos(x) + 10 & \text{if } x > 0 \end{cases}$$

qf22

Ex. F-33 2.6

Determine where $f(x)$ is continuous.

$$f(x) = \begin{cases} 3x^2 - x + 1 & \text{if } x < -2 \\ 15 + \sin(2\pi x) & \text{if } -2 \leq x < 3 \\ 2x - 4 & \text{if } 3 \leq x \end{cases}$$

c2s6

Ex. F-34 2.6

Let $f(x) = \dfrac{x^3 - 9x}{x + 3}$.

(a) What is the domain of f?

(b) Find all points where f is discontinuous.

c2s6

2.6

(c) For each point where f is discontinuous, classify the type of discontinuity as removable, jump, infinite, or other.

c2s6

Ex. F-35 2.6

Let $f(x) = \dfrac{\sqrt{2x^2+1}-1}{x^2(x-3)}$.

(a) What is the domain of f?

(b) Find all points where f is discontinuous.

(c) For each point where f is discontinuous, classify the type of discontinuity as removable, jump, infinite, or other.

c2s6

Ex. F-36 2.6

Find the values of the constants a and b that make f continuous for all real numbers.

$$f(x) = \begin{cases} ax^2 - x & \text{if } x < 4 \\ 6 & \text{if } x = 4 \\ x^3 + bx & \text{if } x > 4 \end{cases}$$

c2s6

Ex. F-37 2.6

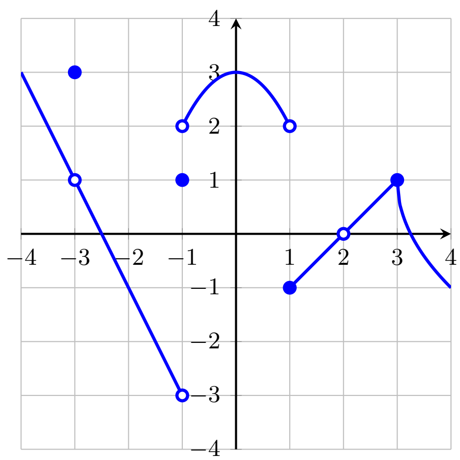
Find the values of the constants a and b that make f continuous for all real numbers.

$$f(x) = \begin{cases} ax + 2b & \text{if } x \le 0 \\ x^2 + 3a - b & \text{if } 0 < x \le 2 \\ 3x - 5 & \text{if } x > 2 \end{cases}$$

c2s6

Ex. F-38 2.6

The figure below shows the graph of $y = f(x)$. Find all values of x in the interval $(-4, 4)$ at which f is not continuous.



unit2

Ex. F-39 2.6

Find the values of the constants a and b that make f continuous at $x = 9$.

$$f(x) = \begin{cases} \sin(2\pi x) - 2ax & \text{if } x < 9 \\ b & \text{if } x = 9 \\ \dfrac{x-9}{\sqrt{x}-3} & \text{if } x > 9 \end{cases}$$

unit2

Ex. F-40 2.6

Consider the function $f(x)$, where a and b are unspecified constants.

$$f(x) = \begin{cases} \dfrac{2x}{\sin(ax)} & \text{if } x < 0 \\ x - 4 & \text{if } 0 \leq x < 5 \\ b & \text{if } x = 5 \\ \dfrac{4 - \sqrt{3x+1}}{x-5} & \text{if } x > 5 \end{cases}$$

(a) Find the value of a so that f is continuous at $x = 0$, or show that no such value exists.

(b) Find the value of b so that f is continuous at $x = 5$, or show that no such value exists.

other

Ex. F-41 2.6

Consider the function

$$f(x) = \begin{cases} ax^2 - 3b & \text{if } x \leq -1 \\ \cos(\pi x) + ax & \text{if } -1 < x < 2 \\ 2b - x^3 & \text{if } x \geq 2 \end{cases}$$

where a and b are unspecified constants. For what values of a and b, if any, is f continuous for all x?

other

Ex. F-42 2.6 ⋆Challenge

Consider $f(x) = \dfrac{\tan(2x)}{|5x|}$.

(a) Where is f not continuous?

(b) Is it possible to redefine f at $x = 0$ to make f continuous there? Explain your answer.

Hint: For the limit of f as $x \to 0$, examine the one-sided limits first.

challenge

Ex. F-43 2.6 ⋆Challenge

Find the values of the constants a and b that make f continuous at $x = 0$. *You may assume* $a > 0$.

$$f(x) = \begin{cases} \dfrac{1 - \cos(ax)}{x^2} & , \quad x < 0 \\ 2a + b & , \quad x = 0 \\ \dfrac{x^2 - bx}{\sin(x)} & , \quad x > 0 \end{cases}$$

challenge

3 Chapter 3: Derivatives

§3.1, 3.2: Introduction to the Derivative

Ex. G-1 **3.1/3.2** Sp18 **Exam**

The parts of this question are independent of each other.

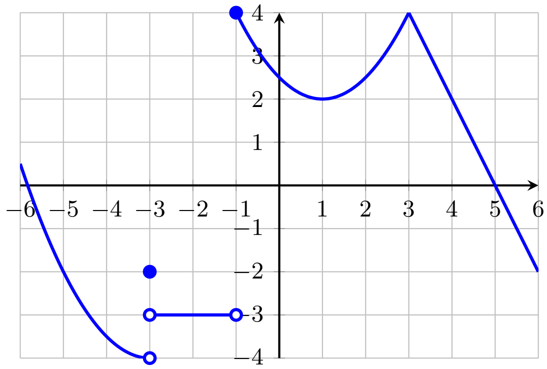
(a) Given the function $g(x)$, state the definition of $g'(4)$.

(b) Let $F(x) = \dfrac{1}{3x-5}$. Calculate $F'(2)$ directly from the definition. Show all work. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

exs18

Ex. F-2 **2.6, 3.1/3.2** Sp18 **Exam**

For each part, use the graph of $y = f(x)$ below.



(a) Find where $f(x)$ is *not* continuous in the interval $(-5, 5)$.

(b) Find where $f(x)$ is *not* differentiable in the interval $(-5, 5)$.

(c) Find where $f'(x) = 0$ in the interval $(-5, 5)$.

(d) Find where $f'(x) < 0$ in the interval $(-5, 5)$.

exs18

Ex. G-2 **3.1/3.2** Sp18 **Exam**

Find an equation of each line that is both tangent to the graph of $f(x) = 4x^2 - 3x - 1$ and parallel to the line $y = 13x - 5$.

exs18

Ex. G-3 **3.1/3.2** Sp19 **Exam**

Let $g(x) = 6 - \dfrac{9}{x}$. Calculate $g'(3)$ directly from the limit definition of the derivative. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

exs19

Ex. G-4 **3.1/3.2** Sp20 **Exam**

Let $f(x) = \dfrac{x+8}{x-3}$. Use the limit definition of derivative to calculate $f'(2)$. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

exs20

Ex. G-5 **3.1/3.2** Sp20 **Exam**

Which statement is true about the graph of $f(x) = |x| + 91$ at the point $(0, 91)$?

(a) The graph has a tangent line at $y = 91$.

(b) The graph has infinitely many tangent lines.

(c) The graph has no tangent line.

(d) The graph has two tangent lines: $y = x + 91$ and $y = -x + 91$.

(e) None of the above statements is true.

exs20

Ex. G-6 **3.1/3.2, 4.1, 4.9** Sp20 **Exam**

Suppose the derivative of f is $f'(x) = 3x^2 - 6x - 9$ and that $f(1) = 10$.

(a) Find an equation of the line tangent to the graph of $y = f(x)$ at $x = 1$.
(b) Find the critical points of f.
(c) Where does f have a local minimum value? local maximum value?
(d) Calculate $f(0)$.
(e) Calculate the absolute maximum value of f on the interval $[0, 6]$. At what x-value does it occur?

exs20

Ex. G-7 **3.1/3.2** Su20 **Exam**

Explain the relationship between $f'(3)$ and the line tangent to the graph of $y = f(x)$ at $x = 3$.

exsu20

Ex. G-8 **3.1/3.2** Su20 **Exam**

Suppose $f'(7)$ exists. What can be said about the limit $\lim_{x\to 7} f(x)$?

exsu20

Ex. G-9 **3.1/3.2** Su20 **Exam**

Consider the following limit.

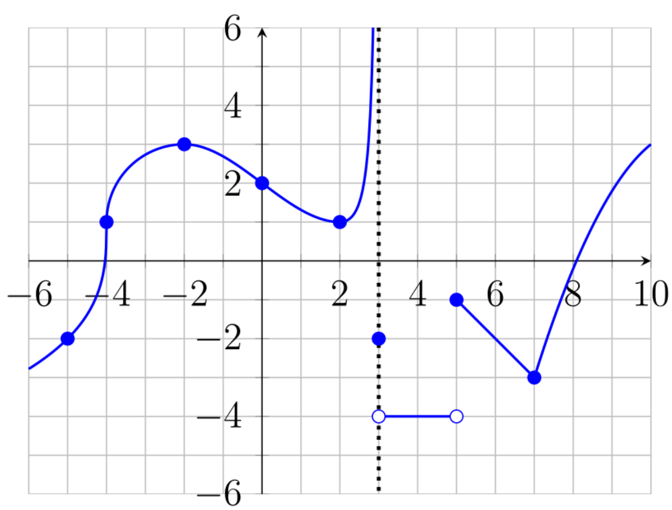
$$\lim_{h\to 0}\left(\frac{(4+h)^{3/2} - 8}{h}\right)$$

Use the limit definition of derivative to identify this limit as the derivative of some function $f(x)$ at the point $x = a$. Then calculate the value of the limit.

exsu20

Ex. G-10 **3.1/3.2** Su20 **Exam**

Use the graph of $y = f(x)$ below to answer the following questions.



(a) In the interval $(-6, 10)$, where is f not differentiable?
(b) Calculate a reasonable estimate of $f'(0)$. Explain your reasoning.
(c) In the interval $(-6, 10)$, where is $f'(x) = 0$?
(d) In the interval $(-6, 10)$, where is $f'(x) < 0$?
(e) In the interval $(-6, 10)$, where is $f'(x) > 0$?

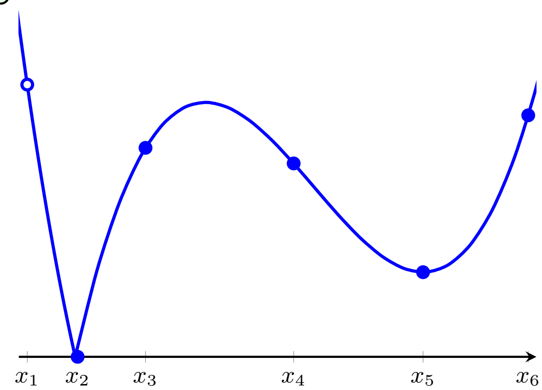
exsu20

Ex. G-11 **3.1/3.2** Fa20 **Exam**

Consider the graph of $y = f(x)$ below.

exf20

3.1/3.2 Fa20 **Exam**



(a) For which values of x is $f'(x) \geq 0$? Choose from x_1, x_2, x_3, x_4, x_5, and x_6. Select all that apply.

(b) For which values of x does $f'(x)$ not exist? Choose from x_1, x_2, x_3, x_4, x_5, and x_6. Select all that apply.

(c) Give a brief, one-sentence explanation of your answer to part (b).

exf20

Ex. G-12 **3.1/3.2** Fa20 **Exam**

Consider the following limit.

$$\lim_{x \to \frac{\pi}{8}} \left(\frac{\tan(2x) - 1}{x - \frac{\pi}{8}} \right)$$

(a) Use the limit definition of derivative to identify this limit as the derivative of some function $f(x)$ at the point $x = a$. You must explicitly identify f and a.

(b) Use your identifications in part (a) to calculate the given limit. Show all work.

exf20

Ex. G-13 **3.1/3.2** Sp21 **Exam**

The following limit represents the derivative of a function f at a point a.

$$f'(a) = \lim_{h \to 0} \left(\frac{5\ln(e^4 + h) - 20}{h} \right)$$

(a) Find a possible function $f(x)$.

(b) For your choice of f in part (a), find a possible value of a.

(c) Calculate the value of the limit. Explain your calculation briefly in one sentence.

exs21

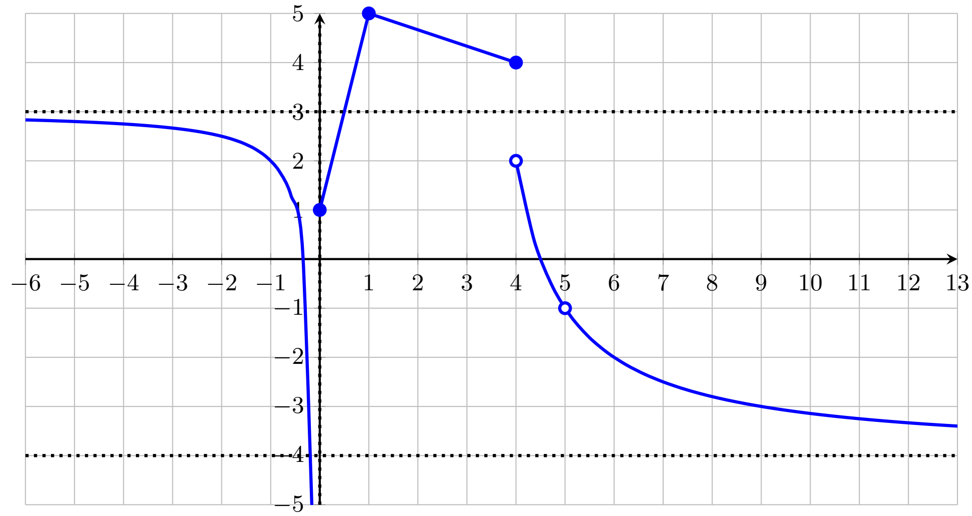
Ex. E-8 **2.5, 2.6, 3.1/3.2** Sp21 **Exam**

Use the graph of f below to answer the following questions. Dashed lines indicate the location of asymptotes.

exs21

2.5, 2.6, 3.1/3.2

Sp21 **Exam**



(a) Calculate $\lim_{x\to\infty} f(x)$.

(b) Calculate $\lim_{x\to-\infty} f(x)$.

(c) List the values of x where f is not continuous.

(d) List the values of x where f is not differentiable.

(e) What is the sign of $f'(-1)$? (choices: positive, negative, zero, does not exist)

(f) What is the sign of $f'(0.5)$? (choices: positive, negative, zero, does not exist)

exs21

Ex. G-14 **3.1/3.2, 3.3/3.4/3.5/3.9**

Fa21 **Exam**

The following limit represents the derivative of a function f at a point a.

$$f'(a) = \lim_{h\to 0}\left(\frac{9\tan\left(\frac{\pi}{6}+h\right) - \frac{9}{\sqrt{3}}}{h}\right)$$

(a) Find a possible pair for f and a.

(b) Calculate the value of the limit.

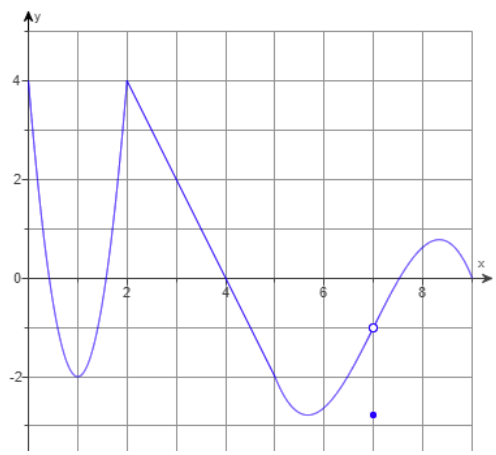
exf21

Ex. G-15 **3.1/3.2**

Fa21 **Exam**

For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

(a) $f'(1)$ (b) $f'(2)$ (c) $f'(3.5)$ (d) $f'(7)$



exf21

Ex. G-16 **3.1/3.2** Fa21 **Exam**

Let $f(x)$ and $g(x)$ be functions such that $f'(-8) = g'(-8)$ and the line tangent to the graph of f at $x = -8$ is $y = -7x + 6$. For each part, compute the desired value, if possible.

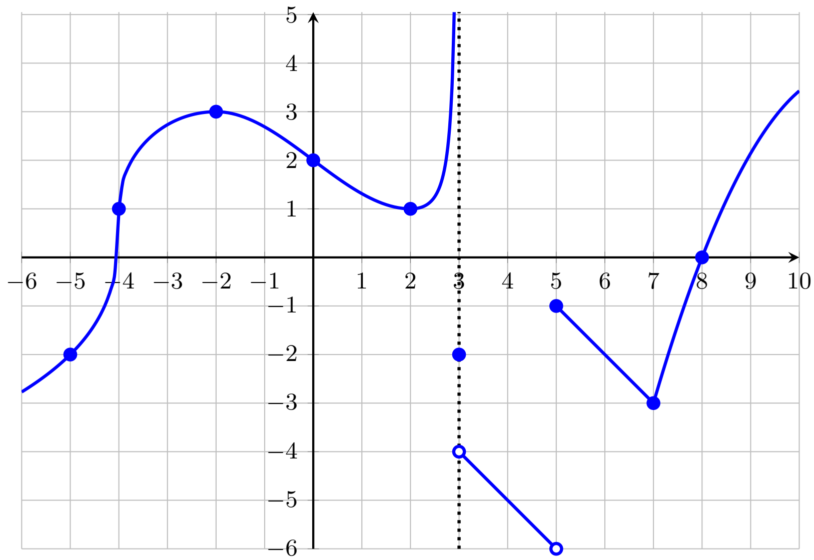
(a) $f(-8)$ (b) $f'(-8)$ (c) $g(-8)$ (d) $g'(-8)$

exf21

Ex. G-17 **3.1/3.2** Sp22 **Exam**

For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

(a) $f'(-4)$ (b) $f'(-2)$ (c) $f'(0)$ (d) $f'(5)$ (e) $f'(8)$



exs22

Ex. G-18 **3.1/3.2** Sp22 **Exam**

For both parts below, $f(x) = \sqrt{2x+1}$.

(a) Use the limit definition of the derivative to calculate $f'(4)$.

(b) Find an equation for the line tangent to the graph of $y = f(x)$ at $x = 4$.

exs22

Ex. G-19 **3.1/3.2** Su22 **Exam**

Find the x-coordinate of each point on the graph of $y = 6x^3 - 9x^2 - 16x + 5$ at which the tangent line is perpendicular to the line $x + 20y = 10$.

exsu22

Ex. G-20 **3.1/3.2, 3.3/3.4/3.5/3.9** Su22 **Exam**

Suppose that an equation to the tangent line to $y = f(x)$ at $x = 9$ is $y = 3x - 20$. Let $g(x) = xf(x^2)$.

(a) Calculate $f(9)$ and $f'(9)$. Explain.

(b) Calculate $g'(x)$.

(c) Find the tangent line to $y = g(x)$ at $x = -3$.

exsu22

Ex. G-21 **3.1/3.2** Su22 **Exam**

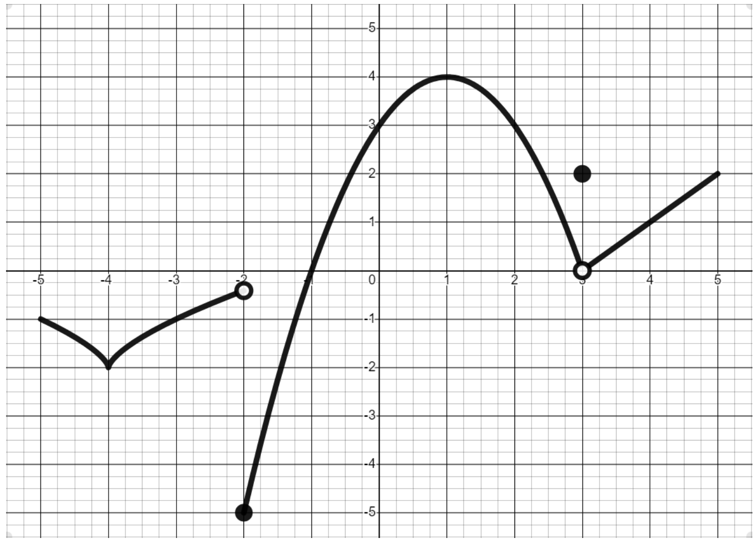
Let $f(x) = \dfrac{4}{x-6} + 3$. Use the limit definition of derivative to calculate $f'(8)$. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

exsu22

Ex. G-22 **3.1/3.2** Fa22 **Exam**

For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

(a) $f'(-3)$
(b) $f'(-2)$
(c) $f'(-1)$
(d) $f'(1)$
(e) $f'(3)$



exf22

Ex. G-23 **3.1/3.2** Fa22 **Exam**

Let $f(x) = \dfrac{8x}{x+5}$.

(a) Calculate $f'(x)$ by any method.
(b) Use the limit definition of derivative to calculate $f'(3)$. ***Hint:*** Use your answer from part (a) to check your final answer.

exf22

Ex. G-24 **3.1/3.2** Sp18 **Quiz**

Let $f(x) = \dfrac{3-x}{1+x}$. Use the limit definition of derivative to calculate $f'(1)$.

If you simply quote a rule, you will receive zero credit. You must use the definition of derivative.

qs18

Ex. G-25 **3.1/3.2** Sp20 **Quiz**

Let $f(x) = x^{-1} - 3x^{-2}$. Use the limit definition of the derivative to calculate $f'(1)$. *(If you simply use shortcut rules, you will receive no credit.)*

qs20

Ex. G-26 **3.1/3.2** Sp20 **Quiz**

Let $f(x) = 3x^2 - 5$. Use the limit definition of derivative to find $f'(x)$.

qs20

Ex. G-27 **3.1/3.2** Su22 **Quiz**

Let $f(x) = \dfrac{x^2-3}{x-1}$. Use the limit definition of derivative to calculate $f'(2)$.

qsu22

Ex. G-28 **3.1/3.2** Su22 **Quiz**

Use the limit definition of derivative to find an equation of the tangent line to $f(x) = 2x^2 + x + 5$ at $x = -1$.

qsu22

Ex. G-29 **3.1/3.2** Fa22 **Quiz**

The limit below is equal to the derivative of some function $f(x)$ at some point $x = a$. Identify both the function f

qf22

3.1/3.2 Fa22 Quiz

and the value of a. No work is required.

$$f'(a) = \lim_{h\to 0}\left(\frac{\frac{1}{(3+h)^2+1} - \frac{1}{10}}{h}\right)$$

qf22

Ex. G-30 3.1/3.2 Fa22 Quiz

Let $f(x) = 2x^2 - 6x + 10$.

(a) Use the limit definition of derivative to calculate $f'(-1)$.

(b) Find the tangent line to $y = f(x)$ at $x = -1$.

qf22

Ex. G-31 3.1/3.2

Suppose the line described by $y = 5x - 9$ is tangent to the graph of $y = f(x)$ at $x = 4$. For each part, calculate the value or explain why there is not enough information to do so.

Note: The function $f(x)$ is unknown. You can't assume that $f(x) = 5x - 9$.

(a) $f(4)$ (b) $f(3)$ (c) $f'(4)$ (d) $f'(3)$

c3s1

Ex. G-32 3.1/3.2

For each part, use the limit definition of the derivative to calculate the derivative of f at $x = 5$. Then find an equation for the line tangent to the graph of $y = f(x)$ at $x = 5$.

(a) $f(x) = 2x - 1$

(b) $f(x) = (2x - 1)^2$

(c) $f(x) = \sqrt{2x - 1}$

(d) $f(x) = \dfrac{1}{2x - 1}$

(e) $f(x) = \dfrac{1}{\sqrt{2x - 1}}$

c3s1

Ex. G-33 3.1/3.2

Let $f(x) = 3\sqrt{x}$. Use the limit definition of the derivative to find $f'(x)$. Show all work.

unit3

Ex. G-34 3.1/3.2

Let $f(x) = \dfrac{x+2}{x-3}$. Use the limit definition of derivative to find $f'(2)$.

unit3

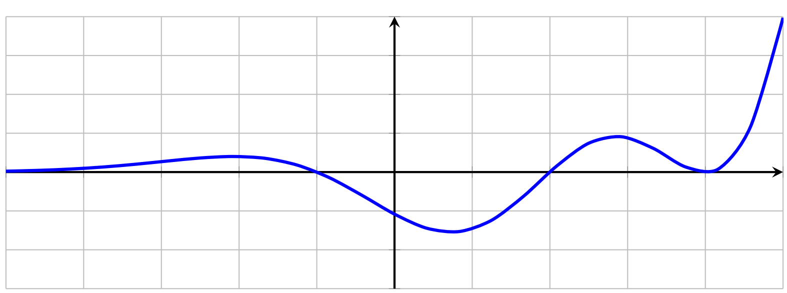
Ex. G-35 3.1/3.2

Let $f(x) = \dfrac{3x + 12}{x^2 - 1}$. Calculate $f'(2)$ directly from the definition of the derivative. You are not allowed to use any shortcut rules.

other

Ex. G-36 3.1/3.2 ⋆Challenge

The graph of $y = f(x)$ is given below. Sketch a graph of $y = f'(x)$. *Only the general shape is important. The graph does not have to be to scale.*



challenge

Ex. G-37 **3.1/3.2** **⋆Challenge**

Consider the following function, where c is an unspecified constant

$$f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2 + 2x & \text{if } 0 \leq x < 1 \\ 6x - x^2 + c & \text{if } x \geq 1 \end{cases}$$

(a) Show precisely that $f'(0)$ does not exist.

(b) Find the value of c that makes f differentiable at $x = 1$ or show that no such value exists.

challenge

Ex. G-38 **3.1/3.2** **⋆Challenge**

Use the limit definition of derivative to find the derivative of $f(x) = x^{2/3}$.

challenge

§3.3, 3.4, 3.5, 3.9: Rules for Computing Derivatives

Ex. H-1 3.3/3.4/3.5/3.9, 3.7 Sp18 Exam

For each part, calculate $f'(x)$. *After calculating the derivative, do not simplify your answer.*

(a) $f(x) = \dfrac{x^{-1}x^{8/3}}{4\sqrt[3]{x^2}}$ (b) $f(x) = \left(x + \sqrt{5x-6}\right)^{1/4}$ (c) $f(x) = \dfrac{x^2 e^x}{\ln(x) - \cos(x)}$

exs18

Ex. H-2 3.3/3.4/3.5/3.9 Fa18 Exam

Calculate $f'(x)$ where f is the function below.

$$f(x) = \left(\frac{x^8 \sin(3x)}{\ln(x) - \ln(11)}\right)^{2/3}$$

After calculating the derivative, do not simplify your answer.

exf18

Ex. H-3 3.3/3.4/3.5/3.9, 3.7 Fa18 Exam

Suppose f and g are differentiable for all x. For each part, use the table below or explain why there is not enough information.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-1	-4	4	2
1	-1	-3	2	-4
2	-4	3	1	-1

(a) Let $F(x) = \dfrac{f(x)}{g(x)}$. Calculate $F'(0)$. (b) Let $G(x) = f\Big(xg(x)\Big)$. Calculate $G'(1)$.

exf18

Ex. H-4 3.3/3.4/3.5/3.9 Sp19 Exam

For each part, calculate $f'(x)$. *Do not simplify your answers.*

(a) $f(x) = e^x \sin(x)$ (b) $f(x) = \dfrac{\ln\left(e^{4x}+6\right)}{9\tan(x) - \pi^9}$

exs19

Ex. H-5 3.3/3.4/3.5/3.9 Sp19 Exam

Find the slope of the line tangent to the graph of $y = 3\ln(x) - 6\sqrt{x}$ at $x = 3$.

exs19

Ex. H-6 3.3/3.4/3.5/3.9 Fa19 Exam

For each part, calculate $f'(x)$. *Do not simplify your answers.*

(a) $f(x) = \dfrac{\ln(x)}{10 - x^3}$ (b) $f(x) = \sqrt{\cos(3+x^5)}$

exf19

Ex. H-7 3.3/3.4/3.5/3.9 Fa19 Exam

Find all points on the graph of $f(x) = x\ln(x)$ where the tangent line is horizontal.

exf19

Ex. H-8 3.3/3.4/3.5/3.9 Sp20 Exam

For each part, calculate $f'(x)$. *Do not simplify your answers.*

(a) $f(x) = 2x^2 - \dfrac{1}{5x} - 8\sqrt{x} + 14\pi^{3/2}$

(b) $f(x) = \left(\dfrac{x^4 - 20x}{x^3+20}\right)^{2/3}$

(c) $f(x) = \sin\left(12x - x^9\right)\ln(x)$

(d) $f(x) = \dfrac{e^{5\sec(6x)+1}}{7}$

exs20

Ex. H-9 **3.3/3.4/3.5/3.9** Sp20 **Exam**

Find the x-coordinate of each point on the graph of $f(x) = 3x + \dfrac{10}{x}$ where the tangent line is parallel to the line $y = 20 - 2x$.

exs20

Ex. H-10 **3.3/3.4/3.5/3.9** Su20 **Exam**

Let $f(x) = x^{15}e^{2-5x}$. Find the x-coordinate of each point where the tangent line to f is horizontal.

exsu20

Ex. H-11 **3.3/3.4/3.5/3.9** Su20 **Exam**

Let $f(x) = 3x^5 - 2x^3 + 7x - 16$. Find an equation of the tangent line to f at $x = -1$.

exsu20

Ex. H-12 **3.3/3.4/3.5/3.9** Fa20 **Exam**

Consider the function $f(x) = x^3 - 6x + c$, where c is an unspecified constant. Suppose the line $102x - y = 609$ is tangent to the graph of $y = f(x)$ at the point P in the first quadrant.

(a) What is the value of $f'(x)$ at the point P? Give a brief, one-sentence explanation.

(b) Find the x-coordinate of P.

(c) Find the y-coordinate of P.

(d) Find the value of c.

exf20

Ex. H-13 **3.3/3.4/3.5/3.9** Fa20 **Exam**

Let $f(x) = \dfrac{8e^x}{x-3}$. Find the equation of each horizontal tangent line of f.

exf20

Ex. H-14 **3.3/3.4/3.5/3.9** Fa20 **Exam**

Suppose $f(1) = -8$ and $f'(1) = 12$. Let $F(x) = x^3 f(x) + 10$. Find an equation of the tangent line to F at $x = 1$.

exf20

Ex. H-15 **3.3/3.4/3.5/3.9** Sp21 **Exam**

Suppose that an equation of the tangent line to f at $x = 5$ is $y = 3x - 8$. Let $g(x) = \dfrac{f(x)}{x^2 + 10}$.

(a) Calculate $f(5)$ and $f'(5)$.

(b) Calculate $g(5)$ and $g'(5)$.

(c) Write down an equation of the tangent line to g at $x = 5$.

exs21

Ex. H-16 **3.3/3.4/3.5/3.9, 3.7** Sp21 **Exam**

Suppose $f(2) = -7$ and $f'(2) = 3$.

(a) Let $g(x) = \cos(x)f(x)$. Calculate $g'(2)$.

(b) Let $h(x) = e^{2f(x)+3}$. Calculate $h'(2)$.

exs21

Ex. H-17 **3.3/3.4/3.5/3.9** Sp21 **Exam**

Let $f(x) = x^2 + bx + c$, where b and c are unspecified constants. An equation of the tangent line to f at $x = 3$ is $12x + y = 10$.

(a) Calculate $f(3)$ and $f'(3)$. Your answers must not contain the letters b or c.

(b) Calculate the value of b.

(c) Calculate the value of c.

exs21

Ex. G-14 **3.1/3.2, 3.3/3.4/3.5/3.9** Fa21 Exam

The following limit represents the derivative of a function f at a point a.

$$f'(a) = \lim_{h\to 0}\left(\frac{9\tan\left(\frac{\pi}{6}+h\right)-\frac{9}{\sqrt{3}}}{h}\right)$$

(a) Find a possible pair for f and a.

(b) Calculate the value of the limit.

exf21

Ex. H-18 **3.3/3.4/3.5/3.9, 3.7** Sp22 Exam

For each part, calculate $f'(x)$. After calculating the derivative, do not simplify your answer.

(a) $f(x) = 3x^{13} + 7\sqrt{x} - \dfrac{5}{x^3} + 12$ (b) $f(x) = \dfrac{e^x - 2\sin(x)}{\ln(x)+x^3}$ (c) $f(x) = 2x^4\cos(3e^x)$

exs22

Ex. H-19 **3.3/3.4/3.5/3.9** Sp22 Exam

For both parts below, suppose the line tangent to the graph of $y = f(x)$ at $x = 5$ is $y = 2x - 3$.

(a) Calculate $f(5)$ and $f'(5)$.

(b) Let $g(x) = xf(x) + 14$. Find an equation of the line tangent to the graph of $y = g(x)$ at $x = 5$.

exs22

Ex. H-20 **3.3/3.4/3.5/3.9, 3.7** Su22 Exam

For each part, calculate the derivative. After calculating the derivative, do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\tan\left(\dfrac{\ln(x)}{2x-5}\right)\right)$ (b) $\dfrac{d}{dx}\left(3x^7\cos(x) - 8e^{3x}\right)$ (c) $\dfrac{d}{dx}\left(10x^{12} - \dfrac{3}{x^3} + \sqrt[4]{x}\right)$

exsu22

Ex. G-20 **3.1/3.2, 3.3/3.4/3.5/3.9** Su22 Exam

Suppose that an equation to the tangent line to $y = f(x)$ at $x = 9$ is $y = 3x - 20$. Let $g(x) = xf(x^2)$.

(a) Calculate $f(9)$ and $f'(9)$. Explain.

(b) Calculate $g'(x)$.

(c) Find the tangent line to $y = g(x)$ at $x = -3$.

exsu22

Ex. H-21 **3.3/3.4/3.5/3.9** Sp18 Quiz

Let $g(x) = x^2\ln(x)$. Find an equation of the tangent line at $x = e$.

qs18

Ex. H-22 **3.3/3.4/3.5/3.9** Sp20 Quiz

Let $f(x) = \dfrac{50e^x}{x^2+1}$. Find an equation of the line tangent to the graph of $y = f(x)$ at $x = 3$.

qs20

Ex. H-23 **3.3/3.4/3.5/3.9, 3.7** Su22 Quiz

Calculate each derivative below. Do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\dfrac{x\sin(x)}{\pi^3+\ln(x)}\right)$ (b) $\dfrac{d}{dx}\left(\left(\sqrt{5x-8}+x^2\right)^{1/3}\right)$

qsu22

Ex. H-24 **3.3/3.4/3.5/3.9** Fa22 Quiz

For each part, calculate the derivative. You do not have to show work and there is no partial credit.

(a) $\dfrac{d}{dx}\left(\cos(x) - \dfrac{5}{x^7}\right)$ (b) $\dfrac{d}{dx}\left(8\sin(x)\ln(x)\right)$ (c) $\dfrac{d}{dx}\left(\dfrac{2x^4}{10-3x}\right)$

qf22

Ex. H-25 3.3/3.4/3.5/3.9 Fa22 Quiz

For each part, calculate the derivative. Do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\sqrt[5]{4\sin(x)+e^{3x-7}}\right)$

(b) $\dfrac{d}{dx}\left(\dfrac{2x^4\tan(x)}{3x+10}\right)$

qf22

Ex. H-26 3.3/3.4/3.5/3.9 Fa22 Quiz

Find the x-coordinate of each point on the graph of $y=3x^2+\dfrac{60}{x}$ where the tangent line is horizontal.

qf22

Ex. H-27 3.3/3.4/3.5/3.9

For each part, calculate $f'(x)$. *Do not simplify your answer.*

(a) $f(x)=\sqrt{2x}+3x^2+e^4$

(b) $f(x)=\dfrac{4}{x}+\ln(4)$

(c) $f(x)=\dfrac{8x^4-5x^{1/3}+1}{x^2}$

(d) $f(x)=\dfrac{x^2+3}{x-1}$

(e) $f(x)=x^3e^x$

(f) $f(x)=\sqrt{x}\cos(x)-e^x\sin(x)$

(g) $f(x)=\dfrac{\tan(x)+9x^2}{\ln(x)-4x}$

(h) $f(x)=\dfrac{x\sin(x)}{1-e^x\cos(x)}$

c3s3

Ex. H-28 3.3/3.4/3.5/3.9

Use the quotient rule to prove that $\dfrac{d}{dx}(\cot(x))=-\csc(x)^2$.

c3s3

Ex. H-29 3.3/3.4/3.5/3.9

Find the x-coordinate of each point on the graph of the given function where the tangent line is horizontal.

(a) $f(x)=\dfrac{1}{x^2}-\dfrac{1}{x^3}$

(b) $f(x)=(x^2-8)e^x$

(c) $f(x)=\dfrac{1}{\sqrt{x}}(x+9)$

(d) $f(x)=(1-\sin(x))\sin(x)$

c3s3

Ex. H-30 3.3/3.4/3.5/3.9

Find an equation for each line tangent to the graph of $f(x)=\dfrac{3x+5}{x+1}$ that is perpendicular to the line $2x-y=1$.

c3s3

Ex. H-31 3.3/3.4/3.5/3.9

For each part, calculate $f'(x)$. Do not simplify your answer after computing the derivative.

(a) $f(x)=\dfrac{\tan(x)}{\pi-\sec(x)}$

(b) $f(x)=\cos\left(e^{-3x}\right)$

(c) $f(x)=\sqrt{\ln(x^2+4)+x\sin(2x)}$

(d) $f(x)=\dfrac{e^{1/x}}{x^{2/3}+x^{1/3}}$

unit3

Ex. H-32 3.3/3.4/3.5/3.9, 3.7

Some values of g, h, g', and h' are given below. Use this table to answer parts (a) and (b).

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
0	1	7	2	3
2	-3	-9	1	5
4	5	-1	1	-6

(a) Let $f(x)=3g(x)h(x)$. Calculate $f'(2)$.

(b) Let $F(x)=g\left(\sqrt{x}\right)$. Calculate $F'(4)$.

unit3

Ex. H-33 **3.3/3.4/3.5/3.9**

Find an equation of the line normal to the graph of $f(x) = 2x^2 - \ln(x) + 3$ at $x = 1$. (Recall that the normal line is perpendicular to the tangent line.)

unit3

Ex. H-34 **3.3/3.4/3.5/3.9**

Find the x-coordinate of each point on the graph of $y = \dfrac{1}{\sqrt{x}}(x^3 + 15)$ where the tangent line is perpendicular to the line $x + 5y = 1$.

unit3

Ex. H-35 **3.3/3.4/3.5/3.9, 3.7**

Suppose $f(4) = 7$, $f'(4) = -5$, $g(4) = 4$, and $g'(4) = -3$. Let $F(x) = f\left(\dfrac{x^2}{g(x)}\right)$. Calculate $F'(4)$.

other

Ex. H-36 **3.3/3.4/3.5/3.9, 3.7**

Find an equation of the tangent line to $f(x) = 4x\cos(\pi x)$ at $x = \frac{1}{4}$.

other

Ex. H-37 **3.3/3.4/3.5/3.9**

Find the x-coordinate of each point on the graph of $y = x^3 - 7x^2 + x + 4$ such that the tangent line there is parallel to the line $6x - y = 1$.

other

Ex. H-38 **3.3/3.4/3.5/3.9** **⋆Challenge**

Find all points on the graph of $y = \dfrac{2}{x} + 3x$ such that the tangent line there passes through $(6, 17)$.

challenge

Ex. H-39 **3.3/3.4/3.5/3.9** **⋆Challenge**

Find all points P on the graph of $y = 4x^2$ with the property that the tangent line at P passes through the point $(2, 0)$.

challenge

§3.7: The Chain Rule

Ex. H-1 3.3/3.4/3.5/3.9, 3.7 Sp18 Exam

For each part, calculate $f'(x)$. *After calculating the derivative, do not simplify your answer.*

(a) $f(x) = \dfrac{x^{-1}x^{8/3}}{4\sqrt[3]{x^2}}$ (b) $f(x) = \left(x + \sqrt{5x-6}\right)^{1/4}$ (c) $f(x) = \dfrac{x^2 e^x}{\ln(x) - \cos(x)}$

exs18

Ex. H-3 3.3/3.4/3.5/3.9, 3.7 Fa18 Exam

Suppose f and g are differentiable for all x. For each part, use the table below or explain why there is not enough information.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	-1	-4	4	2
1	-1	-3	2	-4
2	-4	3	1	-1

(a) Let $F(x) = \dfrac{f(x)}{g(x)}$. Calculate $F'(0)$. (b) Let $G(x) = f\Big(xg(x)\Big)$. Calculate $G'(1)$.

exf18

Ex. I-1 3.7 Sp19 Exam

Suppose $f(4) = -8$ and $f'(4) = 3$. Let $g(x) = f\left(\frac{1}{4}x^2\right)$. Find $g'(4)$ or explain why it is impossible to do so with the given information.

exs19

Ex. I-2 3.7 Fa19 Exam

Find an equation of the line tangent to the graph of $f(x) = \tan(2x)$ at $x = \dfrac{\pi}{8}$.

exf19

Ex. I-3 3.7 Sp20 Exam

Find an equation of the line tangent to the graph of $f(x) = 5e^{2\cos(x)}$ at $x = 3\pi/2$.

exs20

Ex. I-4 3.7 Su20 Exam

For each part, calculate the derivative by any valid method.

(a) $f(x) = x^2\cos(3x) + \dfrac{1}{5x}$ (b) $f(x) = \sqrt{\sin\left(\dfrac{e^x}{x+1}\right)}$

exsu20

Ex. H-16 3.3/3.4/3.5/3.9, 3.7 Sp21 Exam

Suppose $f(2) = -7$ and $f'(2) = 3$.

(a) Let $g(x) = \cos(x)f(x)$. Calculate $g'(2)$.
(b) Let $h(x) = e^{2f(x)+3}$. Calculate $h'(2)$.

exs21

Ex. I-5 3.7 Fa21 Exam

Let $f(x) = x^9 e^{4x}$.

(a) Find $f'(x)$.
(b) Explain how to find where the tangent line to the graph of f is horizontal.
(c) Find where the graph of f has a horizontal tangent line.

exf21

Ex. I-6 3.7 Fa21 Exam

Selected values of the functions f and g and their derivatives are given in the table below. Use these values to complete the questions.

x	1	2	3	4
$f(x)$	4	3	2	1
$f'(x)$	-4	-1	-9	-3
$g(x)$	2	1	3	4
$g'(x)$	1	2	4	5

(a) Suppose $h(x) = 5f(x) - 8g(x)$. Find $h'(1)$.

(b) Suppose $p(x) = x^2 f(x)$. Find $p'(2)$.

(c) Suppose $q(x) = f(x^2)$. Find $q'(2)$.

exf21

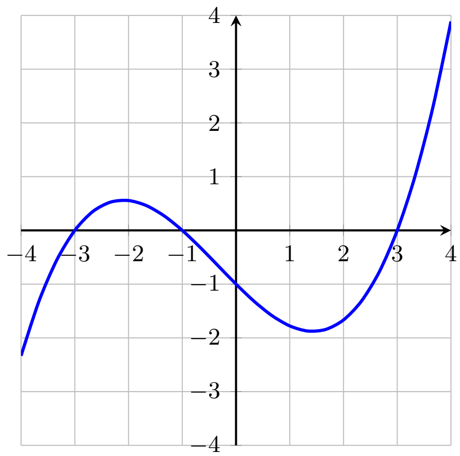
Ex. I-7 3.7 Fa21 Exam

Suppose f is differentiable at x and $g(x) = \dfrac{16\ln(15x)}{6f(x) - \sqrt{x+17}}$. Find $g'(x)$.

exf21

Ex. B-8 2.1/2.2, 3.7, 4.3/4.4 Fa21 Exam

For each part, use the graph of $y = g(x)$.



(a) How many solutions does the equation $g'(x) = 0$ have?

(b) Order the following quantities from least to greatest: $g'(-2.5)$, $g'(-2)$, $g'(0)$, and $g'(4)$. In your answer, write these quantities symbolically; do not give a numerical estimate.

(c) What is the sign of $g''(-3)$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

(d) Let $h(x) = g(x)^2$. What is the sign of $h'(-4)$(negative, positive, or zero)? If there is not enough information to determine the value, explain why.

exf21

Ex. H-18 3.3/3.4/3.5/3.9, 3.7 Sp22 Exam

For each part, calculate $f'(x)$. After calculating the derivative, do not simplify your answer.

(a) $f(x) = 3x^{13} + 7\sqrt{x} - \dfrac{5}{x^3} + 12$ (b) $f(x) = \dfrac{e^x - 2\sin(x)}{\ln(x) + x^3}$ (c) $f(x) = 2x^4\cos(3e^x)$

exs22

Ex. I-8 3.7 Sp22 Exam

Let $h(x) = \dfrac{f(x^2)}{g(x)}$. Use the table of values below to calculate $h'(1)$.

exs22

3.7 Sp22 Exam

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	-4	6	2	3
2	5	-2	-1	9

exs22

Ex. H-20 3.3/3.4/3.5/3.9, 3.7 Su22 Exam

For each part, calculate the derivative. After calculating the derivative, do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\tan\left(\dfrac{\ln(x)}{2x-5}\right)\right)$ (b) $\dfrac{d}{dx}\left(3x^7\cos(x) - 8e^{3x}\right)$ (c) $\dfrac{d}{dx}\left(10x^{12} - \dfrac{3}{x^3} + \sqrt[4]{x}\right)$

exsu22

Ex. I-9 3.7 Fa22 Exam

For each part, calculate the indicated derivative. Do not simplify your answer.

(a) $\dfrac{d}{dx}\left(7x^{10} + \sqrt[3]{x} - \dfrac{8}{x^{20}} + \sec(8x)\right)$ (b) $\dfrac{d}{dx}\left(\dfrac{\ln\left(x^3+30\right)}{8x}\right)$ (c) $\dfrac{d}{dx}\left(\sin\left(xe^{-5x}\right)\right)$

exf22

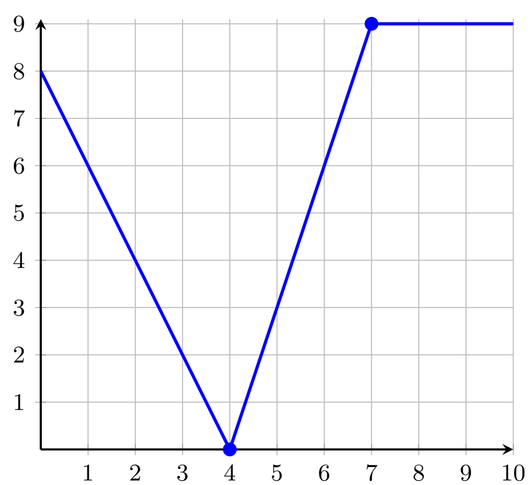
Ex. I-10 3.7 Fa22 Exam

Find the coordinates of all points on the graph of $f(x) = x\sqrt{14-x^2}$ where the tangent line is horizontal. You must give both the x- and y-coordinate of each such point.

exf22

Ex. I-11 3.7 Fa22 Exam

The graph of $y = f(x)$ is given below.



(a) Calculate $f'(6)$. Briefly explain how you found your answer.

(b) Let $g(x) = 9xf(2x)$. Find an equation of the line tangent to the graph of $y = g(x)$ at $x = 3$.

exf22

Ex. I-12 3.7 Sp18 Quiz

Calculate $\dfrac{d}{dx}\left(4x^3e^{\sin(2x)}\right)$. *After computing the derivative, do not simplify your answer.*

qs18

Ex. I-13 3.7 Sp20 Quiz

For each part, find $f'(x)$. After computing the derivative, do not simplify your answer.

(a) $f(x) = \sqrt{\tan(x^3)}$ (b) $f(x) = x^{3/4}\ln\left(\sin(x) + x + e^3\right)$

qs20

Ex. I-14 **3.7** Sp20 **Quiz**

Let $f(x) = x^{12}e^{5-3x}$. Find the x-coordinate of each point at which the graph of $y = f(x)$ has a horizontal tangent line.

qs20

Ex. I-15 **3.7** Su22 **Quiz**

Find the x-coordinate of each point on the graph of $y = x^3e^{-5x}$ where the tangent line is horizontal.

qsu22

Ex. H-23 **3.3/3.4/3.5/3.9, 3.7** Su22 **Quiz**

Calculate each derivative below. Do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\dfrac{x\sin(x)}{\pi^3 + \ln(x)}\right)$

(b) $\dfrac{d}{dx}\left(\left(\sqrt{5x-8} + x^2\right)^{1/3}\right)$

qsu22

Ex. I-16 **3.7** Fa22 **Quiz**

Find the x-coordinate of each point on the graph $y = \left(x^2 + x - 1\right)e^{3x}$ where the tangent line is horizontal.

qf22

Ex. I-17 **3.7**

For each part, calculate $f'(x)$ *Do not simplify your answer.*

(a) $f(x) = \sqrt{\sin(x)}$

(b) $f(x) = \sin(\sqrt{x})$

(c) $f(x) = \sqrt{\sin(\sqrt{x})}$

(d) $f(x) = (x^3 - 3x + 2)^2$

(e) $f(x) = \dfrac{1}{(3x+1)^2}$

(f) $f(x) = (2x + \sec(x))^2$

(g) $f(x) = e^{-2x}\sin(x)$

(h) $f(x) = \dfrac{\ln(2x+1)}{(2x+1)^2}$

(i) $f(x) = (\tan(x) + 1)^4\cos(2x)$

(j) $f(x) = \left(\dfrac{6}{9-2x}\right)^8$

(k) $f(x) = \left(\sin\left((4x-5)^2\right)\right)^4$
(*Some authors write this function as* $f(x) = \sin^4(4x-5)^2$.)

(l) $f(x) = \sqrt[3]{\sin(x)\cos(x)}$

(m) $f(x) = \sqrt{\dfrac{x^2-1}{x^3+x}}$

(n) $f(x) = \ln(\ln(x))$

(o) $f(x) = \sin(\sin(\sin(x)))$

(p) $f(x) = \left(x + \left(x + \sin(x)^2\right)^3\right)^4$

(q) $f(x) = |x|$
(*Hint: Recall* $|x| = \sqrt{x^2}$.)

c3s7

Ex. I-18 **3.7**

Find the x-coordinate of each point at which the graph of $y = f(x)$ has a horizontal tangent line.

(a) $f(x) = (2x^2 - 7)^3$

(b) $f(x) = x^2e^{1-3x}$

(c) $f(x) = \ln(3x^4 + 6x^2 - 4x^3 - 12x + 6)$

(d) $f(x) = \dfrac{(e^{3x} + e^{-3x})^2}{e^{3x}}$

c3s7

Ex. I-19 **3.7**

Suppose g and h are differentiable functions. Selected values of g, h, and their derivatives are given below.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
2	1	7	2	3
4	−3	−9	1	5
16	5	−1	1	−6

Define the function f by the formula

$$f(x) = g\left(\sqrt{x}\right)h\left(x^2\right)$$

(a) Calculate $f(4)$ or explain why there is not enough information to do so.

(b) Calculate $f'(4)$ or explain why there is not enough information to do so.

c3s7

Ex. H-32 **3.3/3.4/3.5/3.9, 3.7**

Some values of g, h, g', and h' are given below. Use this table to answer parts (a) and (b).

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
0	1	7	2	3
2	−3	−9	1	5
4	5	−1	1	−6

(a) Let $f(x) = 3g(x)h(x)$. Calculate $f'(2)$.

(b) Let $F(x) = g\left(\sqrt{x}\right)$. Calculate $F'(4)$.

unit3

Ex. I-20 **3.7**

For each part, calculate $f'(x)$.

(a) $f(x) = \tan(3x^2 + e)$

(b) $f(x) = e^{x/(x+1)}$

unit3

Ex. I-21 **3.7**

For each part, calculate $f'(x)$.

(a) $f(x) = \sin\left(7xe^{-3x}\right)$

(b) $f(x) = \sqrt{\dfrac{2\ln(x)}{\tan(3x) - \tan(3)}}$

other

Ex. H-35 **3.3/3.4/3.5/3.9, 3.7**

Suppose $f(4) = 7$, $f'(4) = -5$, $g(4) = 4$, and $g'(4) = -3$. Let $F(x) = f\left(\dfrac{x^2}{g(x)}\right)$. Calculate $F'(4)$.

other

Ex. H-36 **3.3/3.4/3.5/3.9, 3.7**

Find an equation of the tangent line to $f(x) = 4x\cos(\pi x)$ at $x = \frac{1}{4}$.

other

§3.8: Implicit Differentiation

Ex. J-1 3.8 Fa17 Exam

Find all points on the following curve at which the tangent line is horizontal.

$$2x^2 - 4xy + 7y^2 = 45$$

Hint: Find a second equation that such points must satisfy. Then solve a system of two equations for x and y.

exf17

Ex. J-2 3.8 Sp18 Exam

Find an equation of the line tangent to the following curve at the point $(2, 0)$.

$$x^3 + e^{xy} = 3y + 9$$

exs18

Ex. J-3 3.8 Fa18 Exam

Find an equation of the line tangent to the following curve at $(8, 1)$.

$$\sin\left(\frac{\pi x}{y}\right) = x - 8y$$

exf18

Ex. J-4 3.8 Sp19 Exam

Find an equation of the line tangent to the following curve at the point $(1, 1)$.

$$\frac{5x}{y} = 4x + y^3$$

exs19

Ex. J-5 3.8 Fa19 Exam

Find an equation of the line tangent to the following curve at the origin.

$$\sin(x + 2y) + 9x + 1 = e^y$$

Ex. J-6 3.8 Fa19 Exam

Find $\dfrac{dy}{dx}$ for a general point on the curve described by the following equation. *Do not simplify your answer.*

$$x^3y^2 + (x + y)^2 = 100$$

Ex. J-7 3.8 Sp20 Exam

A particle in the fourth quadrant is moving along a path described by the equation

$$x^2 + xy + 2y^2 = 16$$

such that at the moment its x-coordinate is 2, its y-coordinate is decreasing at a rate of 5 cm/sec. At what rate is its x-coordinate changing at that time?

exs20

Ex. J-8 3.8 Sp20 Exam

Find an equation of line tangent to the following curve at the origin.

$$\sin(x + 3y) + 9x + 1 = e^y$$

exs20

Ex. J-9 3.8 Sp20 Exam

Consider the curve described by the equation

$$3x^2 + 2xy + 4y^2 = 132$$

exs20

3.8 Sp20 Exam

At any point on this curve, we have

$$\frac{dy}{dx} = \frac{-3x - y}{x + 4y}$$

(a) Describe in two or three sentences the steps you should take to find the points on the curve where the tangent line is horizontal. *Your answer may contain either English, mathematical symbols, or both.*

(b) What is the rightmost (i.e., greatest x-coordinate) point on the curve where the tangent line is horizontal?

(c) Describe in one or two sentences how parts (a) and (b) would change if instead you wanted to find the points where the tangent line is vertical. You do not have to solve the problem again, but only describe generally what you would do differently. *Your answer may contain either English, mathematical symbols, or both.*

exs20

Ex. J-10 3.8 Sp20 Exam

Find an equation of the line tangent to the following curve at $(1, 7)$.

$$\ln(xy + x - 7) = 2x + 4y - 30$$

exs20

Ex. J-11 3.8 Sp20 Exam

Consider the curve described by the equation

$$5x^2 - 4xy + y^2 = 8$$

At any point on this curve, we have

$$\frac{dy}{dx} = \frac{-5x + 2y}{-2x + y}$$

(a) Describe in two or three sentences the steps you should take to find each point on the curve where the tangent line is parallel to the line $y = x$. *Your answer may contain either English, mathematical symbols, or both.*

(b) What is the leftmost (i.e., least x-coordinate) point on the curve where the tangent line is parallel to $y = x$?

(c) Describe in one or two sentences how parts (a) and (b) would change if instead you wanted to find the points where the tangent line is perpendicular to the line $y = 4$. You do not have to solve the problem again, but only describe generally what you would do differently. *Your answer may contain either English, mathematical symbols, or both.*

exs20

Ex. J-12 3.8 Sp20 Exam

Consider the curve described by the following equation.

$$e^{12x+2y} = 6y - 3xy + 1$$

(a) Find $\dfrac{dy}{dx}$ at a general point on this curve.

(b) Calculate the slope of the line tangent to the curve at $(2, -12)$.

(c) There is a point on the curve close to the origin with coordinates $(0.07, b)$, and the line tangent to the curve at the origin is $y = 3x$. Use linear approximation to estimate the value of b.

exs20

Ex. J-13 3.8 Su20 Exam

Consider the curve described by the equation

$$x^4 - x^2y + y^4 = 1$$

(a) Find $\dfrac{dy}{dx}$ at a general point on the curve.

(b) Find an equation of the line tangent to the curve at the point $(-1, 1)$.

exsu20

Ex. J-14 **3.8** Su20 **Exam**

On an online exam, a student uses logarithmic differentiation to find the first derivative of

$$f(x) = (3 + \sin(x))^{2+x^2}$$

They type the following two lines for their work.

$$\begin{aligned} y &= (3 + \sin(x))^{2+x^2} \\ \ln(y) &= \ln(\cdots \end{aligned}$$

Unfortunately, the student runs out of time and is unable to submit the rest of their work. Oh no! Find $f'(x)$ by completing the student's work.

exsu20

Ex. J-15 **3.8** Fa20 **Exam**

Consider the following curve, where a and b are unspecified constants.

$$ax^2y - 3xy^2 + 4x = b$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{3y^2 - 2axy - 4}{ax^2 - 6xy}$.

(b) Suppose the tangent line to the curve at the point $(1, 1)$ is $y = 1 + 5(x - 1)$. Use part (a) to find the value of a.

(c) Use your answer to part (b) to find the value of b.

exf20

Ex. J-16 **3.8** Sp21 **Exam**

Consider the curve defined by the equation below, where a and b are unspecified constants.

$$\sqrt{xy} = ay^3 + b$$

Suppose the equation of the tangent line to the curve at the point $(3, 3)$ is $y = 3 + 4(x - 3)$.

(a) What is the value of $\frac{dy}{dx}$ at $(3, 3)$?

(b) Calculate a and b.

exs21

Ex. J-17 **3.8** Fa21 **Exam**

Consider the curve defined by the following equation, where A and B are unspecified constants.

$$Ax^2 - 8xy = B\cos(y) + 3$$

(a) Find a formula for $\frac{dy}{dx}$.

(b) Suppose the point $(8, 0)$ is on the curve. Find an equation that A and B must satisfy.

(c) Suppose the tangent line to the curve at the point $(8, 0)$ is $y = 6x - 48$. Find the values of A and B.

exf21

Ex. J-18 **3.8** Sp22 **Exam**

Consider the curve described by the following equation:

$$12x^2 + 6xy + y^2 = 20$$

Find all points on the curve where the tangent line is horizontal. Write your answer as a comma-separated list of coordinate pairs.

Hint: Find a second equation that such points must satisfy.

exs22

Ex. J-19 3.8 Su22 Exam

Find all points on the graph of the following equation where the tangent line is vertical.

$$x^2 - 2xy + 10y^2 = 450$$

exsu22

Ex. J-20 3.8 Fa22 Exam

Consider the following curve.

$$\cos(5x + y - 5) = 8xe^y + y - 7$$

(a) Calculate $\dfrac{dy}{dx}$ for a general point on the curve.

(b) Find an equation of the line tangent to the curve at the point $(1, 0)$.

exf22

Ex. J-21 3.8 Sp20 Quiz

Find an equation of the line tangent to the graph of $xe^y = x^3 + (y-1)^2 - 1$ at the point $(0, 2)$.

qs20

Ex. J-22 3.8 Su22 Quiz

Suppose x and y are implicitly related by the following equation.

$$5 + xy^2 = \frac{y}{2 - x^3}$$

Find $\dfrac{dy}{dx}$ for a general point on the curve.

qsu22

Ex. J-23 3.8 Su22 Quiz

Suppose x and y are implicitly related by the following equation.

$$6x^2 - 3xy + 2y^2 = 52$$

Find all points (both x- and y-coordinates) on the curve where the tangent line is horizontal.

qsu22

Ex. J-24 3.8 Fa22 Quiz

Find $\dfrac{dy}{dx}$ for a general point on the following curve.

$$x\sin(y) + 10 = \ln(y^2 + x)$$

qf22

Ex. J-25 3.8 Fa22 Quiz

Find the slope of the line tangent to the given curve at the point $(1, \frac{1}{4})$.

$$x\tan(\pi y) = 16y^2 + 3\ln(x)$$

qf22

Ex. J-26 3.8

For each part, find $\dfrac{dy}{dx}$ for a general point on the curve described by the given equation.

(a) $x^2 + y^4 = 12x + y$

(b) $y + \dfrac{1}{xy} = x^2$

(c) $\sin(x+y) = x + \cos(y)$

(d) $\ln\left(\dfrac{x-y}{xy}\right) = \dfrac{1}{y}$

(e) $6x^2 + 3xy + 2y^2 + 17y = 6$

c3s8

Ex. J-27 3.8

Find an equation of the line tangent to the following curve at $\left(\frac{1}{e-2}, 1\right)$.

$$xe^y = 2xy + y^3$$

c3s8

Ex. J-28 3.8

Find an equation of the line tangent to the following curve at $(0, \pi)$.

$$\sin(x - y) = xy$$

c3s8

Ex. J-29 3.8

Consider the curve given by the following equation.

$$x^2 + xy + 3y^2 = 99$$

(a) Find all points on the graph of the curve where the tangent line is horizontal.

(b) Find all points on the graph of the curve where the tangent line is vertical.

c3s8

Ex. J-30 3.8

Find the slope of the tangent line to the curve $x^3 - y^3 = y - 1$ at the point $(1, 1)$.

unit3

Ex. J-31 3.8

Find the slope of the tangent line to the curve $x^3 + xy + y^2 = 7$ at $(1, 2)$.

unit3

Ex. J-32 3.8

Find an equation of the line normal to the curve $5x^2y + 2y^3 = 22$ at the point $(2, 1)$.

unit3

Ex. J-33 3.8

Find an equation of the line tangent to the curve $2x^2 - xy + 5y^2 = 24$ at the point $(-1, 2)$.

other

Ex. J-34 3.8

Find an equation of the line tangent to the curve $\sin(x - y) = 4e^{xy} - 4e^9$ at the point $(3, 3)$.

other

Ex. J-35 3.8 ⋆Challenge

Find all tangent lines to the graph of $9x^2 - 18xy + y^2 = 1800$ that are perpendicular to the line $x + 3y = 10$.

challenge

Ex. J-36 3.8, 4.6 ⋆Challenge

Consider the curve described by the equation

$$\frac{x - y^3}{y + x^2} = x - 12$$

(a) Find an equation for the line tangent to this curve at $(-1, 4)$.

(b) There is a point on the curve with coordinates $(-1.1, b)$. Use linear approximation to estimate b. Round to three decimal places.

(c) There is a point on the curve with coordinates $(a, 4.2)$. Use linear approximation to estimate a. Round to three decimal places.

challenge

Ex. J-37 3.8 ⋆Challenge

Suppose $x^2 + y^2 = R^2$, where R is a constant. Find $\frac{d^2y}{dx^2}$ and fully simplify your answer as much as possible.

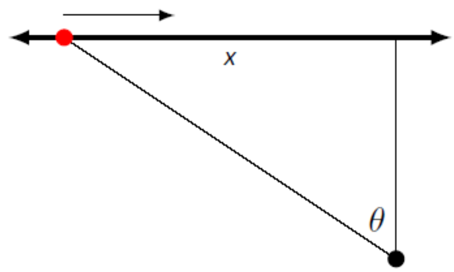
challenge

§3.11: Related Rates

Ex. K-1 3.11 Fa17 Exam

A camera is located 5 feet from a straight wire along which a bead is moving at 6 feet per second. The camera automatically turns so that it is pointed at the bead at all times. How fast is the camera turning when the bead is 12 feet from passing closest to the camera?

You must give correct units as part of your answer.



exf17

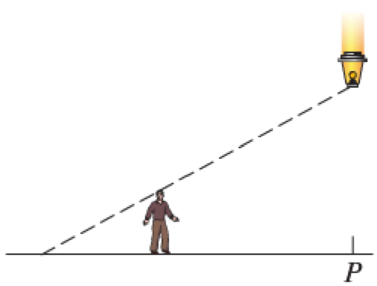
Ex. K-2 3.11 Sp18 Exam

The total surface area of a cube is changing at a rate of 12 in^2/s when the length of one of the sides is 10 in. At what rate is the volume of the cube changing at that time?

exs18

Ex. K-3 3.11 Fa18 Exam

A person 6 feet tall stands 10 feet from point P, which is directly beneath a lantern hanging 30 feet above the ground. At the moment when the lantern is 9 feet above the ground, the lantern is falling at a rate of 4 ft/sec. At what rate is the length of the person's shadow changing at that moment?



exf18

Ex. K-4 3.11 Sp19 Exam

A child flies a kite at a constant height of 30 feet and the wind is carrying the kite horizontally away from the child at a rate of 5 ft/sec. At what rate must the child let out the string when the kite is 50 feet away from the child?

You must give correct units as part of your answer.

exs19

Ex. K-5 3.11 Fa19 Exam

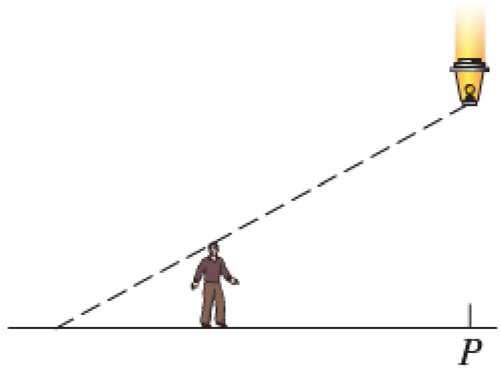
A spherical snowball melts in such a way that it always remains a sphere, and its volume decreases at 8 cm^3/sec. At what rate is the surface area of the snowball changing when its surface area is 40π cm^2? *You must give correct units as part of your answer.*

Ex. K-6 3.11 Sp20 Exam

A 6-ft tall person is initially standing 12 ft from point P directly beneath a lantern hanging 42 ft above the ground, as shown in the diagram below. The person then begins to walk towards point P at 5 ft/sec. Let D denote the distance between the person's feet and the point P. Let S denote the length of the person's shadow.

exs20

3.11 Sp20 Exam



(a) Write an equation that relates D and S.

(b) Write an equation that expresses the English sentence "*The person then begins to walk towards point P at 5 ft/sec.*"

(c) Is the length of the person's shadow increasing, decreasing or remaining constant?

(d) At what rate is the length of the person's shadow changing when the person is 8 ft from point P? Include correct units as part of your answer.

exs20

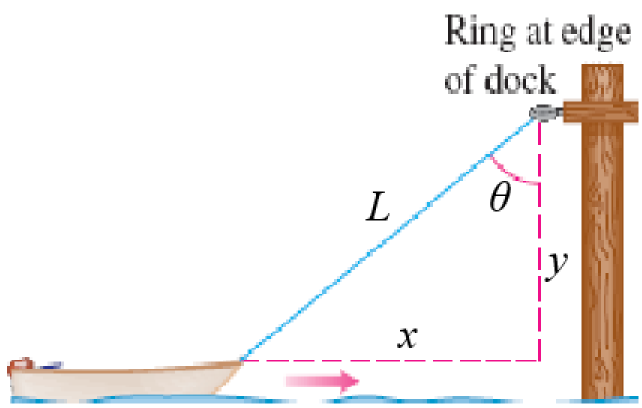
Ex. K-7 3.11 Sp20 Exam

The volume of a cube is decreasing at the rate of 300 cm^3/sec at the moment its total surface area is 150 cm^2. What is the rate of change of the length of one edge of the cube at this moment?

exs20

Ex. K-8 3.11 Sp20 Exam

A boat is pulled toward a dock by a rope through a ring on the dock 4 ft above the front of the boat. The rope is hauled in at the rate of 12 ft/sec.



(a) Which of the marked variables (x, y, L, and θ) are changing over time?

(b) Write a mathematical equation that expresses the English sentence *"The rope is hauled in at the rate of 12 ft/sec"*.

(c) Is $\cos(\theta)$ increasing, decreasing, or constant?

(d) Write a mathematical expression for *"the rate at which the boat approaches the dock"*.

(e) How fast in ft/sec is the boat approaching the dock when the rope is 5 ft long?

exs20

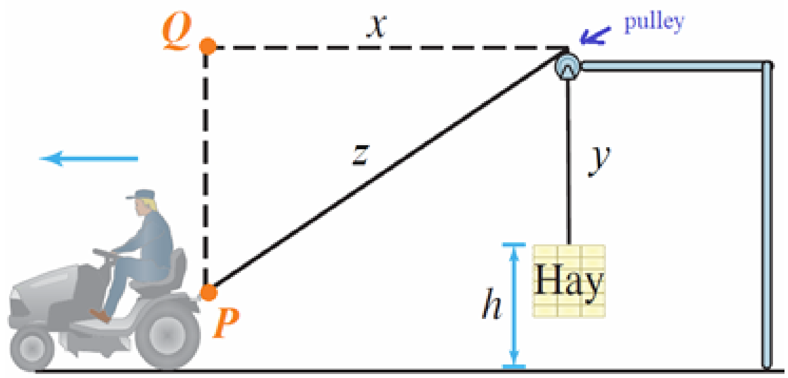
Ex. K-9 3.11 Sp20 Exam

A farmer's tractor pulls a rope of length 12 m attached to a bale of hay through a pulley is 8 m above the ground. The vertical distance between the tractor and the pulley (the distance from P to Q) is 7 m. The tractor is moving to the left at rate of 2 m/sec, which causes the bale of hay to rise off the ground.

exs20

3.11

Sp20 Exam



(a) The rate of change (with respect to time) of which variable is equal to the speed of the tractor?

(b) Use the Pythagorean theorem to find an equation that holds for all time and involves only the variables x and z.

(c) Use the fact that the length of the rope is constant to find an equation that holds for all time and involves only the variables z and y.

(d) Use the fact that the height of the pulley is constant to find an equation that holds for all time and involves only the variables h and y.

(e) Combine the equations from parts (b), (c), and (d) to find an equation that holds for all time and involves only the variables x and h.

(f) The rate of change (with respect to time) of which variable is equal to the rate at which the bale of hay is rising?

(g) Find the rate at which the bale of hay is rising off the ground when the horizontal distance between the tractor and the bale of hay is 8 m.

exs20

Ex. K-10 3.11

Su20 Exam

In a right triangle, the base is decreasing in length by 3 cm/sec and the area is increasing by 15 cm^2/sec. (The triangle always remains a right triangle.) At the time when the base is 15 cm in length and the height is 20 cm in length...

(a) ...at what rate is the height changing? (Give a number only.)

(b) ...at what rate is the length of the hypotenuse changing? (Give a number only.)

(c) What are the units of your answer in part (a)?

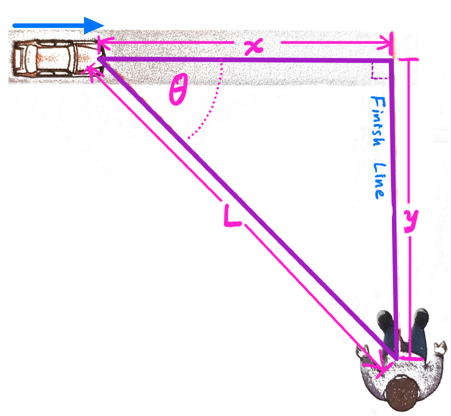
(d) In part (b), is the length of the hypotenuse increasing, decreasing, or staying constant?

exsu20

Ex. K-11 3.11

Fa20 Exam

At a certain moment, a race official is watching a race car approach the finish line along a straight track at some constant, positive speed. Suppose the official is sitting still at the finish line, 20 m from the point where the car will cross.



For parts (a)–(e), the allowed answers are "positive", "negative", "zero", or "not enough information".

exf20

3.11 Fa20 Exam

(a) At the moment described, what is the sign of $\frac{dx}{dt}$?

(b) At the moment described, what is the sign of $\frac{dy}{dt}$?

(c) At the moment described, what is the sign of $\frac{dL}{dt}$?

(d) At the moment described, what is the sign of $\frac{d(\cos(\theta))}{dt}$?

(e) At the moment described, what is the sign of $\frac{d^2x}{dt^2}$?

(f) Suppose the speed of the car is 70 m/sec. At what rate is the distance between the car and the race official changing when the car is 60 m from the finish line? *Your answer must have the correct units. Your answer must be exact. No decimal approximations.*

exf20

Ex. K-12 3.11 Sp21 Exam

A local gym has two cylindrical swimming pools. The larger pool has radius 20 meters and is filled with water. The smaller pool has radius 12 meters and is empty. Water is drained from the large pool and immediately emptied into the small pool. The height of the water in the small pool increases at a rate of 0.2 m/min.
Let V_L, V_S, h_L, and h_S refer to the volume of the large pool, volume of the small pool, height of the large pool, and height of the small pool, respectively.

(a) How are $\dfrac{dV_L}{dt}$ and $\dfrac{dV_S}{dt}$ related?

(b) What is the sign of $\dfrac{dh_L}{dt}$?

(c) Find $\dfrac{dV_S}{dt}$.

(d) Find $\dfrac{dh_L}{dt}$.

exs21

Ex. K-13 3.11 Fa21 Exam

The base of a right triangle is decreasing at a constant rate of 10 cm/sec and in such a way that the triangle always remains a right triangle. At the time when the base is 15 cm and the height is 22 cm, the area of the triangle is increasing by 25 cm^2/sec. Use this information to answer the questions below. Let B denote the base of the triangle.

(a) At the described time, what is the sign of $\frac{dB}{dt}$?

(b) At the described time, what is the sign of $\frac{d^2B}{dt^2}$?

(c) At the described time, at what rate is the height changing?

(d) What are the units of the answer to part (c)?

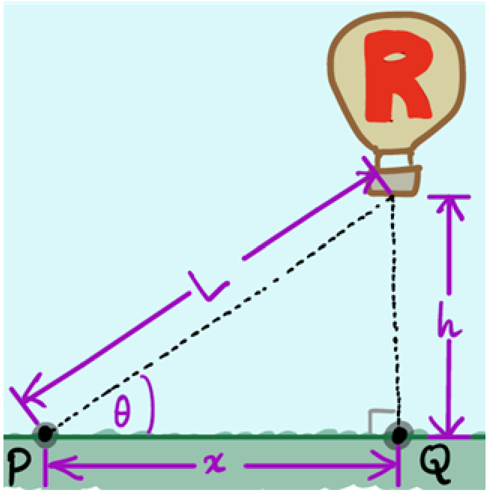
exf21

Ex. K-14 3.11 Fa21 Exam

A hot-air balloon is floating directly above the point Q on the ground and is descending at a constant rate of 10 ft/sec. A camera is on the ground at point P, which is 500 feet from point Q. See the figure below.

exf21

3.11 Fa21 Exam



(a) What is the sign of $\dfrac{dh}{dt}$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

(b) How is $\cos(\theta)$ changing over time? Circle your answer below.

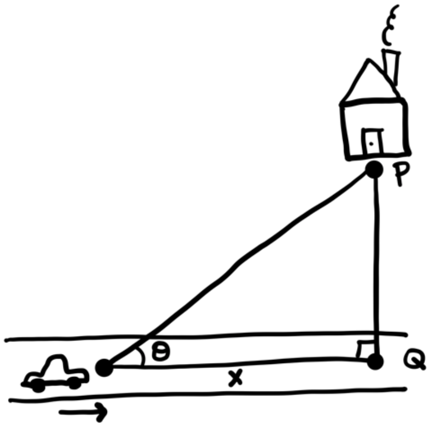
(i) increasing over time
(ii) decreasing over time
(iii) constant over time
(iv) sometimes increasing and sometimes decreasing
(v) not enough information to determine

(c) What is the rate of change of the distance between the camera and the balloon when the balloon is 600 feet above the ground? *You must give correct units as part of your answer.*

exf21

Ex. K-15 3.11 Sp22 Exam

A house sits at point P, which is 20 m from point Q on a straight road. A car travels along the road toward the point Q at 19 m/s. Let x be the distance between the car and point Q, and let θ be the angle between the road and the line of sight from the car to the house. See the figure below.



(a) What is the sign of $\dfrac{dx}{dt}$?

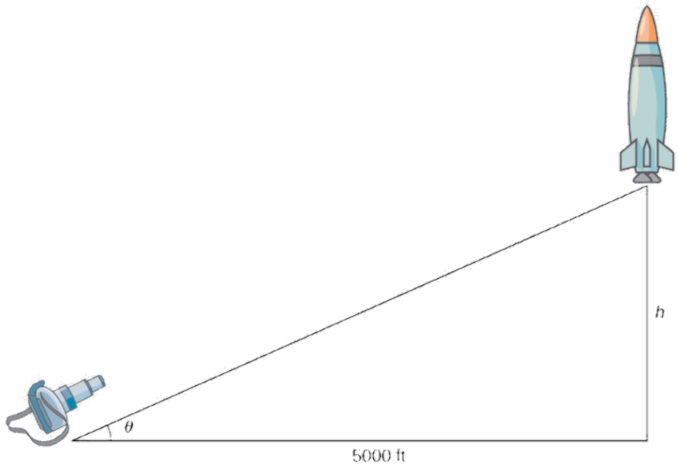
(b) What is the sign of $\dfrac{d\theta}{dt}$?

(c) Find the rate of change of the distance between the car and the house when the car is 45 m from point Q. You must include correct units in your answer. You may leave unsimplified radicals in your answer.

exs22

Ex. K-16 **3.11** Su22 **Exam**

A rocket is launched so that it rises vertically. A camera is positioned 5000 feet from the launch pad and turns so that it stays forces on the rocket. At the moment when the rocket is 12,000 feet above the launch pad, its velocity is 600 feet/sec. Let h be the height of the rocket above the launch pad and let θ be the viewing angle of the camera. See the figure below.



(a) Determine the sign of $\frac{d}{dt}(\cos(\theta))$ at the moment described or determine that there is not enough information to do so.

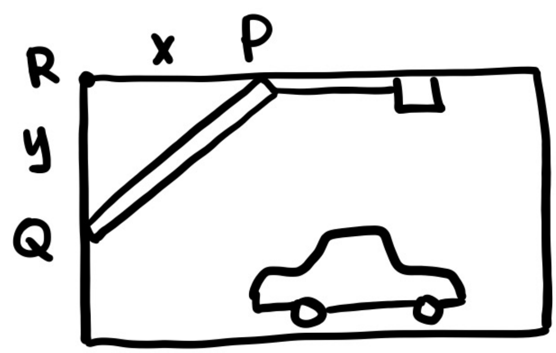
(b) Determine the sign of $\frac{d^2h}{dt^2}$ at the moment described or determine that there is not enough information to do so.

(c) At the moment described, what is the rate at which the camera is turning? That is, what is the rate at which θ is changing over time? *You must include proper units as part of your answer.*

exsu22

Ex. K-17 **3.11** Fa22 **Exam**

A solid 14-foot tall garage door opens via a pulley mechanism. As the pulley opens the garage door, the top of the garage door (point P in the figure) moves to the right at 5 ft/s. At the same time, the bottom of the garage door (point Q in the figure) moves straight up.
As shown in the figure, the point R is the fixed point at the top of the garage door frame, x represents the distance between P and R, and y represents the distance between Q and R.



(a) What is the sign of $\frac{dx}{dt}$?

(b) What is the sign of $\frac{dy}{dt}$?

(c) What is the rate of change of the distance between the points Q and R when the distance between them is 9 feet? You must include correct units in your answer. You may leave unsimplified radicals in your answer.

exf22

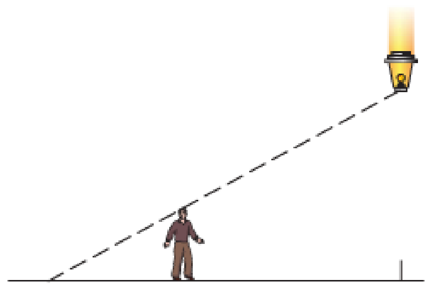
Ex. K-18 **3.11** Sp20 **Quiz**

The image of a certain rectangle of area 30 cm^2 is changing in such a way that its length is decreasing at a rate of 2 cm/sec. and its area remains constant. At what rate is its width changing when its length is 6 cm?

qs20

Ex. K-19 3.11 Su22 Quiz

A 2-meter tall person is initially standing 4 meters from point P directly beneath a lantern hanging 14 meters above the ground, as shown in the figure below. The person then begins to walk towards point P at 1.5 m/sec. Let x denote the distance between the person's feet and the point P. Let y denote the length of the person's shadow.



(a) Write an equation that relates x and y.

(b) Write an equation that expresses the English sentence *"The person begins to walk towards point P at 1.5 m/sec."*

(c) At what rate is the length of the person's shadow changing when the person is 3 meters from point P? *You must include correct units as part of your answer.*

qsu22

Ex. K-20 3.11 Fa22 Quiz

A pebble is dropped into a lake and an expanding circular ripple results. When the radius of the ripple is 8 inches, the area enclosed by the ripple is changing at a rate of 48π in^2/sec. What is the rate at which the radius is changing at this time? *You must include correct units as part of your answer.*

qf22

Ex. K-21 3.11

A rock is dropped into a lake to create an expanding, circular ripple. When the radius of the ripple is 8 inches, the radius is increasing at a rate of 3 in/sec. At what rate is the area enclosed by the ripple changing at this time?

c3s11

Ex. K-22 3.11

Every day, a flight to Los Angeles flies directly over a man's home at a constant altitude of 4 miles and at a constant speed of 400 miles per hour. At what rate is the angle of elevation of the man's line of sight changing with respect to time when the horizontal distance between the approaching plane and the man's location is exactly 3 miles?

c3s11

Ex. K-23 3.11

The volume of a spherical balloon is increasing at constant rate of 3 in^3/s. At what rate is the radius of the balloon changing when the radius is 2 in.?

c3s11

Ex. K-24 3.11

Recall that a baseball diamond is a square of side length 90 ft. The corners of the diamond are labeled, in anti-clockwise order, home plate, first base, second base, and third base. A player runs from home plate to first base at a speed of 20 ft/s. How fast is the player's distance from second base changing when the player is halfway to first base?

c3s11

Ex. K-25 3.11

A particle moves along the elliptical path given by $x^2 + 9y^2 = 13$ in such a way that when it is at the point $(-2, 1)$, its x-coordinate is decreasing at the rate of 7 units per second. How fast is the y-coordinate changing at that instant?

c3s11

Ex. K-26 3.11

The surface area of a sphere is changing at a rate of 16π in^2/s when its radius is 3 in. At what rate is the volume of the sphere changing at that time?

unit4

Ex. K-27 **3.11**

A car traveling north at 40 mi/hr and a truck traveling east at 30 mi/hr leave an intersection at the same time. At what rate will the distance between them be changing 4 hours later?

unit4

Ex. K-28 **3.11**

The altitude of a triangle is increasing at a rate of 1 ft/min. while the area is increasing at a rate of 2 ft/min. At what rate is the base of the triangle changing when the altitude is 10 ft. and the area is 100 ft^2?

unit4

Ex. K-29 **3.11** **⋆Challenge** **!!!**

A water tank in the shape of an inverted cone has height 10 meters and base radius 8 meters. Water flows into the tank at the rate of 32π m^3/min. At what rate is the depth of the water in the tank changing when the water is 5 meters deep?

challenge

4 Chapter 4: Applications of the Derivative

§4.1: Maxima and Minima

Ex. L-1 4.1 Sp18 Exam

Find the minimum and maximum values of $f(x) = 2x^3 - 3x^2 - 12x + 18$ on the interval $[-3, 3]$.

Hint: You may use the factorization $f(x) = (x^2 - 6)(2x - 3)$ to make any required arithmetic easier.

exs18

Ex. L-2 4.1 Fa18 Exam

Let $f(x) = 4(x-3)^{1/3} - \frac{1}{3}x + 1$. *Note:* The domain of f is $(-\infty, \infty)$.

(a) Calculate all critical points of f. *For each number you find, you must clearly indicate in your work why it is a critical point.*

(b) What are the absolute extreme values of f on the interval $[2, 30]$?

exf18

Ex. L-3 4.1 Sp19 Exam

Find all critical points of $f(x) = x - \dfrac{3}{2}(x-8)^{2/3}$ or explain why f has no critical points.

exs19

Ex. L-4 4.1 Sp19 Exam

Find the absolute extreme values of $f(x) = \dfrac{20x}{x^2 + 4}$ on $[-4, 0]$.

exs19

Ex. L-5 4.1 Fa19 Exam

Find all critical points of $f(x) = 2 - \left(x^2 - 2x\right)^{1/3}$ or explain why f has no critical points.

Note: The domain of f is $(-\infty, \infty)$.

exf19

Ex. L-6 4.1 Fa19 Exam

For each part, find the absolute extreme values of $f(x)$ on the given interval.

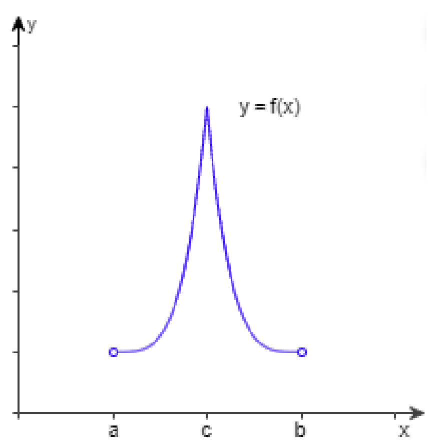
(a) $f(x) = x + \dfrac{9}{x}$ on $[1, 18]$.

(b) $f(x) = (6 - x)e^x$ on $[0, 6]$.
(***Hint:*** $2 < e < 3$.)

exf19

Ex. L-7 4.1 Sp20 Exam

Determine from the given graph whether the function has any absolute extreme values on (a, b).



exs20

Ex. L-8 **4.1, 4.3/4.4** Sp20 Exam

Consider the following function

$$g(x) = \frac{3}{2}x^4 + 8x^3 - 36x^2$$

(a) Where does g have a local minimum on $(-7, 3)$? local maximum?

(b) Where does g have a global minimum on $[-7, 3]$? global maximum?

exs20

Ex. L-9 **4.1** Sp20 Exam

Find all critical points of the function

$$f(x) = 2x^{4/3} - 16x^{2/3} + 24$$

Note: The function f is continuous on the interval $(-\infty, \infty)$.

exs20

Ex. G-6 **3.1/3.2, 4.1, 4.9** Sp20 Exam

Suppose the derivative of f is $f'(x) = 3x^2 - 6x - 9$ and that $f(1) = 10$.

(a) Find an equation of the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Find the critical points of f.

(c) Where does f have a local minimum value? local maximum value?

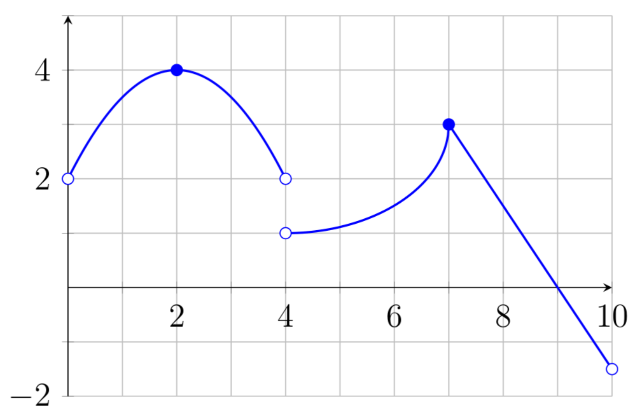
(d) Calculate $f(0)$.

(e) Calculate the absolute maximum value of f on the interval $[0, 6]$. At what x-value does it occur?

exs20

Ex. L-10 **4.1** Su20 Exam

Suppose $f(x)$ is continuous on $[0, 10]$. The figure below shows the graph of $y = f'(x)$ on $[0, 10]$.
Note: The figure does not show a graph of $f(x)$ but rather its derivative.)



Use the graph to answer the following questions. Read each question carefully. Some questions ask about f and others ask about the derivative f'.

(a) Find the absolute maximum of $f'(x)$ on $(0, 10)$ or determine that it does not exist.

(b) Find the absolute minimum of $f'(x)$ on $(0, 10)$ or determine that it does not exist.

(c) Find all critical points of $f(x)$ in $(0, 10)$.

exsu20

Ex. L-11 **4.1** Su20 Exam

Let $f(x) = \dfrac{1 - 2x}{6 + x^2}$. Find the absolute extrema of f on $[-3, 2]$ and where they occur.

exsu20

Ex. L-12 4.1 Su20 Exam

Let $f(x) = x^{1/3}(x-16)^{1/5}$. Find all critical points of f. You must be clear why each of your answers really is a critical point. ***Note:*** The domain of f is $(-\infty, \infty)$.

exsu20

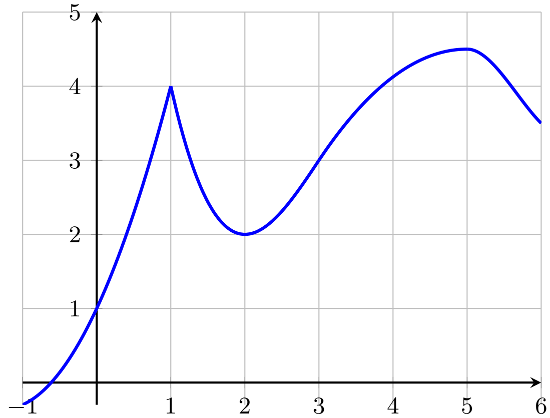
Ex. L-13 4.1 Fa20 Exam

For each part, use the graph of $y = f(x)$. Assume that the domain of f is $(-\infty, \infty)$.

(a) Where does f have a local minimum on the interval $(-1, 6)$?

(b) List all of the critical points of f.

(c) Estimate the absolute maximum of f on $[0, 3]$ or explain why f has no such maximum.



exf20

Ex. L-14 4.1 Fa20 Exam

(You may need a basic calculator for this problem.)

Consider the function

$$f(t) = \frac{a}{t^2 - 3t + 25}$$

where a is an unspecified **positive** constant. Suppose the absolute minimum of f on $[0, 6]$ is 3.

(a) Find the value of a. ***Hint:*** First find the absolute minimum of f on $[0, 6]$ in terms of a.

(b) Calculate the absolute maximum of f on $[0, 6]$.

exf20

Ex. L-15 4.1 Sp21 Exam

Consider the function below, where A is an unspecified, **positive** constant.

$$f(x) = \frac{A}{x - 8\sqrt{x} + 60}$$

For parts (c) and (d) only, assume the absolute minimum of f on $[0, 21]$ is 8.

(a) List all x-values that must be tested to find the absolute extrema of f on $[0, 21]$.

(b) At which x-value does the absolute minimum of f occur on $[0, 21]$?

(c) Find the value of A.

(d) Find the absolute maximum of f on $[0, 21]$ and all x-values at which it occurs.

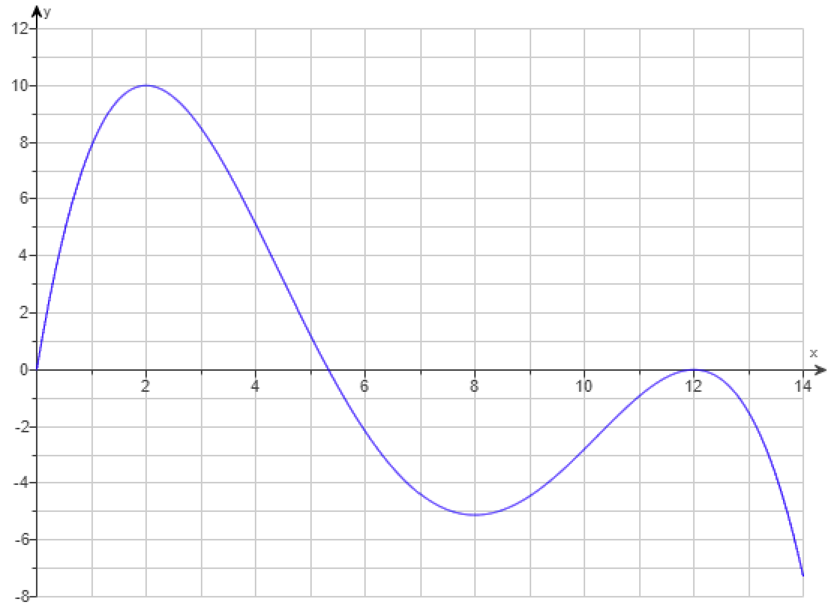
exs21

Ex. L-16 4.1 Sp21 Exam

Use the graph of $y = f(x)$ on $[0, 14]$ below to answer the questions.

exs21

4.1 Sp21 Exam



(a) List the critical points of f in $(0, 14)$.

(b) How many local extrema does f have on $(0, 14)$?

(c) Find the absolute maximum of f and the x-value at which it occurs.

(d) Find the absolute minimum of f and the x-value at which it occurs.

exs21

Ex. L-17 4.1 Fa21 Exam

Find the absolute extreme values of $f(x) = x^3 - 6x^2 + 9x + 20$ on $[-3, 2]$ and the x-value(s) at which they occur.

exf21

Ex. L-18 4.1, 4.3/4.4 Fa21 Exam

Let $f(x) = x^3(3x - 4)$.

(a) Find where relative extrema of f occur (if any). Classify each as a local minimum or a local maximum.

(b) Find the absolute extrema of f on $[-1, 2]$ and the x-values at which they occur.

exf21

Ex. L-19 4.1, 4.6 Fa21 Exam

The parts of this problem are not related.

(a) Suppose that when x units are produced, the total cost is $C(x) = 2x^2 + 10x + 18$ and the selling price per unit is $p(x) = 46 - x$. Find the level of production that maximizes total profit.

(b) Suppose the total cost of producing q units is $C(q) = q^3 + 20q^2 + 200q + 2000$. Use marginal analysis to estimate the cost of the 3rd unit.

exf21

Ex. L-20 4.1 Sp22 Exam

Find the absolute extreme values of $f(x) = x\,(x - 8)^{5/3}$ on the interval $[0, 9]$ and the x-values at which they occur.

exs22

Ex. L-21 4.1, 4.3/4.4 Su22 Exam

Let $f(x) = Ax^B \ln(x)$, where A and B are unspecified constants. Suppose that $(e^5, 10)$ is a point of local extremum for $f(x)$.

(a) Calculate the values of A and B.

(b) Determine whether $(e^5, 10)$ is a point of local minimum or a point of local maximum for $f(x)$. Explain your answer.

exsu22

Ex. L-22 4.1, 4.3/4.4 Su22 Exam

For each part, find the absolute extreme values of the given function on the given interval. If a particular extreme value does not exist, write "DNE" as your answer, and explain why that extreme value does not exist.

(a) $f(x) = \frac{e}{x} + \ln(x)$ on $[1, e^3]$

(b) $g(x) = 12x - x^3$ on $[0, \infty)$

exsu22

Ex. L-23 4.1 Sp18 Quiz

Find the absolute maximum and absolute minimum values of $f(x) = \frac{10x}{x^2+1}$ on the interval $[0, 2]$.

qs18

Ex. L-24 4.1 Su22 Quiz

Find the absolute extrema of $f(x) = 10 + 8x^2 - x^4$ on the interval $[-1, 3]$.

qsu22

Ex. L-25 4.1 Su22 Quiz

Let $f(x) = x^2(5x+9)^{1/5}$. Observe that the domain of f is $(-\infty, \infty)$. Calculate the critical numbers of f. For each critical number you find, explain precisely why your answer is a critical number.

qsu22

Ex. L-26 4.1 Fa22 Quiz

Let $f(x) = 3x^{4/3} - 300x^{1/3}$. Find all critical points of f. You must make clear why each of your answers is a critical point.

qf22

Ex. L-27 4.1 Fa22 Quiz

Find the absolute extrema of $f(x) = \sqrt{2}\sin(x) + \cos^2(x)$ on the interval $[0, \pi]$. *Hint:* You will need the approximation $\sqrt{2} \approx 1.4$.

qf22

Ex. L-28 4.1

For each part, find the absolute extreme values of $f(x)$ on the given interval. *You may use a scientific calculator for parts (j) and (k) only.*

(a) $f(x) = x^4 - 8x^2$ on $[-3, 3]$

(b) $f(x) = x^3 + 3x^2 - 24x - 72$ on $[-4, 4]$

(c) $f(x) = \sqrt{x}(x-5)^{1/3}$ on $[0, 6]$

(d) $f(x) = e^{-x}\sin(x)$ on $[0, 2\pi]$

(e) $f(x) = x\left(\ln(x) - 5\right)^2$ on $[e^{-4}, e^4]$

(f) $f(x) = 2x^3 - 9x^2 + 12x$ on $[0, 3]$

(g) $f(x) = \frac{1-x}{x^2+3x}$ on $[1, 4]$

(h) $f(x) = x - 2\sin(x)$ on $[0, 2\pi]$

(i) $f(x) = (x - x^2)^{1/3}$ on $[-1, 2]$

(j) $f(x) = x^3 - 24\ln(x)$ on $[\frac{1}{2}, 3]$

(k) $f(x) = 3e^x - e^{2x}$ on $[-\frac{1}{2}, 1]$

c4s1

Ex. L-29 4.1

Find the absolute extreme values of $f(x) = 3x^4 - 4x^3 - 12x^2$ on $[-2, 1]$.

unit4

Ex. L-30 4.1

Find the absolute extreme values of $f(x) = x^2(x+5)^3$ on $[-6, 0]$.

unit4

Ex. L-31 4.1

Find the absolute minimum and maximum of $f(x) = (6x+1)e^{3x}$ on the interval $[-1000, 1000]$.

unit4

Ex. L-32 4.1, 4.9

The marginal revenue of a certain product is $R'(x) = -9x^2 + 17x + 30$, where x is the level of production. Assume $R(0) = 0$. Find the market price that maximizes revenue.

unit4

Ex. L-33 4.1

Calculate the absolute extreme values of $f(x) = \dfrac{225 - 75x^2}{5x + x^3}$ on $[-5, -1]$.

other

§4.3, 4.4: What Derivatives Tell Us and Graphing Functions

Ex. M-1 4.3/4.4 Fa17 Exam

Consider the function $f(x) = (x-5)(x+10)^2 = x^3 + 15x^2 - 500$.

(a) Calculate all x- and y-intercepts of f.

(b) Find where f is increasing and find where f is decreasing. Then calculate the x- and y-coordinates of all local extrema, classifying each as either a local minimum or a local maximum.

(c) Find where f is concave up and find where f is concave down. Then calculate the x- and y-coordinates of all inflection points.

(d) Sketch the graph of $y = f(x)$ on the provided grid. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

exf17

Ex. M-2 4.3/4.4 Sp18 Exam

Suppose $f(x)$ satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

domain of f:	$[-8, 8]$
specific points on graph:	$f(-2) = -3$ and $f'(-6) = 0$
asymptotes of f:	$x = -2$ and $y = -3$
f is decreasing on:	$[-8, -2)$, $(-2, 2)$
f is increasing on:	$(2, 8]$
f is concave down on:	$(-1, 1)$
f is concave up on:	$[-8, -1)$, $(1, 8]$

exs18

Ex. M-3 4.3/4.4 Sp18 Exam

Consider the function f and its derivatives below.

$$f(x) = \frac{x^2}{x^2-1} \quad , \quad f'(x) = \frac{-2x}{(x^2-1)^2} \quad , \quad f''(x) = \frac{6x^2+2}{(x^2-1)^3}$$

(a) Find all horizontal asymptotes of f.

(b) Find all vertical asymptotes of f. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of f.

(c) Find where f is decreasing and find where f is increasing. Then calculate all points of local extrema, classifying each as either a local minimum, a local maximum, or neither.

(d) Find where f is concave down and find where f is concave up. Then calculate all points of inflection.

exs18

Ex. M-4 4.3/4.4 Fa18 Exam

Consider the function f and its derivatives below.

$$f(x) = \frac{2x^3+3x^2-1}{x^3} \quad , \quad f'(x) = \frac{3-3x^2}{x^4} \quad , \quad f''(x) = \frac{6x^2-12}{x^5}$$

For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

(a) Find all horizontal asymptotes of f.

(b) Find all vertical asymptotes of f. Then for each asymptote, find the corresponding one-sided limits of f.

(c) Find where f is decreasing, where f is increasing, and where f has a local extremum.

(d) Find where f is concave down, where f is concave up, and where f has an inflection point.

exf18

Ex. M-5 **4.3/4.4** Sp19 **Exam**

Consider the function f and its derivatives below.

$$f(x) = 2x + \frac{8}{x^2} \quad , \quad f'(x) = \frac{2\left(x^3 - 8\right)}{x^3} \quad , \quad f''(x) = \frac{48}{x^4}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

exs19

Ex. M-6 **4.3/4.4** Fa19 **Exam**

Find the x-coordinate of each inflection point, if any, of $f(x) = x^3 - 12x^2 + 5x - 10$.

Ex. M-7 **4.3/4.4** Fa19 **Exam**

Consider the function f and its derivatives below.

$$f(x) = \frac{3x^3 - 2x + 48}{x} \quad , \quad f'(x) = \frac{6(x^3 - 8)}{x^2} \quad , \quad f''(x) = \frac{6(x^3 + 16)}{x^3}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

4.3/4.4 Fa19 Exam

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

Ex. M-8 4.3/4.4 Sp20 Exam

For each part, sketch the graph of a function that satisfies the given properties.

(a) $f(x)$ is decreasing for all x; $f''(x) < 0$ for $x < 13$; $f''(x) > 0$ for $x > 13$.

(b) $f(x)$ has a local minimum at $x = a$ where $f'(a) = 0$.

(c) $f(x)$ has a local maximum at $x = b$ where $f'(b)$ is undefined.

exs20

Ex. M-9 4.3/4.4 Sp20 Exam

The first two derivatives of the function f are given below.

$$f'(x) = \frac{x}{(x-6)^2(x+48)} \quad , \quad f''(x) = \frac{-2(x+12)^2}{(x-6)^3(x+48)^2}$$

(Do not attempt to find a formula for $f(x)$.)

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

exs20

Ex. L-8 **4.1, 4.3/4.4** Sp20 **Exam**

Consider the following function

$$g(x) = \frac{3}{2}x^4 + 8x^3 - 36x^2$$

(a) Where does g have a local minimum on $(-7, 3)$? local maximum?

(b) Where does g have a global minimum on $[-7, 3]$? global maximum?

exs20

Ex. M-10 **4.3/4.4** Sp20 **Exam**

Consider the function f and its first two derivatives below.

$$f(x) = \frac{99e^x}{(x-25)^{47}} + 98 \quad , \quad f'(x) = \frac{99e^x(x-72)}{(x-25)^{48}} \quad , \quad f''(x) = \frac{99e^x\left((x-72)^2+47\right)}{(x-25)^{49}}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

exs20

Ex. M-11 **4.3/4.4** Su20 **Exam**

Suppose f is continuous for all x and its first derivative is given by $f'(x) = (x-4)^2(x+2)$.

(a) Where is f decreasing?

(b) A student writes "since $f'(4) = 0$, there is a local extremum (either min or max) at $x = 4$". Is the student correct? Explain.

(c) Where is f concave up?

(d) Find the x-coordinate of each inflection point of f.

exsu20

Ex. M-12 **4.3/4.4** Su20 **Exam**

Suppose $f(x)$ satisfies all of the following properties.

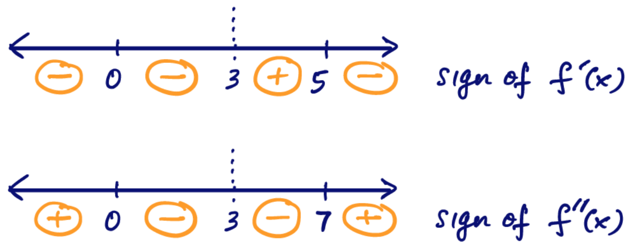
- $f(x)$ is continuous and differentiable on $(-\infty, 3) \cup (3, \infty)$
- $x = 3$ is a vertical asymptote of $f(x)$
- $\lim\limits_{x\to\infty} f(x) = 1$
- the only x-values for which $f'(x) = 0$ are $x = 0$ and $x = 5$

exsu20

4.3/4.4 Su20 Exam

- the only x-values for which $f''(x) = 0$ are $x = 0$ and $x = 7$

A sign chart for the first and second derivatives of f are given below.



Use this information to answer the following questions about $f(x)$. ***Note:*** Do not attempt to find a formula for $f(x)$.

(a) Where is f increasing?

(b) Where is f concave down?

(c) At which x-value(s) does f have a local minimum?

(d) At which x-value(s) does f have a local maximum?

(e) Calculate $\lim_{x\to 3^+} f(x)$ or determine there is not enough information to do so.

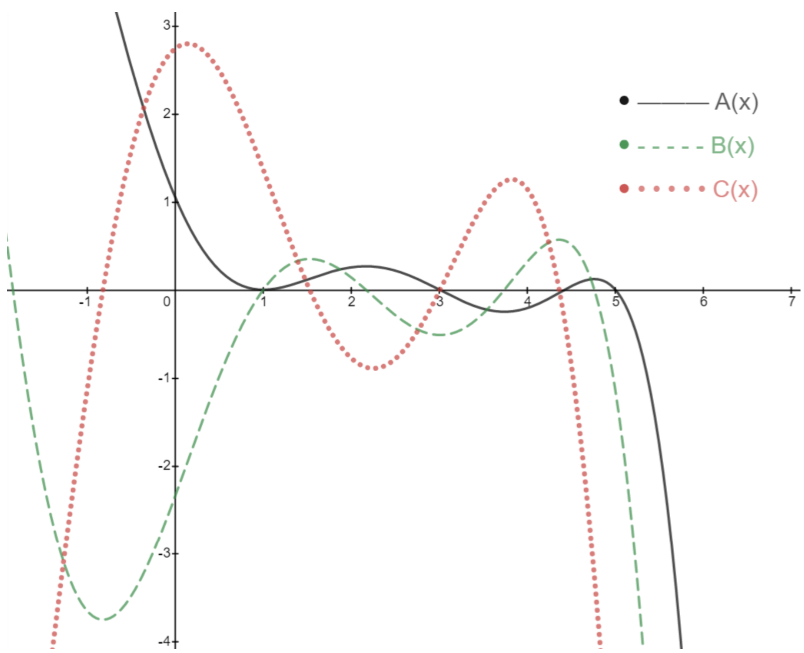
(f) Calculate $\lim_{x\to -\infty} f(x)$ or determine there is not enough information to do so.

(g) Sketch a possible graph of $y = f(x)$. Clearly mark and label all of the following: local minima, local maxima, inflection points, vertical asymptotes, horizontal asymptotes. *Your graph does not have to be to scale, but the shape must be correct.*

exsu20

Ex. M-13 4.3/4.4 Fa20 Exam

The figure below shows the graphs of f, f', and f''. Identify which graph is that of f''.



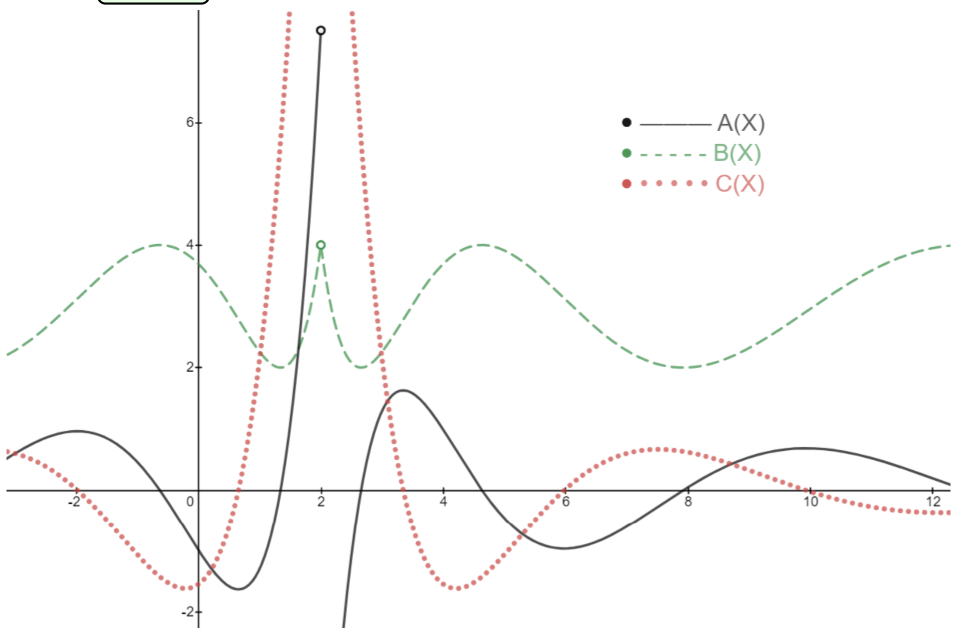
exf20

Ex. M-14 4.3/4.4 Fa20 Exam

The figure below shows the graphs of f, f', and f''. Identify which graph is which.

exf20

4.3/4.4 Fa20 Exam



exf20

Ex. M-15 4.3/4.4 Fa20 Exam

Suppose $f(x)$ satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

Information from $f(x)$:

- $\lim_{x\to-\infty} f(x) = 1$
- $\lim_{x\to\infty} f(x) = 6$
- $x = -3$ is a vertical asymptote for f

Information from $f'(x)$:

- $f'(x) > 0$ on $(2, \infty)$
- $f'(x) < 0$ on $(-\infty, -3)$ and $(-3, 2)$
- $f'(2) = 0$

Information from $f''(x)$:

- $f''(x) > 0$ on $(-3, 5)$
- $f''(x) < 0$ on $(-\infty, -3)$ and $(5, \infty)$
- $f''(5) = 0$

exf20

Ex. M-16 4.3/4.4 Fa20 Exam

The first and second derivative of f are given below. You may assume that $f(x)$ has a vertical asymptote at $x = 25$ only, but do not attempt to calculate $f(x)$ explicitly.

$$f'(x) = \frac{(x+2)^{1/5}}{(x-25)^2} \quad , \quad f''(x) = \frac{-9(x+5)}{5(x-25)^3(x+2)^{4/5}}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

exf20

4.3/4.4 Fa20 Exam

where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

exf20

Ex. M-17 4.3/4.4 Sp21 Exam

Consider the function $f(x)$ whose second derivative is given.

$$f''(x) = \frac{(x-2)^2(x-5)^3}{(x-9)^5}$$

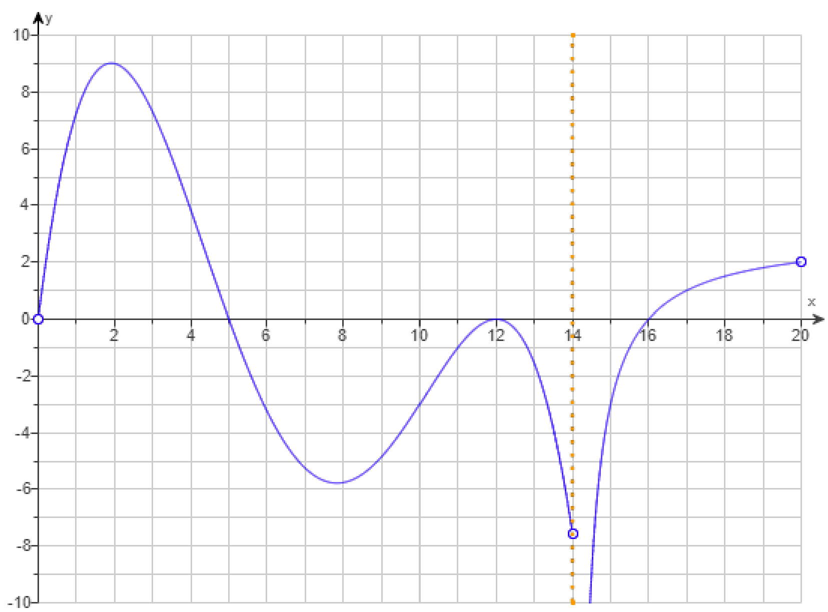
You may assume the domain of $f(x)$ is $(-\infty, 9) \cup (9, \infty)$.

Find where $f(x)$ is concave down, where $f(x)$ is concave up, and where $f(x)$ has an inflection point. Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

exs21

Ex. M-18 4.3/4.4 Sp21 Exam

Use the graph of $y = f'(x)$ below to answer the questions. You may assume that $f'(x)$ has a vertical asymptote at $x = 14$ and that the domain of f is $(0, 14) \cup (14, 20)$.



Note: You are given a graph of the first derivative of f, not a graph of f.

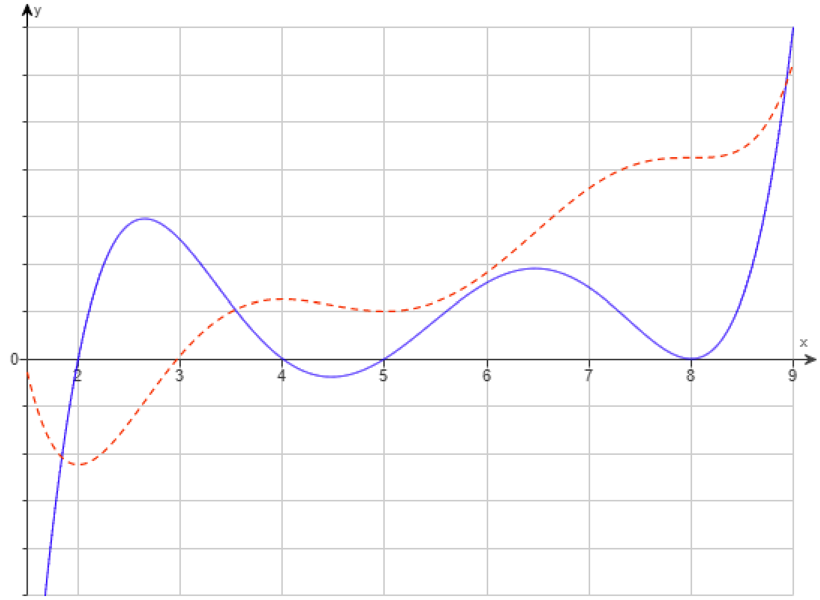
(a) Find the critical points of f.

(b) Find where f is decreasing, where f is increasing, where f has a local minimum, and where f has a local maximum. Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

exs21

Ex. M-19 **4.3/4.4** Sp21 **Exam**

The figure below shows the graphs of two functions. One function is $f(x)$ and the other is $f'(x)$, but you are not told which is which.



(a) Which graph is that of $y = f(x)$?

(b) Explain your answer to part (a) based on the behavior of the graphs at $x = 4$ only.

(c) Explain your answer to part (a) based on the behavior of the graphs near $x = 3.5$ only.

exs21

Ex. M-20 **4.3/4.4** Fa21 **Exam**

Consider the function f and its derivatives below.

$$f(x) = \frac{x-3}{x^2-6x-16} \quad , \quad f'(x) = \frac{-(x-3)^2-25}{(x^2-6x-16)^2} \quad , \quad f''(x) = \frac{2(x-3)\left((x-3)^2+75\right)}{(x^2-6x-16)^3}$$

Find where f is concave down and where f is concave up; write your answers using interval notation. Also find the x-coordinate of each inflection point of f.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

exf21

Ex. M-21 **4.3/4.4** Fa21 **Exam**

Suppose f is differentiable on $(-\infty, 1) \cup (1, \infty)$ and satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

(i) $\lim\limits_{x\to-\infty} f(x) = -3$; $\lim\limits_{x\to\infty} f(x) = \infty$; $\lim\limits_{x\to1^-} f(x) = -\infty$; $\lim\limits_{x\to1^+} f(x) = \infty$;

(ii) $f'(x) > 0$ on $(-\infty, -2)$ and $(5, \infty)$; $f'(x) < 0$ on $(-2, 1)$ and $(1, 5)$; $f'(-2) = f'(5) = 0$

(iii) $f''(x) > 0$ on $(-\infty, -7)$ and $(1, \infty)$; $f''(x) < 0$ on $(-7, 1)$; $f''(-7) = 0$

exf21

Ex. M-22 **4.3/4.4** Fa21 **Exam**

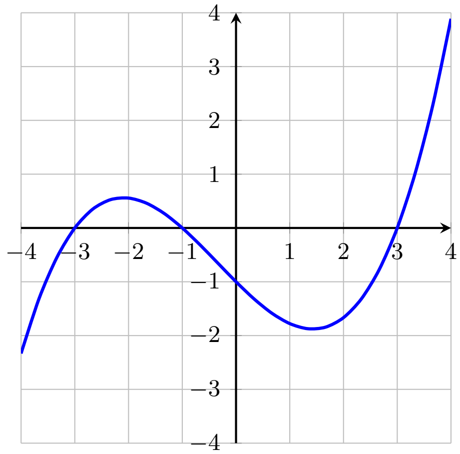
Let $f(x) = -e^{-x}\left(x^2 - 5x - 23\right)$. Find all critical points of f. Then find where f is decreasing and where f is increasing; write your answers using interval notation. Also find where relative extrema of f occur.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

exf21

Ex. B-8 **2.1/2.2, 3.7, 4.3/4.4** Fa21 **Exam**

For each part, use the graph of $y = g(x)$.



(a) How many solutions does the equation $g'(x) = 0$ have?

(b) Order the following quantities from least to greatest: $g'(-2.5)$, $g'(-2)$, $g'(0)$, and $g'(4)$. In your answer, write these quantities symbolically; do not give a numerical estimate.

(c) What is the sign of $g''(-3)$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

(d) Let $h(x) = g(x)^2$. What is the sign of $h'(-4)$(negative, positive, or zero)? If there is not enough information to determine the value, explain why.

exf21

Ex. L-18 **4.1, 4.3/4.4** Fa21 **Exam**

Let $f(x) = x^3(3x - 4)$.

(a) Find where relative extrema of f occur (if any). Classify each as a local minimum or a local maximum.

(b) Find the absolute extrema of f on $[-1, 2]$ and the x-values at which they occur.

exf21

Ex. M-23 **4.3/4.4** Fa21 **Exam**

Consider the function $g(x)$, whose first two derivatives are given below. ***Note:*** Do not attempt to calculate $g(x)$. Also assume that $g(x)$ has the same domain as $g'(x)$.

$$g'(x) = \frac{8x^{17}}{x - 32} \qquad g''(x) = \frac{128x^{16}(x - 34)}{(x - 32)^2}$$

Fill in the table below with information about the graph of $y = g(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

exf21

4.3/4.4 Fa21 **Exam**

where g is decreasing	
where g is increasing	
x-coordinate(s) of local minima of g	
x-coordinate(s) of local maxima of g	
where g is concave down	
where g is concave up	
x-coordinate(s) of inflection point(s) of g	

exf21

Ex. M-24 **4.3/4.4** Sp22 **Exam**

Let $f(x) = 4x^5 - 20x^4 + 7x + 32$. Find where f is concave down and where f is concave up; write your answer using interval notation. Also find where inflection points of f occur.

exs22

Ex. M-25 **4.3/4.4** Sp22 **Exam**

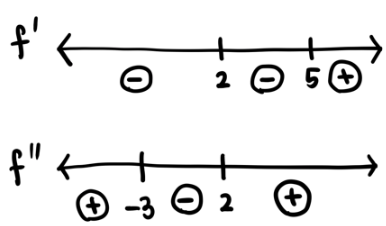
Suppose $f(x)$ satisfies all of the following properties. Sign charts for f' and f'' are also given below. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

(i) f is continuous and differentiable on $(-\infty, 2) \cup (2, \infty)$

(ii) $\lim_{x\to-\infty} f(x) = \infty$; $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to 2^-} f(x) = -\infty$; $\lim_{x\to 2^+} f(x) = \infty$

(iii) the only x-value for which $f'(x) = 0$ is $x = 5$

(iv) the only x-value for which $f''(x) = 0$ is $x = -3$



exs22

Ex. M-26 **4.3/4.4** Sp22 **Exam**

Let $f(x) = \dfrac{x^2 + 21}{x - 2}$. Find where f is decreasing and where f is increasing; write your answer using interval notation. Also find where the local extrema of f occur.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

exs22

Ex. L-21 **4.1, 4.3/4.4** Su22 **Exam**

Let $f(x) = Ax^B \ln(x)$, where A and B are unspecified constants. Suppose that $(e^5, 10)$ is a point of local extremum for $f(x)$.

(a) Calculate the values of A and B.

(b) Determine whether $(e^5, 10)$ is a point of local minimum or a point of local maximum for $f(x)$. Explain your answer.

exsu22

Ex. L-22 **4.1, 4.3/4.4** Su22 **Exam**

For each part, find the absolute extreme values of the given function on the given interval. If a particular extreme value does not exist, write “DNE” as your answer, and explain why that extreme value does not exist.

(a) $f(x) = \frac{e}{x} + \ln(x)$ on $[1, e^3]$

(b) $g(x) = 12x - x^3$ on $[0, \infty)$

exsu22

Ex. M-27 **4.3/4.4** Su22 **Exam**

Consider the function f and its derivatives below.

$$f(x) = \frac{x^2}{x-7} \qquad f'(x) = \frac{x(x-14)}{(x-7)^2} \qquad f''(x) = \frac{98}{(x-7)^3}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write “NONE” as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

exsu22

Ex. M-28 **4.3/4.4, 4.7** Su22 **Exam**

Let $f(x) = x^2e^x$.

(a) Calculate the vertical and horizontal asymptotes of f.

(b) Calculate the critical points of f. Then use the Second Derivative Test to classify each critical point of f as a local minimum or a local maximum. Show your work and label your answers clearly. ***Hint:*** The second derivative of f is $f''(x) = (x^2 + 4x + 2)e^x$.

exsu22

Ex. M-29 **4.3/4.4** Su22 **Quiz**

Consider the function g and its derivatives below.

$$g(x) = x^2 - \frac{27}{x} \qquad g'(x) = 2x + \frac{27}{x^2} \qquad g''(x) = 2 - \frac{54}{x^3}$$

Fill in the table below with information about the graph of $y = g(x)$. For each part, write “NONE” as your answer if appropriate. (You may use the bottom or back of this page for scratch work.) ***You do not have to show work, and each part of the table will be graded with no partial credit.***

qsu22

4.3/4.4 Su22 **Quiz**

vertical asymptote(s) of g:	
horizontal asymptote(s) of g:	
g is increasing on:	
g is decreasing on:	
g is concave up on:	
g is concave down on:	
x-coordinate(s) of relative maxima	
x-coordinate(s) of relative minima	
x-coordinate(s) of inflection point(s)	

qsu22

Ex. M-30 **4.3/4.4** Fa22 **Quiz**

Consider the function f and its derivatives below.

$$f(x) = \frac{x^4}{3-x} \qquad f'(x) = \frac{x^3(12-3x)}{(3-x)^2} \qquad f''(x) = \frac{6x^2((x-4)^2+2)}{(3-x)^3}$$

Fill in the table below with information about the graph of $y = f(x)$. Write your answers using interval notation if appropriate. For each part, write "NONE" as your answer if appropriate.

vertical asymptote(s) of f	
horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

qf22

Ex. M-31 **4.3/4.4**

For each part, do all of the following.

(i) Find all vertical asymptotes and horizontal asymptotes of $f(x)$.

(ii) Find where $f(x)$ is decreasing and where $f(x)$ is increasing. Also find and classify all local extrema of $f(x)$.

c4s3

4.3/4.4

(iii) Find where $f(x)$ is concave down and where $f(x)$ is concave up. Also find all inflection points of $f(x)$.

(iv) Sketch a graph of $y = f(x)$.

(a) $f(x) = \frac{1}{3}x^3 - 9x + 2$

(b) $f(x) = (x+1)^2(x-5)$

(c) $f(x) = \dfrac{x}{x^2+1}$

(d) $f(x) = x - \sin(2x)$ (on $[0, \pi]$ only)

(e) $f(x) = 1 + 2x + 18x^{-1}$

(f) $f(x) = 1 - \dfrac{x}{4-x}$

(g) $f(x) = 10x^3 - x^5$

(h) $f(x) = \dfrac{1}{x^3+8}$

(i) $f(x) = \dfrac{x^3}{x-1}$

(j) $f(x) = \dfrac{1}{x^3-3x}$

c4s3

Ex. M-32 4.3/4.4

Sketch the graph of a function f that satisfies all of the following conditions.

- $f'(x) > 0$ when $x < 2$ and when $2 < x < 5$
- $f'(x) < 0$ when $x > 5$
- $f'(2) = 0$
- $f''(x) < 0$ when $x < 2$ and when $4 < x < 7$
- $f''(x) > 0$ when $2 < x < 4$ and when $x > 7$

c4s3

Ex. M-33 4.3/4.4

Sketch the graph of a function f that satisfies all of the following conditions.

- the lines $y = 1$ and $x = 3$ are asymptotes
- f is increasing for $x < 3$ and $3 < x < 5$, and f is decreasing elsewhere
- the graph of $y = f(x)$ is concave up for $x < 3$ and for $x > 7$
- the graph of $y = f(x)$ is concave down for $3 < x < 7$
- $f(0) = f(5) = 4$ and $f(7) = 2$

c4s3

Ex. M-34 4.3/4.4

Consider the function

$$f(x) = e^{-x^2/2}$$

Find where f is concave down and find where f is concave up. Then find all inflection points (x- and y- coordinates). *Write "NONE" for your answer if appropriate.*

unit4

Ex. M-35 4.3/4.4

Consider the function

$$f(x) = \frac{1}{x^2 - 6x}$$

Find all vertical asymptotes of f. Then find where f is decreasing and find where f is increasing. Finally determine the x-coordinates of all local extrema of f (and classify them as either a local minimum or a local maximum). *Write "NONE" for your answer if appropriate.*

unit4

Ex. M-36 4.3/4.4

Consider the function f and its derivatives below.

$$f(x) = \frac{(x-1)^2}{(x+2)(x-4)} \quad , \quad f'(x) = \frac{-18(x-1)}{(x+2)^2(x-4)^2} \quad , \quad f''(x) = \frac{54((x-1)^2+3)}{(x+2)^3(x-4)^3}$$

Find the vertical and horizontal asymptotes of f. Then find where f is decreasing, where f is increasing, where f is concave down, and where f is concave up. Calculate the x-coordinates of all local minima, local maxima, and points of inflection.

unit4

Ex. M-37 **4.3/4.4**

Consider the function f and its derivatives given below.

$$\boxed{f(x) = \frac{1}{(x+4)^2(x-6)^2}} \quad \boxed{f'(x) = \frac{-4(x-1)}{(x+4)^3(x-6)^3}} \quad \boxed{f''(x) = \frac{20((x-1)^2+5)}{(x+4)^4(x-6)^4}}$$

(i) Find all vertical asymptotes and horizontal asymptotes of $f(x)$.

(ii) Find where $f(x)$ is decreasing and where $f(x)$ is increasing. Also find and classify all local extrema of $f(x)$.

(iii) Find where $f(x)$ is concave down and where $f(x)$ is concave up. Also find all inflection points of $f(x)$.

(iv) Sketch a graph of $y = f(x)$.

other

Ex. M-38 **4.3/4.4** **⋆Challenge**

Consider the function $f(x) = ax^6e^{-bx}$, where a and b are unspecified constants. Suppose f has a point of local maximum at $(2, 64e^{-2})$. Find the values of a and b.

challenge

Ex. M-39 **4.3/4.4** **⋆Challenge** **!!!**

Consider the function $f(x) = (x-3a)(x+2a)^4$, where a is an unspecified **positive** constant. Answer all of the following in terms of a.

(a) where is f decreasing?

(b) where is f increasing?

(c) where does f have a local minimum?

(d) where does f have a local maximum?

(e) where is f concave down?

(f) where is f concave up?

(g) where does f have an inflection point?

Finally, sketch a graph of $y = f(x)$. Your horizontal scale should be in terms of a and your vertical scale should be in terms of a^5.

challenge

Ex. M-40 **4.3/4.4** **⋆Challenge** **!!!**

Let $f(x) = \dfrac{e^x}{4+x^3}$. Answer all of the following.

(a) what are the vertical asymptotes of f?

(b) what are the horizontal asymptotes of f?

(c) where is f decreasing?

(d) where is f increasing?

(e) where does f have a local minimum?

(f) where does f have a local maximum?

challenge

Ex. M-41 **4.3/4.4** **⋆Challenge**

Let $f(x) = \sqrt[3]{x^3 - 48x}$.

(i) Find all vertical asymptotes and horizontal asymptotes of $f(x)$.

(ii) Find where $f(x)$ is decreasing and where $f(x)$ is increasing. Also find and classify all local extrema of $f(x)$.

(iii) Find where $f(x)$ is concave down and where $f(x)$ is concave up. Also find all inflection points of $f(x)$.

(iv) Sketch a graph of $y = f(x)$.

challenge

§4.5: Optimization Problems

Ex. N-1 4.5 Fa17 Exam

A wire of length 51 cm is cut into two pieces. One piece is bent into a square. The other piece is bent into a rectangle whose length is two times its width. How should the wire be cut and the pieces assembled so that the total area enclosed by both pieces is a minimum?

You must use calculus-based methods in your work. You must also justify that your answer really does give the minimum.

exf17

Ex. N-2 4.5 Sp18 Exam

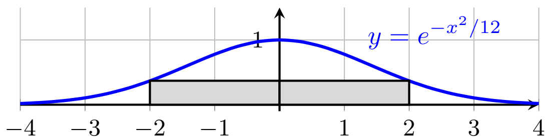
You are constructing a rectangular box with a total surface area (six sides) of 450 in^2. The length of the box is three times its width. Find the dimensions of the box, measured in inches, with the largest possible volume.

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.

exs18

Ex. N-3 4.5 Fa18 Exam

Find the maximum possible area of a rectangle inscribed in the region between the graph of $f(x) = e^{-x^2/12}$ and the x-axis. *You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.*



exf18

Ex. N-4 4.5 Sp19 Exam

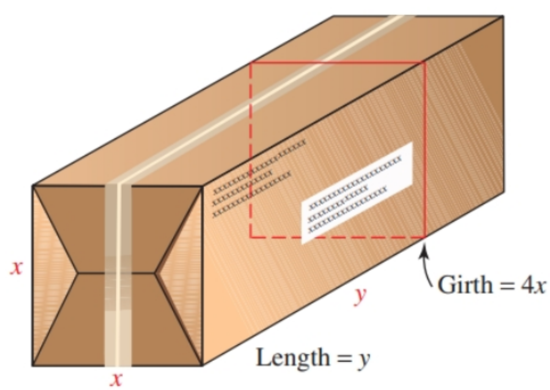
The cost of producing x units is $C(x) = 2x^2 + 5x + 8$. Find the level of production (value of x) that minimizes the average cost. ***Hint:*** Average cost is $AC(x) = \frac{C(x)}{x}$.

exs19

Ex. N-5 4.5 Sp19 Exam

According to postal regulations, the sum of the girth and length of a parcel may not exceed 90 inches. What are the dimensions (in inches) of the parcel with the largest possible volume that can be sent, if the parcel is a rectangular box with two square sides?

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.



exs19

Ex. N-6 4.5 Fa19 Exam

When x units of a certain product are produced, the total cost is $C(x) = 5x^2 + 104x + 80$. Find the level of production which minimizes the average cost per unit.

Ex. N-7 4.5 Fa19 Exam

A rectangular container with a closed top and a square base is to be constructed. The top and all four sides of the container are to be made of material that costs $\$2/\text{ft}^2$, and the bottom is to be made of material that costs $\$3/\text{ft}^2$. Find the container with the largest volume that can be constructed for a total cost of \$60.

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.

Ex. N-8 4.5 Sp20 Exam

Let x be the level of production for a certain commodity. The marginal cost is modeled by the function

$$\frac{dC}{dx} = 3x^2 + 2x$$

and the market price is modeled by the function

$$p(x) = 144 - 2x$$

Suppose that the cost of producing the 1st unit of the commodity is 70.

(a) What is the cost of producing the first 3 units of the commodity?

(b) What is the level of production that maximizes the total profit?

exs20

Ex. N-9 4.5 Sp20 Exam

Suppose the local post office has a policy that all packages must be shaped like a rectangular box with a sum of length, width, and height not exceeding 144 inches. You plan to construct such a package whose length is 2 times its width. Find the dimensions of the package with the largest volume. For this problem, let L, W, and H be the length, width, and height of the package, respectively.

(a) What is the objective function for this problem in terms of L, W, and H?

(b) There are two constraints for this problem. In terms of L, W, and H, give the constraint equation which corresponds to...

 (i) ...the policy set by the post office.

 (ii) ...your specific plan to construct such a package.

(c) Find the objective function in terms of W only.

(d) What is the interval of interest for the objective function?

(e) Find the values of L, W, and H that give the largest volume.

(f) Suppose the post office adds the additional requirement that the width W of the package must be no smaller than 36 inches and no larger than 40 inches. With this additional policy, what is the width of the package with the largest volume?

exs20

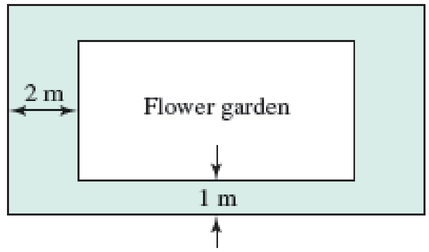
Ex. N-10 4.5 Sp20 Exam

A local park has hired you to construct a rectangular flower garden surrounded by a grass border that is 1 m wide on two sides and 2 m wide on the other two sides. (See the figure below.) The area of the garden only (the small rectangle) must be 126 m^2.

Your primary task is to find the dimensions of the garden that give the smallest possible combined area of the garden and the grass border. For this problem, let W be the horizontal width of the garden and let H be the vertical height of the garden.

exs20

4.5 Sp20 Exam



(a) What is the objective function for this problem in terms of W and H?
(b) What is the constraint equation for this problem in terms of W and H?
(c) Find the objective function in terms of W only.
(d) What is the interval of interest for the objective function?
(e) Find the values of W and H that minimize the total combined area.
(f) What horizontal width W of the garden will *maximize* the total area?

exs20

Ex. N-11 4.5 Su20 Exam

Farmer Brown wants to create a rectangular pen that must enclose exactly 1800 ft^2. The fencing along the north and south sides of the fence costs \$10/ft and the fencing along the east and west sides costs \$5/ft. (The cost is different because some parts of the fence have to be taller than other parts.) Let x denote the length of the north side and let y denote the length of the east side.

(a) What are the dimensions and total cost of the cheapest pen?
(b) Justify that your answer really does give the cheapest pen.

exsu20

Ex. N-12 4.5 Su20 Exam

In a certain video game, the player may adjust the values of their character's *Intelligence* (denoted by x) and *Dexterity* (denoted by y). These power values must be non-negative but can be any real number (they need not be whole numbers). The player cannot arbitrarily adjust their power, but rather these values must satisfy the equation $x^2+y^2 = 100$. The total damage done (denoted by D) by the spell *Thunderbolt* is given by $D = x + 3y$.

(a) How should the player adjust their power so that *Thunderbolt* does the most possible damage?
(b) What is the minimum possible damage that *Thunderbolt* will do, regardless of how the player adjusts their character's power? How should a player adjust these power values to achieve the minimum possible damage?

exsu20

Ex. N-13 4.5 Fa20 Exam

A rectangular box with a square base and no top is being constructed to hold a volume of 150 cm^3. The material for the base of the container costs \$6/cm^2 and the material for the sides of the container costs \$2/cm^2. Find the dimensions of the cheapest possible container.

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.

exf20

Ex. N-14 4.5 Sp21 Exam

An airline policy states that all baggage must be shaped like a rectangular box with the sum of the length, width, and height not exceeding 122 inches. You plan to purchase a bag from a company that makes customized bagged whose height must be 3 times its width. Find the dimensions of the baggage with the largest volume. (Let L, W, and H be the length, width, and height of the baggage, respectively.)

(a) Before using any constraints particular to this problem, find the objective function in terms of L, W, and H.
(b) There are two constraints for this problem. One constraint is from the airline and the other is from the baggage company. Find these constraints.
(c) Write the objective function in terms of W only.

exs21

4.5 Sp21 Exam

(d) Find the interval of interest for the objective function in part (c).

(e) Find the dimensions of the baggage with the largest volume.

exs21

Ex. N-15 4.5 Fa21 Exam

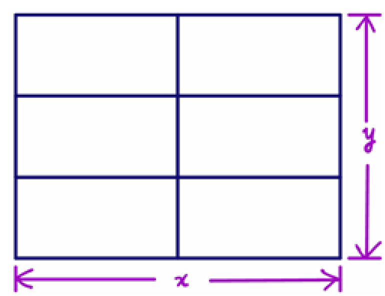
A storage shed with a volume of 1500 ft^3 is to be built in the shape of a rectangular box with a square base. The material for the base costs \$ 6/ft^2, the material for the roof costs \$ 9/ft^2, and the material for the sides costs \$2.50/ft^2. Find the dimensions of the cheapest shed. As you work, fill in the answer boxes below. Let x represent the length of the base of the shed.

objective function in terms of x:	
interval of interest:	
dimensions of cheapest shed (in ft):	______ × ______ × ______ length of base, width of base, height of shed

exf21

Ex. N-16 4.5 Fa21 Exam

Farmer Green is building an enclosure that must have a total area of 48 m^2. The pen will also be subdivided into 6 pens of equal area, as shown on the right. Find the dimensions of the enclosure that will require the least amount of fencing. *As you work, fill in the answer boxes below. You must use calculus-based methods in your work. You must also justify that your answer really does give the least fencing.*



constraint equation in terms of x and y:	
objective function in terms of x only:	
interval of interest:	
dimensions of desired enclosure (in meters):	______ × ______ total length (x), total width (y)

exf21

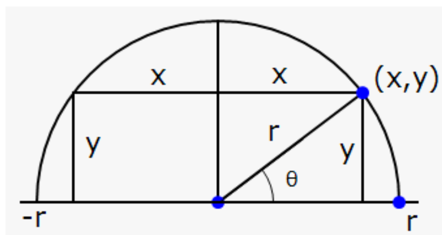
Ex. N-17 4.5 Sp22 Exam

A rectangle (with base $2x$ and height y) is constructed with its base on the diameter of a semicircle with radius 5 and with its two other vertices on the semicircle. Find the dimensions of the rectangle with the maximum possible area. *As you work, fill in the answer boxes below. You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.*

exs22

4.5 Sp22 Exam

constraint equation in terms of x and y:	
objective function in terms of x only:	
interval of interest:	
dimensions of rectangle:	______ × ______ $2x$ (base) y (height)

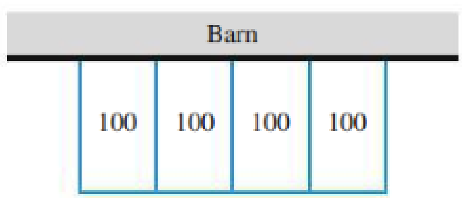


exs22

Ex. N-18 4.5 Su22 Exam

A rancher plans to make four identical and adjacent rectangular pens against a barn, each with an area of 100 m^2 (see the figure below). What are the dimensions of each pen that minimize the amount of fence that must be used? ***Note:*** No fencing is needed on the side of the pen that borders the barn (the north side of the pen).

As you work, fill in the answer boxes below. You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.



constraint equation(s):	
objective function in one variable only:	
interval of interest:	
dimensions of one pen:	______ × ______ horizontal dimension vertical dimension

exsu22

Ex. N-19 4.5 Su22 Quiz

An airline policy states that all carry-on baggage must be box-shaped with a sum of length, width, and height not exceeding 60 in. Suppose the length of a particular carry-on is three times its width. Under the airline's policy, what are the dimensions of such a carry-on with the greatest volume?

qsu22

4.5 Su22 Quiz

You must use calculus-based methods to solve this problem, and you must demonstrate that your answer really does give the greatest volume.

qsu22

Ex. N-20 4.5 Fa22 Quiz

A rectangle is constructed with its lower two vertices on the x-axis and its upper two vertices on the parabola $y = 75 - 3x^2$. Find the dimensions of the rectangle with the greatest area.

In your work, you must clearly define your variables, identify any constraint equations, and identify your objective function (in terms of one variable). You must also verify that your answer really does give a maximum.

qf22

Ex. N-21 4.5

Find the largest possible product of two numbers whose sum is 180.

c4s5

Ex. N-22 4.5

The sum of two numbers is 10. Find the smallest possible value for the sum of their squares.

c4s5

Ex. N-23 4.5

Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 4, assuming that one side of the rectangle lies on the diameter of the semicircle.

c4s5

Ex. N-24 4.5

A farmer is constructing a rectangular fence along a straight river. The side of the rectangle bordering the river does not need any fencing. If the farmer has 1000 feet of fencing, what is the largest possible area he may enclose?

c4s5

Ex. N-25 4.5

A farmer with 1600 feet of fencing wants to enclose a rectangular area and then divide it into four equal-area pens with fencing parallel to one side of the rectangle. What is the largest possible area that a single pen can enclose?

c4s5

Ex. N-26 4.5

Consider the construction of a rectangular aquarium that must hold a volume of 4000 in^3. The length of the base must be twice the width of the base. The top and bottom bases of the tank cost \$1.50/in^2. Each of the sides of the tank costs \$3/in^2. Find the dimensions (length, width, height) of the tank with the least cost.

c4s5

Ex. N-27 4.5

Suppose that the total cost of producing s widgets is $C(x) = x^3 + 9x^2 + 18x + 200$ and the selling price per unit is $p(x) = 45 - 2x^2$. At what price should the widgets be sold to maximize total profit?

c4s5

Ex. N-28 4.5

Suppose the total cost of manufacturing x widgets is $C(x) = 3x^2 + 5x + 75$. What level of production minimizes the average cost per unit?

c4s5

Ex. N-29 4.5

The total cost of producing x widgets is

$$C(x) = x^3 - 6x^2 + 15x$$

and the selling price per unit is fixed at $p(x) = 6$. Show that if you want to set a level of production to maximize total profit, the best you can do is break even.

c4s5

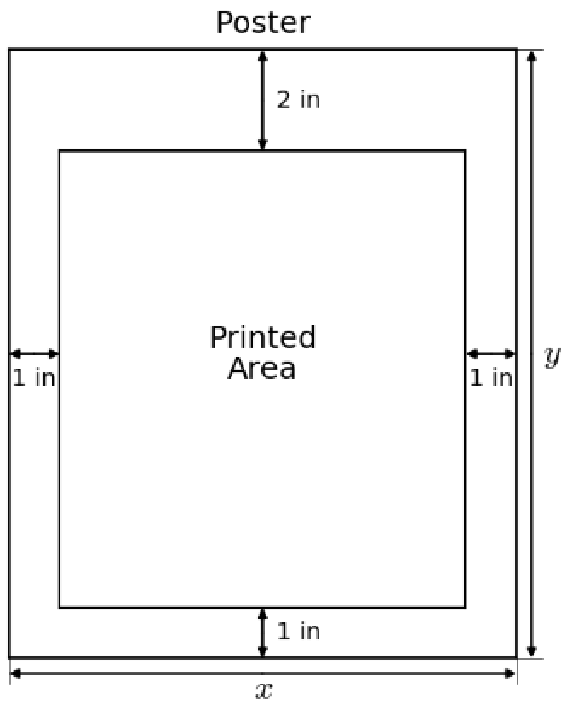
Ex. N-30 4.5

If x units are produced, then the total cost is $C(x) = x^3 + 4x^2 + 60x + 200$ and the selling price per unit is $p(x) = 100 - 3x$. Find the level of production that maximizes the total profit.

unit4

Ex. N-31 4.5

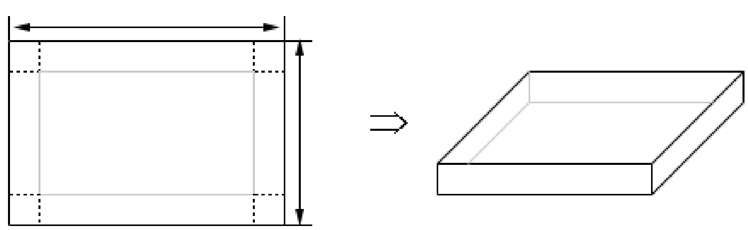
A poster is to have a total area of 150 in^2, which includes a central printed area, 1-inch margins at the bottom and sides, and a 2-inch margin at the top. What poster dimensions (in inches) will give the largest printed area? Use calculus to justify your answer.
You must demonstrate that your answers really are the optimal dimensions.



unit4

Ex. N-32 4.5

A piece of cardboard that is 24 inches wide and 15 inches long is to be used to construct a box with an open top. To do this, congruent squares are cut from each corner of the cardboard, and the flaps are folded up and taped to form the sides of the box. What is the largest possible volume of such a box? Use calculus to justify your answer.
You must demonstrate that your answers really are the optimal dimensions.



unit4

Ex. N-33 4.5

A cylindrical can must have a volume of 32π cm^3. The cost of each of the top and bottom is \$6/cm^2 and the cost of the curved side surface is \$3/cm^2. Find the radius and height of the least expensive can. Justify that your answer does, in fact, give the minimum cost.

unit4

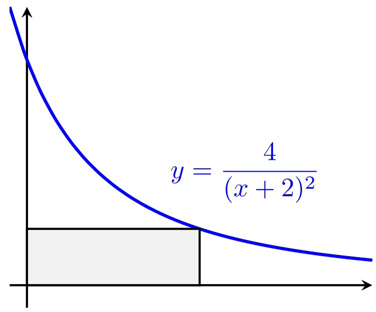
Ex. N-34 4.5

Find the maximum possible area of a rectangle inscribed in the region below the graph of $y = \dfrac{4}{(x+2)^2}$ and in the

other

4.5

first quadrant.



other

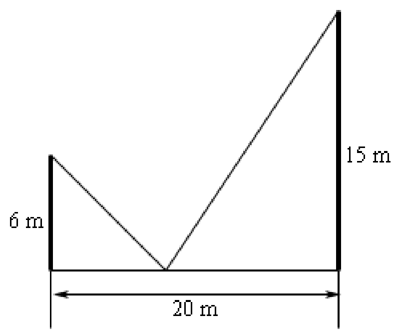
Ex. N-35 4.5 ⋆Challenge !!!

Find the equation of the line through $(2, 4)$ that cuts off the least area from the first quadrant. (Observe that this cut off region is a triangle.)

challenge

Ex. N-36 4.5 ⋆Challenge !!!

Two poles, one 6 meters tall and one 15 meters tall, are 20 meters apart. A length of wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



challenge

§4.6: Linear Approximation and Differentials

Ex. O-1 4.6 Fa17 Exam

Use a linear approximation to estimate $\sqrt{33}$.

exf17

Ex. O-2 4.6 Sp18 Exam

At a certain factory, the daily output is

$$Q(L) = 1500L^{2/3}$$

where L denotes the size of the labor force measured in worker-hours. Currently $1,000$ worker-hours of labor are used each day. Use a linear approximation to estimate the effect on the daily output if the labor force is cut to 975 worker-hours.

exs18

Ex. O-3 4.6 Fa18 Exam

The concentration of a certain drug in the bloodstream t hours after the drug is injected is modeled by the following formula.

$$C(t) = \frac{100t}{t^2+1}$$

(The concentration is measured in micrograms per milliliter.) Use a linear approximation to estimate the change in the concentration over the time period from 2 to 2.1 hours after injection. Also indicate whether the concentration increases or decreases.

exf18

Ex. O-4 4.6 Sp19 Exam

Use a linear approximation to estimate $\sqrt{35.9}$. Do not simplify your answer.

exs19

Ex. O-5 4.6 Sp19 Exam

The cost of producing x units is $C(x) = 3x^2 + 4x + 1000$. Use marginal analysis to estimate the cost of producing the 41st unit.

exs19

Ex. O-6 4.6 Fa19 Exam

Note: The parts of this problem are not related!

(a) Use linear approximation to estimate the value of $\sqrt{79}$.

(b) A manufacturer's total cost to produce x units is $C(x) = 25\ln(x^2+16)$. Use marginal analysis to estimate the cost of the 4th unit.

Ex. O-7 4.6 Sp20 Exam

Use linear approximation or differentials to estimate the value of $\dfrac{1}{\sqrt[3]{8.48}}$.

exs20

Ex. O-8 4.6 Sp20 Exam

Suppose the cost of manufacturing x units is given by $C(x) = x^3 + 5x^2 + 12x + 50$.

(a) What is the exact cost of producing the 3rd unit?

(b) Using marginal analysis, estimate the cost of producing the 3rd unit.

exs20

Ex. O-9 4.6 Sp20 Exam

Use linear approximation to estimate the value of $(0.98)^3 - 5(0.98)^2 + 4(0.98) + 10$.

exs20

Ex. O-10 4.6 Sp20 Exam

If x units are produced, the total cost is $C(x) = x^2 + 15x + 24$ and the selling price per unit is

$$p(x) = \frac{156}{x^2 - 4x + 16}$$

(a) What is the exact cost of producing the 3rd unit?
(b) Using marginal analysis, estimate the revenue from the 3rd unit sold.

exs20

Ex. O-11 4.6 Sp20 Exam

Suppose the cost (in dollars) of manufacturing q units is given by

$$C(q) = 6q^2 + 34q + 112$$

Use marginal analysis to estimate the cost of producing the 5th unit.

exs20

Ex. O-12 4.6 Su20 Exam

Given that x units of a commodity are sold, the selling price per unit is $p(x) = \dfrac{5000}{x^2 + 64}$.

(a) Calculate the revenue function.
(b) Calculate the exact revenue derived from the 7th unit.
(c) Using marginal analysis, estimate the revenue derived from the 7th unit.

exsu20

Ex. O-13 4.6 Su20 Exam

The total number of gallons in a water tank at t hours is given by $N(t) = 40t^{2/5}$. Use a linear approximation to estimate the number of gallons added to the water between $t = 32$ and $t = 35$.

exsu20

Ex. O-14 4.6 Fa20 Exam

Suppose f is differentiable on $(-\infty, \infty)$, $f(5) = 3$, and $f'(5) = -7$. Use linear approximation to estimate $f(5.1)$.

exf20

Ex. O-15 4.6 Fa20 Exam

Use linear approximation to estimate $\sqrt[3]{29} - \sqrt[3]{27}$. *Your final answer must be exact and may not contain any radicals.*

exf20

Ex. O-16 4.6 Sp21 Exam

Use the identity $4^2 + \sqrt{4} = 18$ and linear approximation to estimate $(3.81)^2 + \sqrt{3.81}$.

exs21

Ex. O-17 4.6 Sp21 Exam

The total cost (in dollars) of producing x items is modeled by the function $C(x) = x^2 + 4x + 3$, and the price per item (in dollars) is $p(x) = \dfrac{98x + 49}{x + 3}$.

(a) Calculate the exact cost of producing the 5th item.
(b) Using marginal analysis, estimate the revenue derived from producing the 5th item.

exs21

Ex. O-18 4.6 Fa21 Exam

Use linear approximation to estimate $\tan\left(\frac{\pi}{4} + 0.12\right) - \tan\left(\frac{\pi}{4}\right)$.

exf21

Ex. L-19 4.1, 4.6 Fa21 Exam

The parts of this problem are not related.

(a) Suppose that when x units are produced, the total cost is $C(x) = 2x^2 + 10x + 18$ and the selling price per unit is $p(x) = 46 - x$. Find the level of production that maximizes total profit.

(b) Suppose the total cost of producing q units is $C(q) = q^3 + 20q^2 + 200q + 2000$. Use marginal analysis to estimate the cost of the 3rd unit.

exf21

Ex. O-19 4.6 Sp18 Quiz

Use a linear approximation to estimate the value of $\dfrac{1}{\sqrt[4]{0.96}}$.

qs18

Ex. O-20 4.6 Sp20 Quiz

Use a linear approximation to estimate the value of $(2.01)^5 - 5 \cdot (2.01)^3 + 9$.

qs20

Ex. O-21 4.6 Sp20 Quiz

The total cost of producing x units is $C(x) = 10x^3 + 500x^2 + 1000x + 24000$.

(a) Write a numerical expression equal to the exact cost of the 11th unit.

(b) Use marginal analysis to estimate the cost of the 11th unit. Write your answer as an exact decimal or as a fraction of integers.

qs20

Ex. O-22 4.6 Su22 Quiz

Use linear approximation to estimate the number $\dfrac{1}{(2.9)^2}$. Do not simplify your answer.

qsu22

Ex. O-23 4.6 Su22 Quiz

The position of a particle at time t is given by $x(t) = 5 + 20t^{3/5} + t$. Use linear approximation to estimate the change in the particle's position between $t = 32$ and $t = 35$. Do not simplify your answer.

qsu22

Ex. O-24 4.6 Su22 Quiz

If x units of a certain product are being produced, the marginal cost is

$$\frac{dC}{dx} = 5 + 12x + 20x^{3/2}$$

Suppose the total cost of producing 1 unit is 100 (measured in thousands of dollars). Calculate the total cost of producing 4 units.

qsu22

Ex. O-25 4.6

For each part, use a linear approximation to estimate the given value. *Each answer should be an exact rational number.*

(a) $e^{0.1}$

(b) $\ln(1.04)$

(c) $\dfrac{1}{\sqrt[3]{25}}$

(d) $\left(\sec\left(\frac{\pi}{4} - 0.02\right)\right)^2$

(e) $\sqrt{96}$

(f) $(5.01)^3 - 2(5.01) + 3$

c4s6

Ex. O-26 4.6

When the level of production is q units, the total cost (in dollars) is $C(q) = q^5 - 2q^3 + 3q^2 - 2$. The current level of production is 3 units, and the manufacturer is planning to increase this to 3.01 units. Use a linear approximation to estimate how the total cost will change as a result.

c4s6

Ex. O-27 4.6

When the level of production is q units, the total cost (in dollars) is $C(q) = 3q^2 + q + 500$.

(a) What is the exact cost of manufacturing the 41st unit?

(b) Use marginal analysis to estimate the cost of manufacturing the 41st unit.

c4s6

Ex. O-28 4.6

The total revenue from selling x units of a certain product is $R(x) = 40 - \frac{200}{x+5}$. Using marginal analysis, estimate the revenue from selling the 6th unit.

unit4

Ex. O-29 4.6

Use a linear approximation to estimate the value of $(16.32)^{1/4}$.

unit4

Ex. O-30 4.6

Use linear approximation to estimate $(33.6)^{1/5}$.

unit4

Ex. O-31 4.6

Use linear approximation to estimate $\sec\left(\frac{\pi}{6} + 0.12\right) - \sec\left(\frac{\pi}{6}\right)$.

other

Ex. J-36 3.8, 4.6 ⋆Challenge

Consider the curve described by the equation

$$\frac{x - y^3}{y + x^2} = x - 12$$

(a) Find an equation for the line tangent to this curve at $(-1, 4)$.

(b) There is a point on the curve with coordinates $(-1.1, b)$. Use linear approximation to estimate b. Round to three decimal places.

(c) There is a point on the curve with coordinates $(a, 4.2)$. Use linear approximation to estimate a. Round to three decimal places.

challenge

Ex. O-32 4.6 ⋆Challenge

The acceleration (measured in m/s^2) of a particle moving along the x-axis is given by

$$a(t) = 14t^{3/4} - 6t^2 + 1$$

and the particle is at rest (zero velocity) when $t = 1$. Use a linear approximation to estimate the particle's change in position between $t = 16$ and $t = 16.02$.

challenge

§4.7: L'Hôpital's Rule

Ex. P-1 4.7 Fa17 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0} (1-\sin(4x))^{6/x}$

(b) $\lim_{x\to 1} \left(\dfrac{xe^{4x}+4e^4-5e^4x}{(x-1)^2}\right)$

exf17

Ex. P-2 4.7 Sp18 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 1} \left(\dfrac{x^{1/4}-1}{e^{2x}-e^2}\right)$

(b) $\lim_{x\to 1} \left((x-1)\tan\left(\dfrac{\pi x}{2}\right)\right)$

exs18

Ex. P-3 4.7 Fa18 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0} \left(\dfrac{1-\cos(9x)}{x^2}\right)$

(b) $\lim_{x\to 0} (1-3x)^{5/x}$

exf18

Ex. E-2 2.5, 4.7 Sp19 Exam

The parts of this problem *are* related!

(a) Show that $\lim_{x\to\infty} \left(\dfrac{x}{x-3}\right) = 1$.

(b) Calculate the following limit or show it does not exist.

$$\lim_{x\to\infty} \left(\frac{x}{x-3}\right)^x$$

Hint: First use part (a) to identify the appropriate indeterminate form.

exs19

Ex. P-4 4.7 Fa19 Exam

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to\pi} \left(\dfrac{1+\cos(x)}{(x-\pi)^2}\right)$

(b) $\lim_{x\to\infty} \left(1-\dfrac{12}{x}\right)^{5x}$

(c) $\lim_{x\to 1} \left(\dfrac{x^3-2x^2-5x+6}{x^3+x^2+x-3}\right)$

(d) $\lim_{x\to 4^+} \left(\dfrac{2x-x^2}{x-4}\right)$

Ex. P-5 4.7 Sp20 Exam

Calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 0^+} \left(\sqrt{12x+9}-\sqrt{2x+4}\right)^{1/x}$$

exs20

Ex. P-6 4.7 Sp20 Exam

Suppose you want to compute a limit that is in the form of a quotient, i.e., a limit of the form:

$$\lim_{x\to a} \left(\frac{f(x)}{g(x)}\right)$$

exs20

4.7 Sp20 **Exam**

Suppose you have already determined that L'Hospital's Rule is applicable. Explain the next step in your calculation, i.e., how do you apply L'Hospital's Rule? *Your answer may contain either English, mathematical symbols, or both.*

exs20

Ex. P-7 **4.7** Sp20 **Exam**

Each of the following limits is written in the form of a quotient. Which limits can be calculated using L'Hospital's Rule directly, i.e., by applying L'Hospital's Rule as the immediately next step without any other algebra or modification? Select all that apply.

(a) $\lim_{x\to\pi}\left(\frac{\sin(7x)}{x}\right)$

(b) $\lim_{x\to 2}\left(\frac{x^3+3x-14}{x^2-5x+6}\right)$

(c) $\lim_{x\to\infty}\left(\frac{x^{-1}+5}{x^{-2}+8}\right)$

(d) $\lim_{x\to 9^-}\left(\frac{x^{3/2}+x-36}{x-\sqrt{x}-6}\right)$

(e) $\lim_{x\to\infty}\left(\frac{e^x+10}{e^x-3}\right)$

(f) $\lim_{x\to-\infty}\left(\frac{e^x+10}{e^x-3}\right)$

exs20

Ex. D-2 **2.4, 4.7** Sp20 **Exam**

Which of the following limits are equal to $+\infty$? Select all that apply.

(a) $\lim_{x\to 5^-}\left(\frac{x^2+25}{5-x}\right)$

(b) $\lim_{x\to 5^+}\left(\frac{x^2+25}{5-x}\right)$

(c) $\lim_{x\to -3^-}\left(\frac{x^3}{|x+3|}\right)$

(d) $\lim_{x\to 0^-}\left(\frac{x^4-2x-5}{\sin(x)}\right)$

(e) $\lim_{x\to 1^+}\left(\frac{x^6-x^2}{x-1}\right)$

exs20

Ex. P-8 **4.7** Sp20 **Exam**

Calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to\infty}\left(5x^3+2x^2+8\right)^{1/\ln(x)}$$

exs20

Ex. P-9 **4.7** Sp20 **Exam**

Suppose you have determined

$$\lim_{x\to a} f(x) = 0 \text{ and } \lim_{x\to a} g(x) = \infty$$

and you want to calculate the following limit:

$$L = \lim_{x\to a}\left(f(x)g(x)\right)$$

You recall that to calculate L, you have to use L'Hospital's Rule. What is the next step you must take before you are able to apply L'Hospital's Rule directly to the limit L? *Your answer may contain either English, mathematical symbols, or both.*

exs20

Ex. P-10 **4.7** Sp20 **Exam**

Which of the following are indeterminate forms? Recall that in this course, we have learned that limits with indeterminate forms may often be computed using L'Hospital's Rule.

(a) $\frac{0}{0}$

(b) $0\cdot\infty$

(c) $\frac{\infty}{-\infty}$

(d) $\frac{0}{\infty}$

(e) 2^{∞}

(f) $3\cdot(-\infty)$

(g) $\infty\cdot(-\infty)$

(h) ∞^0

(i) ∞^{∞}

exs20

Ex. P-11 **4.7** Sp20 **Exam**

A student is asked to calculate the following limit using l'Hospital's Rule and to show all their work.

$$L = \lim_{x\to 0}\left(\frac{\sin(2x)+17x^2+2x}{4x^2+\tan(x)}\right)$$

exs20

4.7 Sp20 Exam

The student decides to cheat, so they find the solution online (shown below) and they submit the work as their own!

$$\begin{aligned}
L &= \lim_{x\to 0}\left(\frac{\sin(2x)+17x^2+2x}{4x^2+\tan(x)}\right) && (1)\\
&= \lim_{x\to 0}\left(\frac{2\cos(2x)+34x+2}{8x+\sec(x)^2}\right) && (2)\\
&= \lim_{x\to 0}\left(\frac{-4\sin(2x)+34}{8+2\sec(x)^2\tan(x)}\right) && (3)\\
&= \frac{-4\sin(0)+34}{8+2\sec(0)^2\tan(0)} && (4)\\
&= \frac{0+34}{8+0} && (5)\\
&= \frac{17}{4} && (6)
\end{aligned}$$

Unfortunately, this solution contains an error, and so the student lost all credit for the problem. The student was also later determined to be responsible for cheating, and so they earned a grade of 0 on the entire exam!

Your task is to find and correct the error(s). Answer the following questions.

(a) There may be several errors in this solution. Which line is the first incorrect line?

(b) Explain the error in the first incorrect line in your own words.

(c) Calculate the correct value of L (the original limit).

exs20

Ex. P-12 4.7 Su20 Exam

Calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 1}\left(\frac{\tan(\pi x)}{\sqrt{2+x^3}-\sqrt{2+x}}\right)$$

exsu20

Ex. P-13 4.7 Su20 Exam

Consider the following limit.

$$L = \lim_{x\to -3}(4+x)^{7/(6+2x)}$$

(a) What indeterminate form does this limit have?

(b) Explain why l'Hospital's rule cannot be used on this limit in its current form.

(c) Calculate the value of L.

exsu20

Ex. P-14 4.7 Fa20 Exam

Consider the limit $L = \lim_{x\to 2^-}((x-2)\ln(2-x))$.

(a) Does this limit have an indeterminate form? If so, which indeterminate form?

(b) Explain why l'Hospital's rule cannot be used on this limit in its current form.

(c) Write the limit in an equivalent form to which l'Hospital's rule may be applied.

Note: You are not required to calculate the limit; do not attempt to do so.

exf20

Ex. P-15 4.7 Fa20 Exam

Suppose $f'(x)$ is continuous with $f(3)=2$ and $f'(3)=-8$. Calculate the following limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 1}\left(\frac{2x^4-f(3x^{1/4})}{x^2-4x+3}\right)$$

exf20

Ex. P-16 **4.7** Sp21 **Exam**

Suppose $f''(x)$ is continuous. You are also given the following values:

$$f\left(\frac{1}{8}\right) = 20 \quad , \quad f'\left(\frac{1}{8}\right) = -22$$

Calculate the following limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 8}\left(\frac{20 - f\left(\frac{1}{x}\right)}{x^2+x-72}\right)$$

exs21

Ex. D-13 **2.4, 2.5, 4.7** Fa21 **Exam**

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\displaystyle\lim_{x\to 1}\left(\frac{x^4-x}{\ln(77x-76)}\right)$

(b) $\displaystyle\lim_{x\to -\infty}\left(\frac{\sqrt{36x^2+63}}{31x}\right)$

(c) $\displaystyle\lim_{x\to 2^+} f(x)$, with $f(x) = \begin{cases} 1+4x & x \le 2 \\ \dfrac{x^2-4}{x-2} & x > 2 \end{cases}$

(d) $\displaystyle\lim_{x\to 5^-}\left(\frac{\cos(\pi x)}{x^2-25}\right)$

exf21

Ex. D-14 **2.4, 4.7** Fa21 **Exam**

For each part, find all vertical asymptotes of the given function.

(a) $f(x) = \dfrac{x^2-8x+15}{x^2-9}$

(b) $g(x) = \dfrac{e^{x+3}-1}{x^2-9}$

exf21

Ex. P-17 **4.7** Sp22 **Exam**

For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\displaystyle\lim_{x\to \pi}\left(\frac{\cos(6x)-1}{(x-\pi)^2}\right)$

(b) $\displaystyle\lim_{x\to 0}\left(e^{2x}+3x\right)^{1/x}$

exs22

Ex. M-28 **4.3/4.4, 4.7** Su22 **Exam**

Let $f(x) = x^2e^x$.

(a) Calculate the vertical and horizontal asymptotes of f.

(b) Calculate the critical points of f. Then use the Second Derivative Test to classify each critical point of f as a local minimum or a local maximum. Show your work and label your answers clearly. ***Hint:*** The second derivative of f is $f''(x) = (x^2+4x+2)e^x$.

exsu22

Ex. P-18 **4.7** Su22 **Exam**

Let $f(x) = \dfrac{x\sin(Ax)}{\sin^2(2x)}$, where A is a constant. Suppose $\displaystyle\lim_{x\to 0} f(x) = -6$. Calculate A.

exsu22

Ex. P-19 **4.7** Sp18 **Quiz**

Calculate the following limit or show it does not exist.

$$\lim_{x\to 0}\left(\frac{x-\ln(1+x)}{1-\cos(2x)}\right)$$

qs18

Ex. P-20 4.7 Sp20 Quiz

For each part, calculate the limit or show that it does not exist. If the limit is infinite, your answer should be "$+\infty$" or "$-\infty$".

(a) $\displaystyle\lim_{x\to e}\left(\frac{1-\ln(x)}{x^2\ln(x)-e^2}\right)$

(b) $\displaystyle\lim_{x\to 2^+}\left(\frac{\cos(\pi x)}{x^2-6x+8}\right)$

qs20

Ex. P-21 4.7 Su22 Quiz

Calculate the limit or show that it does not exist. If the limit is infinite, write "$+\infty$" or "$-\infty$" as your answer, instead of "does not exist", as appropriate.

$$\lim_{x\to 0}\left(\frac{xe^{-2x}+\cos(x)-1-x}{x^2}\right)$$

qsu22

Ex. P-22 4.7 Fa22 Quiz

Compute $\displaystyle\lim_{x\to 0}\left(\frac{e^{-5x}-1}{\ln(1+13x)}\right)$. If the limit is infinite, write "$+\infty$" or "$-\infty$" instead of "DNE".

qf22

Ex. P-23 4.7 Fa22 Quiz

Calculate the limit below or determine it does not exist.

$$\lim_{x\to 0}\left(\frac{\cos(5x)-\cos(4x)}{3x^2}\right)$$

qf22

Ex. P-24 4.7

For each part, calculate the limit or show that it does not exist.

(a) $\displaystyle\lim_{x\to 0}\left(\frac{e^{2x}-1-2x-2x^2}{x^3}\right)$

(b) $\displaystyle\lim_{x\to 1}\left(\frac{x^3-1}{x^4-x}\right)$

(c) $\displaystyle\lim_{x\to 2}\left(\frac{x^3-8}{x^4-x}\right)$

(d) $\displaystyle\lim_{x\to\infty}\left(\frac{x-1}{x+2}\right)$

(e) $\displaystyle\lim_{x\to\infty}\left(\frac{x-1}{x+2}\right)^x$

(f) $\displaystyle\lim_{x\to\frac{\pi}{2}}\left(\frac{\sec(x)}{\tan(x)}\right)$

(g) $\displaystyle\lim_{x\to 0^+}(\sin(2x)\ln(x))$

(h) $\displaystyle\lim_{x\to 0^+}\left(x^{-4}\ln(x)\right)$

(i) $\displaystyle\lim_{x\to 4}\left(\frac{1}{\sqrt{x}-2}-\frac{4}{x-4}\right)$

(j) $\displaystyle\lim_{x\to 3}\left(\frac{\sqrt{x+1}-2}{x^3-7x-6}\right)$

(k) $\displaystyle\lim_{x\to\infty}\left(\sqrt{x^2+x}-x\right)$

(l) $\displaystyle\lim_{x\to 0}\left(\frac{1}{\sin(x)}-\frac{1}{x}\right)$

(m) $\displaystyle\lim_{x\to 0}(\cos(x))^{3/x^2}$

(n) $\displaystyle\lim_{x\to\infty}\left(\frac{x}{\sqrt{3x^2+4}}\right)$

(o) $\displaystyle\lim_{x\to\infty}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$

(p) $\displaystyle\lim_{x\to 0}(1-\sin(2x))^{1/\tan(3x)}$

(q) $\displaystyle\lim_{x\to 0}\left(\frac{x\sin(x)}{1-\cos(x)}\right)$

(r) $\displaystyle\lim_{x\to\frac{\pi}{2}}\left(\left(x-\tfrac{\pi}{2}\right)\tan(x)\right)$

c4s7

Ex. P-25 4.7

Find the equation of each horizontal asymptote of $f(x)=\dfrac{2e^x-5}{3e^x+2}$.

unit2

Ex. P-26 4.7

For each part, calculate the limit or show that it does not exist. If the limit is infinite, write "∞" or "$-\infty$" as your answer, as appropriate.

(a) $\displaystyle\lim_{x\to 3^-}\left(\frac{x^2+6}{3-x}\right)$

(b) $\displaystyle\lim_{x\to 0}(1-\sin(3x))^{1/x}$

(c) $\displaystyle\lim_{x\to -3}\left((x+3)\tan\left(\frac{\pi x}{2}\right)\right)$

unit4

Ex. P-27 4.7

Calculate each limit.

(a) $\lim_{x\to 0}\left(\frac{\sin(x)^2}{\sin(2x^2)}\right)$

(b) $\lim_{x\to 1}\left(\frac{\ln(x^2+2)-\ln(3)}{x-1}\right)$

unit4

Ex. P-28 4.7

For each part, calculate the limit or show it does not exist.

(a) $\lim_{x\to 2}\left(\frac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x}\right)$

(b) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$

(c) $\lim_{x\to 0}\left(\frac{\sin(5x)-5x}{x^3}\right)$

unit4

Ex. P-29 4.7 ⋆Challenge

Suppose f'' is continuous for all x. Calculate $\lim_{h\to 0}\left(\frac{f(x+5h)+f(x-5h)-2f(x)}{h^2}\right)$.

challenge

Ex. P-30 4.7 ⋆Challenge

Suppose f' is continuous for all x and $f(0)=0$. Calculate $\lim_{x\to 0^+}(1+f(2x))^{4/x}$.

challenge

§4.9: Antiderivatives

Ex. G-6 **3.1/3.2, 4.1, 4.9** Sp20 **Exam**

Suppose the derivative of f is $f'(x) = 3x^2 - 6x - 9$ and that $f(1) = 10$.

(a) Find an equation of the line tangent to the graph of $y = f(x)$ at $x = 1$.
(b) Find the critical points of f.
(c) Where does f have a local minimum value? local maximum value?
(d) Calculate $f(0)$.
(e) Calculate the absolute maximum value of f on the interval $[0, 6]$. At what x-value does it occur?

exs20

Ex. Q-1 **4.9** Su20 **Exam**

Given that x units of a commodity are sold, the marginal cost is

$$\frac{dC}{dx} = 9x^2 + 4x + 15x^{1/4} + 10$$

Suppose the total cost of producing the 1st unit is 100. Calculate the total cost of producing the first 16 units.

exsu20

Ex. Q-2 **4.9** Fa20 **Exam**

Let $V(t)$ denote the volume of water, measured in gallons, in a tank at time t. The tank is initially filled with 5 gallons of water. At $t = 0$, water flows in at a rate in gal/min given by $V'(t) = 0.5(196 - t^2)$ for $0 \le t \le 10$. Find the total amount of water in the tank after 4 minutes.

exf20

Ex. Q-3 **4.9** Sp21 **Exam**

A particle travels along the x-axis with velocity (measured in ft/sec) at any time t (measures in sec) given by

$$v(t) = 4t^3 - 2t + 2$$

The particle is at $x = 3$ when $t = 2$.

(a) Find the position of the particle at any time t.
(b) Find the position of the particle at time $t = 4$.
(c) Find the acceleration of the particle when $t = 4$.

exs21

Ex. Q-4 **4.9** Fa21 **Exam**

For any time $t > 0$, the acceleration of a particle is given by $a(t) = 1 + \dfrac{3}{\sqrt{t}}$, and the particle has velocity $v = -20$ when $t = 1$. Find the velocity of the particle when $t = 16$.

exf21

Ex. Q-5 **4.9** Sp18 **Quiz**

Calculate the following antiderivatives.

(a) $\displaystyle\int (\cos(w) + 2\sin(w) - 3e^w)\, dw$

(b) $\displaystyle\int \frac{3t^3 - \sqrt[3]{t} + 2t}{t^2}\, dt$

qs18

Ex. Q-6 **4.9, 5.3** Su22 **Quiz**

Calculate each of the following. You do not have to simplify your answers.

(a) $\displaystyle\int \left(\frac{3t^2 - \sqrt{t} + 4}{5t}\right) dt$

(b) $\displaystyle\int_{-1}^{3} \left(3x^2 + 2e^x\right)\, dx$

qsu22

Ex. Q-7 4.9 Fa22 **Quiz**

The total number of rabbits in a certain region t weeks after observations have begun is modeled by the equation $N(t) = 200 + 36t^{2/3}$. Use a linear approximation to estimate the increase in the rabbit population between $t = 64$ and $t = 67$.

qf22

Ex. Q-8 4.9 Fa22 **Quiz**

When x units of a product are produced, the derivative of the total cost C (measured in \$) is:

$$\frac{dC}{dx} = 3x^2 + 40x + 100$$

Suppose the total cost of producing 1 unit is \$150. Find the total cost of producing the first 2 units.

qf22

Ex. Q-9 4.9

For each part, find the antiderivative.

(a) $\int (4 - 9x + x^2)\, dx$

(b) $\int \left(12e^x + \sin(x) - \frac{\cos(x)}{4}\right) dx$

(c) $\int (6y - y^3)^2\, dy$

(d) $\int (86t^7 - \sqrt[3]{t})\, dt$

(e) $\int \frac{3t^3 - 6\sqrt{t} - \frac{9}{t}}{t}\, dt$

(f) $\int \left(1 - \frac{1}{u}\right)\left(2 + \frac{3}{\sqrt{u}}\right) du$

c4s9

Ex. Q-10 4.9

The marginal revenue of a certain commodity is $R'(x) = -9x^2 + 24x + 48$. Find the price for which the total revenue is a maximum. (Assume that $R(0) = 0$.)

c4s9

Ex. Q-11 4.9

A particle moves along the x-axis in such a way that its acceleration at time $t > 0$ is

$$a(t) = 1 - \frac{1}{t^2}$$

The particle's velocity when $t = 2$ is $v = 5.5$. What is the net distance the particle travels between $t = 3$ and $t = 6$?

c4s9

Ex. Q-12 4.9

The position of a particle on the x-axis (measured in meters) at time t (measured in seconds) is modeled by the equation $f(t) = 100 + 8t^{3/4} - 5t$. Use a linear approximation to estimate the change in the particle's position between $t = 81$ and $t = 83$.

unit4

Ex. L-32 4.1, 4.9

The marginal revenue of a certain product is $R'(x) = -9x^2 + 17x + 30$, where x is the level of production. Assume $R(0) = 0$. Find the market price that maximizes revenue.

unit4

Ex. Q-13 4.9

The marginal cost (in dollars) of a certain product is $C'(x) = 6x^2 + 30x + 200$. If it costs \$250 to produce 1 unit, how much does it cost to produce 10 units?

unit4

Ex. Q-14 4.9

For each part, find the antiderivative or integral.

unit5

4.9

(a) $\displaystyle\int \frac{2x+\sqrt{x}-1}{x}\,dx$

(b) $\displaystyle\int (2x+3)^{12}\,dx$

(c) $\displaystyle\int_0^1 e^x(1+e^{-2x})\,dx$

(d) $\displaystyle\int_0^{\pi/2} (1+\sin(x))^5\cos(x)\,dx$

unit5

Ex. Q-15 4.9, 5.3, 5.5

For each part, find the antiderivative or integral.

(a) $\displaystyle\int t^2\cos(1-t^3)\,dt$

(b) $\displaystyle\int \sqrt{x-1}\,dx$

(c) $\displaystyle\int_2^3 \frac{\ln(x)}{x}\,dx$

(d) $\displaystyle\int_0^{\ln(3)} e^{2x}\sqrt{e^{2x}-1}\,dx$

unit5

5 Chapter 5: Integration

§5.1, 5.2: Introduction to the Integral

Ex. R-1 5.1/5.2 Sp20 Exam

Suppose f is a continuous function such that all of the following hold:

$$\int_{-1}^{6} f(x)\,dx = -15 \quad , \quad \int_{6}^{9} f(x)\,dx = 14 \quad , \quad \int_{0}^{9} f(x)\,dx = 19$$

Calculate the quantities below or determine there is not enough information.

(a) $\int_{-1}^{9} f(x)\,dx$

(b) $\int_{0}^{6} f(x)\,dx$

(c) $\int_{-1}^{6} |f(x)|\,dx$

(d) $\left|\int_{-1}^{6} f(x)\,dx\right|$

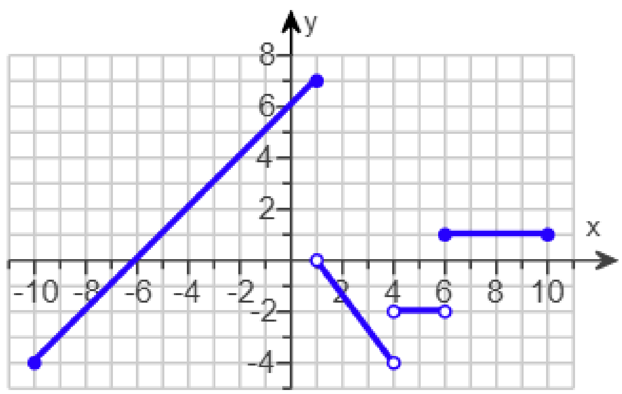
(e) $\int_{-1}^{0} f(x)\,dx$

(f) $\int_{6}^{9} (3f(x)+4)\,dx$

exs20

Ex. R-2 5.1/5.2 Sp20 Exam

Use the graph of $y = f(x)$ to calculate the integrals below.



(a) $\int_{0}^{1} f(x)\,dx$

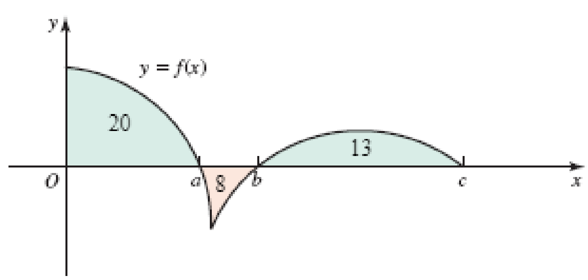
(b) $\int_{1}^{6} f(x)\,dx$

(c) $\int_{-10}^{10} f(x)\,dx$

exs20

Ex. R-3 5.1/5.2 Sp20 Exam

The figure below shows the area of regions bounded by the graph of $y = f(x)$ and the x-axis, where $a = 4$, $b = 6$, and $c = 15$. Evaluate $\int_{a}^{c} (11f(x) - 6)\,dx$.



exs20

Ex. R-4 5.1/5.2 Sp20 Exam

Consider the integral below.

$$\int_{-2}^{1} \sqrt{9-(x-1)^2}\,dx$$

(a) Explain in your own words how you can calculate this integral without using Riemann sums or the fundamental theorem of calculus. ***Hint:*** Try graphing the integrand!

exs20

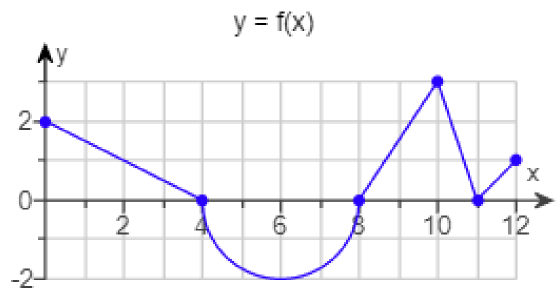
5.1/5.2 Sp20 Exam

(b) Find the exact value of the integral.

exs20

Ex. R-5 5.1/5.2, 5.3 Sp20 Exam

Define the function g by $g(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(x)$ is given below. The graph consists of four line segments and one semicricle. ***Note:*** f and g are different functions!



(a) Calculate $f'(9)$.
(b) Calculate $f'(6)$.
(c) Calculate $g'(6)$.
(d) Calculate $g(11) - g(8)$.
(e) Is the statement "$g(4) > g(0)$" true or false?
(f) Find the critical numbers of g in the interval $(0, 12)$.

exs20

Ex. R-6 5.1/5.2 Su20 Exam

Suppose f is continuous on $[0, 8]$ and has the following integrals:

$$\int_0^3 f(x)\,dx = 2 \qquad \int_3^5 f(x)\,dx = 7 \qquad \int_0^8 f(x)\,dx = 15$$

For each part, calculate the integral or determine there is not enough information to do so.

(a) $\int_0^5 f(x)\,dx$ (b) $\int_5^3 f(x)\,dx$ (c) $\int_5^8 f(x)\,dx$ (d) $\int_3^8 (2f(x) - 6)\,dx$

exsu20

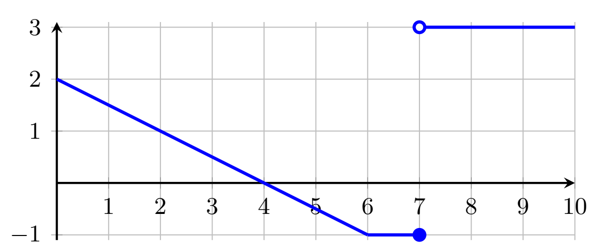
Ex. R-7 5.1/5.2 Su20 Exam

Calculate $\int_0^{\sqrt{10}} \left(x + \sqrt{10 - x^2}\right) dx$ using geometry and properties of integrals only. Do not attempt to use the fundamental theorem of calculus.

exsu20

Ex. R-8 5.1/5.2, 5.3 Fa21 Exam

Let $F(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(t)$ is given below. For each part, use this information to calculate the indicated item.



exf21

5.1/5.2, 5.3 Fa21 Exam

(a) $F(10)$ (b) $F'(6)$ (c) $\int_0^6 |f(t)|\,dt$ (d) $\int_0^4 (f'(t)+5)\,dt$

exf21

Ex. R-9 5.1/5.2 Fa22 Quiz

Let $f(x) = 12 - 3x$. Calculate each of the following integrals using geometry. If you use the Fundamental Theorem of Calculus, you will receive no credit.

(a) $\int_0^5 f(x)\,dx$ (b) $\int_0^5 |f(x)|\,dx$

qf22

Ex. R-10 5.1/5.2, 5.3, 5.5 Fa22 Quiz

Calculate each of the following integrals using any valid method taught in this course. You may need to use basic geometry, the Fundamental Theorem of Calculus, substitution rule, or some combination.

(a) $\int_{-5}^0 \sqrt{25-x^2}\,dx$ (b) $\int_0^1 6x^2(x^3+26)^{1/2}\,dx$ (c) $\int_{-\ln(5)}^{\ln(6)} (2e^x+3)\,dx$

qf22

Ex. R-11 5.1/5.2

For each part, use geometry to calculate the integral.

(a) $\int_{-1}^9 (27-3x)\,dx$

(b) $\int_{-2}^4 (3x+15)\,dx$

(c) $\int_0^{12} (2x-10)\,dx$

(d) $\int_{-3}^5 (|x|-1)\,dx$

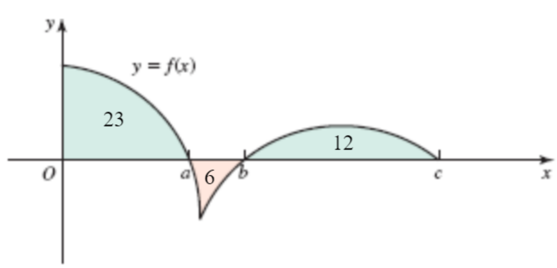
(e) $\int_{-4}^0 \sqrt{16-x^2}\,dx$

(f) $\int_2^{10} \sqrt{64-(x-10)^2}\,dx$

c5s1

Ex. R-12 5.1/5.2

For each part, use the graph below to calculate the integral. Write your answer in terms of a, b, and c, if necessary. If there is not information to calculate the integral, explain why.



(a) $\int_0^a f(x)\,dx$

(b) $\int_0^b f(x)\,dx$

(c) $\int_a^c f(x)\,dx$

(d) $\int_0^c |f(x)|\,dx$

(e) $\int_0^c (2\,|f(x)| + 3f(x))\,dx$

(f) $\int_a^0 f(x)\,dx$

(g) $\int_c^a |f(x)|\,dx$

(h) $\int_0^c (2f(x)+3)\,dx$

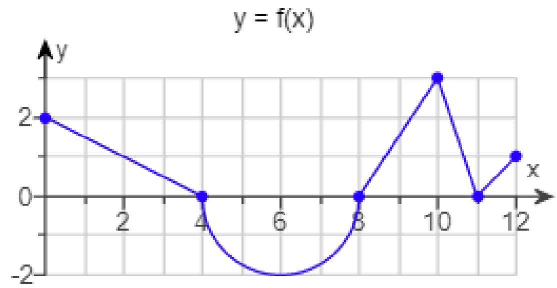
(i) $\int_0^a f(x)^2\,dx$

c5s1

§5.3: Fundamental Theorem of Calculus

Ex. R-5 **5.1/5.2, 5.3** Sp20 **Exam**

Define the function g by $g(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(x)$ is given below. The graph consists of four line segments and one semicricle. ***Note:*** f and g are different functions!



(a) Calculate $f'(9)$.
(b) Calculate $f'(6)$.
(c) Calculate $g'(6)$.
(d) Calculate $g(11) - g(8)$.
(e) Is the statement "$g(4) > g(0)$" true or false?
(f) Find the critical numbers of g in the interval $(0, 12)$.

exs20

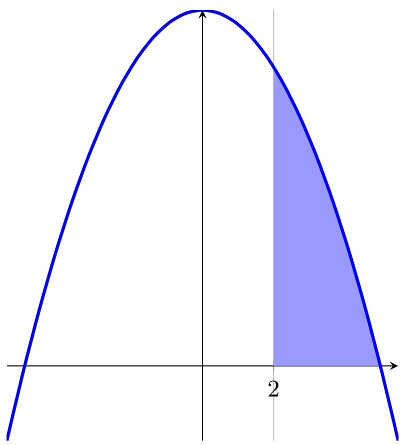
Ex. S-1 **5.3** Su20 **Exam**

Let $f(x) = 5 + \int_{-3}^x t^2 e^t\,dt$. Find an equation of the tangent line to f at $x = -3$.

exsu20

Ex. S-2 **5.3** Su20 **Exam**

The curve $y = 25 - x^2$ is shown in the figure below. Calculate the area of the shaded region.



exsu20

Ex. S-3 **5.3, 5.5** Fa21 **Exam**

The parts of this problem are not related.

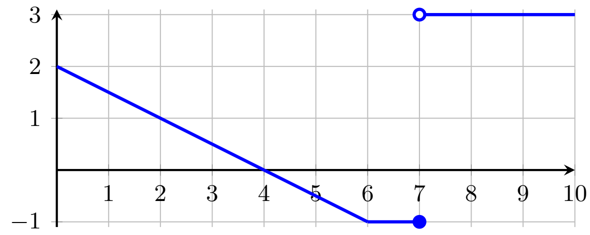
(a) Calculate the integral $\displaystyle\int_2^4 \frac{18t - 3t^2}{t}\,dt$.

(b) Calculate the area of the region below the curve $y = 23\sin(x)\cos^2(x)$ and above the interval $\left[0, \frac{\pi}{2}\right]$ on the x-axis. (Note that $y \geq 0$ on this interval.)

exf21

Ex. R-8 **5.1/5.2, 5.3** Fa21 **Exam**

Let $F(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(t)$ is given below. For each part, use this information to calculate the indicated item.



(a) $F(10)$

(b) $F'(6)$

(c) $\int_0^6 |f(t)|\,dt$

(d) $\int_0^4 (f'(t) + 5)\,dt$

exf21

Ex. Q-6 **4.9, 5.3** Su22 **Quiz**

Calculate each of the following. You do not have to simplify your answers.

(a) $\int \left(\frac{3t^2 - \sqrt{t} + 4}{5t} \right) dt$

(b) $\int_{-1}^{3} \left(3x^2 + 2e^x\right) dx$

qsu22

Ex. S-4 **5.3** Fa22 **Quiz**

Find the area of the region bounded by the graph of $y = (x^4 + 1)^2$, the x-axis, and the lines $x = 0$ and $x = 1$.

qf22

Ex. R-10 **5.1/5.2, 5.3, 5.5** Fa22 **Quiz**

Calculate each of the following integrals using any valid method taught in this course. You may need to use basic geometry, the Fundamental Theorem of Calculus, substitution rule, or some combination.

(a) $\int_{-5}^{0} \sqrt{25 - x^2}\,dx$

(b) $\int_0^1 6x^2(x^3 + 26)^{1/2}\,dx$

(c) $\int_{-\ln(5)}^{\ln(6)} (2e^x + 3)\,dx$

qf22

Ex. S-5 **5.3**

For each part, evaluate the integral using geometry, the Fundamental Theorem of Calculus, or a combination.

(a) $\int_{-3}^{5} (-8)\,dx$

(b) $\int_4^{36} \sqrt{2x}\,dx$

(c) $\int_{-\ln(3)}^{\ln(8)} 5e^x\,dx$

(d) $\int_0^9 \sqrt{x}(x^2 - x + 1)\,dx$

(e) $\int_9^{10} \frac{a}{x}\,dx$

(f) $\int_{-4}^{4} \sqrt{16 - x^2}\,dx$

(g) $\int_{-2}^{5} (2x - |x|)\,dx$

(h) $\int_{-\pi}^{\pi/2} \sin(x)\,dx$

(i) $\left| \int_{-\pi}^{\pi/2} \sin(x)\,dx \right|$

(j) $\int_{-\pi}^{\pi/2} |\sin(x)|\,dx$

c5s3

Ex. S-6 **5.3**

For each part, calculate $F'(x)$.

(a) $F(x) = \int_{-3}^{x} \frac{t^4 - t^2 + 1}{\sqrt{t^6 + 1}}\,dt$

(b) $F(x) = \int_{-\pi}^{x} \sqrt[3]{w}(w^2 - 2w + 5)\,dw$

c5s3

Ex. S-7 **5.3**

Let $f(x) = \begin{cases} 4x - x^2 & \text{if } x \leq 2 \\ \dfrac{8}{x} & \text{if } x > 2 \end{cases}$.

(a) Show that $f(x)$ is continuous on $[-1, 4]$.

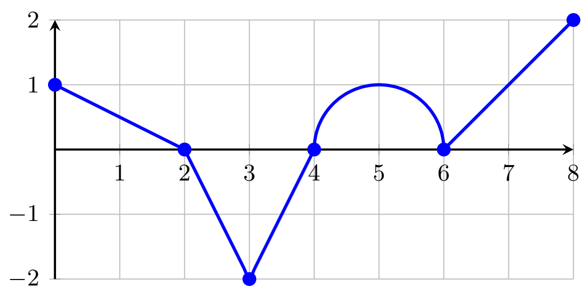
(b) Sketch the region whose net area is given by the integral $\displaystyle\int_{-1}^{4} f(x)\,dx$.

(c) Evaluate $\displaystyle\int_{-1}^{4} f(x)\,dx$.

c5s3

Ex. S-8 **5.3**

Let $g(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(x)$ is given below. This graph consists of four line segments and one semicircle.



(a) Is the statement "$g(4) > g(2)$" true or false? Explain your answer.

(b) Evaluate $g(8)$.

(c) Where is g decreasing and where is g increasing? Where in $(0, 8)$ does g have a local minimum? local maximum?

(d) Where is g concave down and where is g concave up? Where in $(0, 8)$ does g have an inflection point?

c5s3

Ex. S-9 **5.3**

The parts of this question are not related.

(a) Find $F'(x)$ with $F(x) = \displaystyle\int_{-1}^{x} \frac{t^5}{3 + t^6}\,dt$.

(b) Find $\displaystyle\int_{0}^{5} f(t)\,dt$ with $f(x) = \begin{cases} x & \text{if } x < 1 \\ \dfrac{1}{x} & \text{if } x \geq 1 \end{cases}$.

unit5

Ex. Q-15 **4.9, 5.3, 5.5**

For each part, find the antiderivative or integral.

(a) $\displaystyle\int t^2 \cos(1 - t^3)\,dt$

(b) $\displaystyle\int \sqrt{x - 1}\,dx$

(c) $\displaystyle\int_{2}^{3} \frac{\ln(x)}{x}\,dx$

(d) $\displaystyle\int_{0}^{\ln(3)} e^{2x}\sqrt{e^{2x} - 1}\,dx$

unit5

§5.5: Substitution Rule

Ex. T-1 5.5 Sp20 Exam

Note: The parts of this problem are not related.

(a) Suppose we use the fundamental theorem of calculus to calculate an integral as follows:

$$\int_a^b g(u)\,du = G(b) - G(a)$$

What is the relationship between the functions g and G?

(b) Calculate the following definite integral:

$$\int_{e^{-3}}^{e^2} \frac{2\ln(x) - 3}{5x}\,dx$$

(c) Consider the following indefinite integral:

$$J = \int \frac{\ln(x)}{3x^2}\,dx$$

Use the substitution $u = \ln(x)$ to write J as an equivalent indefinite integral in terms of u. *Do not attempt to calculate J.*

exs20

Ex. T-2 5.5 Su20 Exam

Find the unique positive value of a such that $\displaystyle\int_0^a \frac{x}{x^2+1}\,dx = 3$.

exsu20

Ex. S-3 5.3, 5.5 Fa21 Exam

The parts of this problem are not related.

(a) Calculate the integral $\displaystyle\int_2^4 \frac{18t - 3t^2}{t}\,dt$.

(b) Calculate the area of the region below the curve $y = 23\sin(x)\cos^2(x)$ and above the interval $\left[0, \frac{\pi}{2}\right]$ on the x-axis. (Note that $y \geq 0$ on this interval.)

exf21

Ex. R-10 5.1/5.2, 5.3, 5.5 Fa22 Quiz

Calculate each of the following integrals using any valid method taught in this course. You may need to use basic geometry, the Fundamental Theorem of Calculus, substitution rule, or some combination.

(a) $\displaystyle\int_{-5}^0 \sqrt{25 - x^2}\,dx$ (b) $\displaystyle\int_0^1 6x^2(x^3+26)^{1/2}\,dx$ (c) $\displaystyle\int_{-\ln(5)}^{\ln(6)} (2e^x + 3)\,dx$

qf22

Ex. T-3 5.5

For each part, find the antiderivative.

(a) $\displaystyle\int (5x-7)^{14}\,dx$

(b) $\displaystyle\int \frac{x^3}{\sqrt{9 - x^4}}\,dx$

(c) $\displaystyle\int \cos(4-x)\,dx$

(d) $\displaystyle\int x\sqrt{2x+1}\,dx$

(e) $\displaystyle\int \frac{1}{x\ln(x)\ln(\ln(x))}\,dx$

(f) $\displaystyle\int \frac{1}{\sqrt{w}(\sqrt{w}+7)}\,dw$

c5s5

Ex. T-4 5.5

For each part, calculate the integral.

c5s5

5.5

(a) $\displaystyle\int_0^1 \frac{5x^2}{3x^3+2}\,dx$

(b) $\displaystyle\int_{-\frac{\pi}{4}}^{\frac{\pi}{3}} \tan(3\theta)\,d\theta$

(c) $\displaystyle\int_0^2 (e^{3x}-e^{-3x})^2\,dx$

(d) $\displaystyle\int_0^{\ln(2)} \frac{1}{1+e^{-t}}\,dt$

(e) $\displaystyle\int_1^{e^3} \frac{\ln(x)}{x}\,dx$

(f) $\displaystyle\int_{-1}^1 \frac{2x}{2x-9}\,dx$

c5s5

Ex. T-5 5.5

For each part, find the area of the region under the given curve.

(a) $y = t\sqrt{t^2+9}$ on $[0,4]$

(b) $y = x(x-3)^{1/3}$ on $[3,11]$

(c) $y = \sin(2x)^2\cos(2x)$ on $[0,\frac{\pi}{4}]$

(d) $y = \dfrac{e^{\sqrt{x}}}{\sqrt{x}}$ on $[1,9]$

c5s5

Ex. Q-15 4.9, 5.3, 5.5

For each part, find the antiderivative or integral.

(a) $\displaystyle\int t^2\cos(1-t^3)\,dt$

(b) $\displaystyle\int \sqrt{x-1}\,dx$

(c) $\displaystyle\int_2^3 \frac{\ln(x)}{x}\,dx$

(d) $\displaystyle\int_0^{\ln(3)} e^{2x}\sqrt{e^{2x}-1}\,dx$

unit5

6 Chapter 6: Additional Exercises

True or False?

Ex. U-1 True/False Sp19 Exam

For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F $\ln(3) - \ln(11) = \dfrac{\ln(3)}{\ln(11)}$

(b) T F The domain of $f(x) = \sqrt[9]{x-4}$ is all real numbers.

(c) T F The lines $9x + y = 1$ and $x - 9y = 4$ are perpendicular to each other.

(d) T F The equations $2\ln(x) = 0$ and $\ln(x^2) = 0$ have the same solutions.

(e) T F $\cos\left(\dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

exs19

Ex. U-2 True/False Sp20 Exam

4 Each of the following statements describes a scenario in which a certain rectangle is changing over time. For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If two opposite sides of the rectangle increase in length and if the area remains constant, then the other two opposite sides must decrease in length.

(b) T F If the area of the rectangle increases, then all sides of the rectangle must also increase in length.

(c) T F If the length of the rectangle remains the same, then the area and the width of the rectangle cannot change in opposite ways (i.e., one cannot increase while the other decreases).

(d) T F If two opposite sides of the rectangle increase in length and the other two opposite sides decrease in length, then the area of the rectangle must remain constant.

exs20

Ex. U-3 True/False Sp20 Exam

The numbers a, b, and c (which are not necessarily positive) satisfy the formula $a = \dfrac{b}{c}$. The choices below describe scenarios in which the numbers a, b, and c are changing over time. For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

Hint: There is at most one true statement.

(a) T F Suppose a, b, and c are all positive numbers. If a and b are both increasing, then c must also be increasing.

(b) T F Suppose b is a positive number and c is a negative number. If b and c are both increasing, then a must be decreasing.

(c) T F Suppose a, b, and c are all positive numbers. If a is constant, then it is possible for b and c to change in opposite ways (i.e., one can increase while the other decreases).

(d) T F Suppose c is a positive number. If b is constant and c is increasing, then a must be decreasing.

exs20

Ex. U-4 True/False Sp21 Exam

For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim\limits_{x\to a} f(x)$ can be evaluated by direct substitution, then f is continuous at $x = a$.

(b) T F The value of $\lim\limits_{x\to a} f(x)$, if it exists, is found by calculating $f(a)$.

(c) T F If f is not differentiable at $x = a$, then f is also not continuous at $x = a$.

exs21

Ex. U-5 True/False Su22 Exam

For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim_{x\to 1} f(x)$ and $\lim_{x\to 1} g(x)$ both exist, then $\lim_{x\to 1} (f(x)g(x))$ exists.

(b) T F If $f(9)$ is undefined, then $\lim_{x\to 9} f(x)$ does not exist.

(c) T F If $\lim_{x\to 1^+} f(x) = 10$ and $\lim_{x\to 1} f(x)$ exists, then $\lim_{x\to 1} f(x) = 10$.

(d) T F A function is continuous for all x if its domain is $(-\infty, \infty)$.

(e) T F If $f(x)$ is continuous at $x = 3$, then $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$.

(f) T F If $\lim_{x\to 2} f(x)$ exists, then f is continuous at $x = 2$.

(g) T F If $\lim_{x\to 5^-} f(x) = -\infty$, then $\lim_{x\to 5^+} f(x) = +\infty$.

(h) T F A function can have two different horizontal asymptotes.

exsu22

Ex. U-6 True/False Su22 Exam

For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If f is continuous at $x = 3$, then f is differentiable at $x = 3$.

(b) T F If f is differentiable at $x = 3$, then f is continuous at $x = 3$.

(c) T F If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$ for all x.

(d) T F The function $f(x) = |x|$ has two tangent lines at $x = 0$: the lines $y = x$ and $y = -x$.

(e) T F If $f(x) = x^{1/3}$, then $f'(0)$ does not exist.

(f) T F If $f(x) = x^{1/3}$, then there is no tangent line to f at $x = 0$.

(g) T F $\frac{d}{dx}(e^{2x}) = 2xe^{2x-1}$

(h) T F A certain cylindrical tank has a radius of 5 ft. If the height of the water in the tank increases at a constant rate, then the volume of the water in the tank also increases at a constant rate.

exsu22

Ex. U-7 True/False Su22 Quiz

If $f(x)$ is not defined at $x = a$, then which of the following must be true?

(a) $\lim_{x\to a} f(x)$ cannot exist

(b) $\lim_{x\to a^+} f(x)$ must be infinite (either $+\infty$ or $-\infty$)

(c) $\lim_{x\to a} f(x)$ could be 0

(d) none of the above

qsu22

Ex. U-8 True/False Su22 Quiz

If $\lim_{x\to a^-} f(x) = \lim_{x\to a^-} g(x) = 0$, then which of the following is true about $\lim_{x\to a^-} \frac{f(x)}{g(x)}$?

(a) The limit does not exist, and is not infinite.

(b) The limit is infinite (either $+\infty$ or $-\infty$).

(c) The limit must exist.

(d) There is not enough information to say anything about the limit's value.

qsu22

Ex. U-9 **True/False**

Zero or more of the following statements are true for all real numbers a, x, and y. Determine which statements are true and determine which statements are false. For each false statement, find values of a, x, and y that make the statement false.

(a) $a(x+y) = ax + ay$

(b) $a(x+y)^2 = (ax+ay)^2$

(c) $a(x+y)^2 = ax^2 + ay^2$

(d) $a\sqrt{x+y} = \sqrt{a^2x + a^2y}$

(e) $\sin(x+y) = \sin(x) + \sin(y)$

(f) $\cos(ax) = a\cos(x)$

(g) $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

(h) $\dfrac{a}{x+y} = \dfrac{a}{x} + \dfrac{a}{y}$

unit1

Extra Challenges

Ex. A-68 **Algebra/Precalculus** ⋆Challenge

Let $f(x) = \dfrac{2}{3-\sqrt{x}}$. Fully simplify the difference quotient $\dfrac{f(4+h)-f(4)}{h}$ for $h \neq 0$ (i.e., simplify the expression all common factors of h have been canceled.)

challenge

Ex. D-23 **2.4, 2.5** ⋆Challenge

For each function, find all horizontal asymptotes and vertical asymptotes. Then, at each vertical asymptote, calculate both one-sided limits.

(a) $f(x) = \dfrac{4x^3+4x^2-8x}{x^3+3x^2-4}$ (b) $f(x) = \dfrac{4x^3-\sqrt{x^6+17}}{5x^3-40}$

challenge

Ex. F-42 **2.6** ⋆Challenge

Consider $f(x) = \dfrac{\tan(2x)}{|5x|}$.

(a) Where is f not continuous?

(b) Is it possible to redefine f at $x = 0$ to make f continuous there? Explain your answer.

Hint: For the limit of f as $x \to 0$, examine the one-sided limits first.

challenge

Ex. H-38 **3.3/3.4/3.5/3.9** ⋆Challenge

Find all points on the graph of $y = \dfrac{2}{x} + 3x$ such that the tangent line there passes through $(6, 17)$.

challenge

Ex. J-35 **3.8** ⋆Challenge

Find all tangent lines to the graph of $9x^2 - 18xy + y^2 = 1800$ that are perpendicular to the line $x + 3y = 10$.

challenge

Ex. K-29 **3.11** ⋆Challenge **!!!**

A water tank in the shape of an inverted cone has height 10 meters and base radius 8 meters. Water flows into the tank at the rate of 32π m^3/min. At what rate is the depth of the water in the tank changing when the water is 5 meters deep?

challenge

Ex. J-36 **3.8, 4.6** ⋆Challenge

Consider the curve described by the equation

$$\frac{x-y^3}{y+x^2} = x - 12$$

(a) Find an equation for the line tangent to this curve at $(-1, 4)$.

(b) There is a point on the curve with coordinates $(-1.1, b)$. Use linear approximation to estimate b. Round to three decimal places.

(c) There is a point on the curve with coordinates $(a, 4.2)$. Use linear approximation to estimate a. Round to three decimal places.

challenge

Ex. O-32 **4.6** ⋆Challenge

The acceleration (measured in m/s^2) of a particle moving along the x-axis is given by

$$a(t) = 14t^{3/4} - 6t^2 + 1$$

and the particle is at rest (zero velocity) when $t = 1$. Use a linear approximation to estimate the particle's change in position between $t = 16$ and $t = 16.02$.

challenge

Ex. M-38 **4.3/4.4** ⋆Challenge

Consider the function $f(x) = ax^6e^{-bx}$, where a and b are unspecified constants. Suppose f has a point of local maximum at $(2, 64e^{-2})$. Find the values of a and b.

challenge

Ex. P-29 **4.7** ⋆Challenge

Suppose f'' is continuous for all x. Calculate $\lim_{h\to 0}\left(\dfrac{f(x+5h)+f(x-5h)-2f(x)}{h^2}\right)$.

challenge

Ex. P-30 **4.7** ⋆Challenge

Suppose f' is continuous for all x and $f(0) = 0$. Calculate $\lim_{x\to 0^+}(1+f(2x))^{4/x}$.

challenge

Ex. M-39 **4.3/4.4** ⋆Challenge **!!!**

Consider the function $f(x) = (x-3a)(x+2a)^4$, where a is an unspecified **positive** constant. Answer all of the following in terms of a.

(a) where is f decreasing?
(b) where is f increasing?
(c) where does f have a local minimum?
(d) where does f have a local maximum?
(e) where is f concave down?
(f) where is f concave up?
(g) where does f have an inflection point?

Finally, sketch a graph of $y = f(x)$. Your horizontal scale should be in terms of a and your vertical scale should be in terms of a^5.

challenge

Ex. M-40 **4.3/4.4** ⋆Challenge **!!!**

Let $f(x) = \dfrac{e^x}{4+x^3}$. Answer all of the following.

(a) what are the vertical asymptotes of f?
(b) what are the horizontal asymptotes of f?
(c) where is f decreasing?
(d) where is f increasing?
(e) where does f have a local minimum?
(f) where does f have a local maximum?

challenge

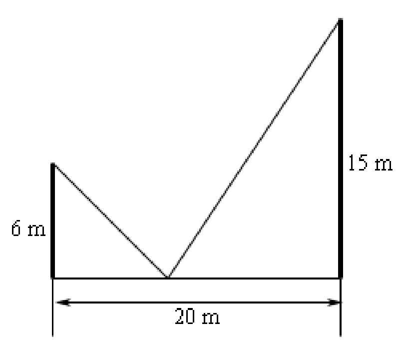
Ex. N-35 **4.5** ⋆Challenge **!!!**

Find the equation of the line through $(2, 4)$ that cuts off the least area from the first quadrant. (Observe that this cut off region is a triangle.)

challenge

Ex. N-36 **4.5** ⋆Challenge **!!!**

Two poles, one 6 meters tall and one 15 meters tall, are 20 meters apart. A length of wire is attached to the top of each pole and it is also staked to the ground somewhere between the two poles. Where should the wire be staked so that the minimum amount of wire is used?



challenge

Ex. B-14 **2.1/2.2** ⋆**Challenge**

Suppose $\lim_{x\to 0}(f(x)+g(x))$ exists. Is it true that $\lim_{x\to 0} f(x)$ and $\lim_{x\to 0} g(x)$ also exist? Explain your answer.

challenge

Ex. E-17 **2.5** ⋆**Challenge**

Find all horizontal asymptotes of $f(x) = \dfrac{2x}{x-\sqrt{x^2+10}}$.

challenge

Ex. F-43 **2.6** ⋆**Challenge**

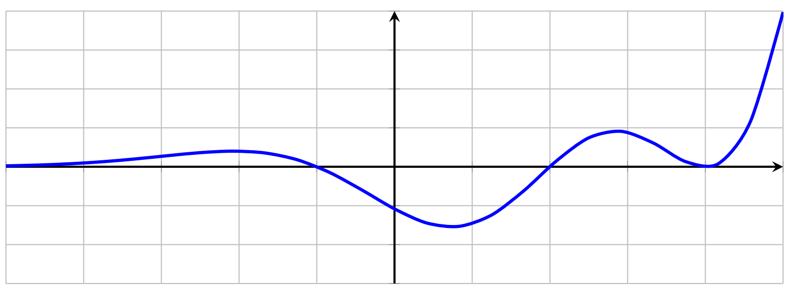
Find the values of the constants a and b that make f continuous at $x = 0$. *You may assume $a > 0$.*

$$f(x) = \begin{cases} \dfrac{1-\cos(ax)}{x^2} & , \quad x < 0 \\ 2a+b & , \quad x = 0 \\ \dfrac{x^2-bx}{\sin(x)} & , \quad x > 0 \end{cases}$$

challenge

Ex. G-36 **3.1/3.2** ⋆**Challenge**

The graph of $y = f(x)$ is given below. Sketch a graph of $y = f'(x)$. *Only the general shape is important. The graph does not have to be to scale.*



challenge

Ex. G-37 **3.1/3.2** ⋆**Challenge**

Consider the following function, where c is an unspecified constant

$$f(x) = \begin{cases} -x^2 & \text{if } x < 0 \\ x^2+2x & \text{if } 0 \le x < 1 \\ 6x-x^2+c & \text{if } x \ge 1 \end{cases}$$

(a) Show precisely that $f'(0)$ does not exist.

(b) Find the value of c that makes f differentiable at $x = 1$ or show that no such value exists.

challenge

Ex. H-39 **3.3/3.4/3.5/3.9** ⋆**Challenge**

Find all points P on the graph of $y = 4x^2$ with the property that the tangent line at P passes through the point $(2, 0)$.

challenge

Ex. J-37 **3.8** ⋆**Challenge**

Suppose $x^2 + y^2 = R^2$, where R is a constant. Find $\frac{d^2y}{dx^2}$ and fully simplify your answer as much as possible.

challenge

Ex. M-41 **4.3/4.4** ⋆**Challenge**

Let $f(x) = \sqrt[3]{x^3 - 48x}$.

(i) Find all vertical asymptotes and horizontal asymptotes of $f(x)$.

(ii) Find where $f(x)$ is decreasing and where $f(x)$ is increasing. Also find and classify all local extrema of $f(x)$.

challenge

4.3/4.4 ⋆Challenge

(iii) Find where $f(x)$ is concave down and where $f(x)$ is concave up. Also find all inflection points of $f(x)$.

(iv) Sketch a graph of $y = f(x)$.

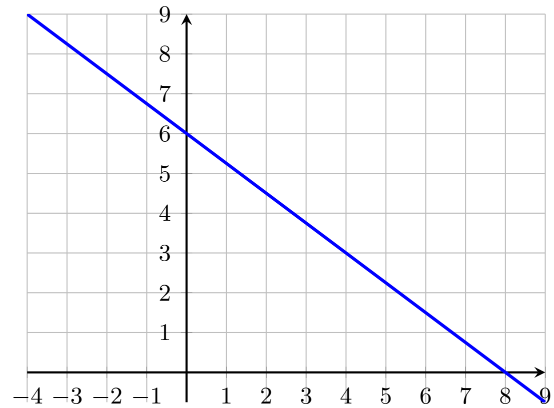
challenge

7 Chapter 7: Sample Exams (Set A)

Sample Precalculus Exam A

A-25. For each part, use the graph of $y = g(x)$ given below and let $f(x) = 8x^2 - 4x + 15$.

(a) Find an expression for $g(x)$.
(b) Calculate the y-intercept of the graph of $y = f(g(x))$.
(c) Calculate $g(f(x))$.



A-26. A 100-gram sample of a radioactive substance decays to 65% of its initial mass in 15 hours. Recall that the mass of the sample M at time t satisfies $M(t) = M_0 e^{kt}$ for some constants M_0 and k.

(a) Find the growth constant k.
(b) Find the mass of the sample after 22 hours.
(c) Find the time in hours when the sample will have a mass of 41 grams.

A-27. A rectangular box is constructed according to the following rules.

- The length of the box is 5 times its width.
- The volume of the box is 110 cubic feet.

Let L, W, and H be the length, width, and height of the box (measured in feet), respectively.

(a) Write an equation in terms of L, W, and H that expresses the first constraint.
(b) Write an equation in terms of L, W, and H that expresses the second constraint.
(c) Write an expression for $S(W)$, the total surface area of the box as a function of W.
(d) Suppose the rules also require that the sum of the box's length and width be less than 78 feet. What is the domain of $S(W)$ in this context?

A-28. Suppose $\log_{16}(x) = A$ and $\log_{16}(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_{16}\left(\frac{4x^7}{\sqrt[9]{y}}\right)$$

A-29. Let $f(x) = \sqrt{3x}$ and assume $h \neq 0$. Fully simplify each of the following expressions:

(a) $f(x+h)$ (b) $f(x+h) - f(x)$ (c) $\dfrac{f(x+h) - f(x)}{h}$

A-30. Consider the function $f(x) = \dfrac{x-6}{x^2 - 9x + 20}$.

(a) Solve the equation $f(x) = 0$.
(b) List all numbers that are not in the domain of $f(x)$.
(c) Solve the inequality $f(x) > 0$ and write your answer using interval notation.

A-31. Find all solutions to the following equation in the interval $[0, 2\pi)$.

$$2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$$

A-32. Complete each of the following algebra exercises.

(a) Fully factor the polynomial $5x^4 + 25x^3 - 180x^2$.

(b) Solve the rational equation below.

$$\frac{4}{x+5} + \frac{9x}{x^2-25} = \frac{6}{x-5}$$

(c) Simplify the complex fraction below by writing it as a simple fraction.

$$\frac{\frac{4}{x} - \frac{2}{xy}}{8 + \frac{7}{y}}$$

Sample Midterm Exam #1A

F-25. On the axes provided, sketch the graph of a function $f(x)$ that satisfies all of the following properties. ***Note:*** Make sure to read these properties carefully!

- the domain of $f(x)$ is $[-10, 7) \cup (7, 10]$
- $\lim_{x\to -8} f(x)$ exists but f is discontinuous at $x = -8$
- $\lim_{x\to -5^+} f(x) = f(-5)$ but $\lim_{x\to -5} f(x)$ does not exist
- $\lim_{x\to 2^-} f(x) = 4$ and f is continuous at $x = 2$
- the line $x = 5$ is a vertical asymptote for f (***Note:*** $x = 5$ is in the domain of f.)
- $\lim_{x\to 7} f(x) = +\infty$ (***Note:*** $x = 7$ is not in the domain of f.)

C-25. For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 8} \left(\frac{(x-2)^2 - 36}{x-8} \right)$ (b) $\lim_{x\to 5} \left(\frac{40 - 8x}{\sqrt{19-3x} - 2} \right)$ (c) $\lim_{x\to 2^-} \left(\frac{4+x}{x^2+x-6} \right)$

F-26. Consider the function below, where a and b are unspecified constants.

$$f(x) = \begin{cases} \dfrac{\sin(4x)\sin(6x)}{x^2} & x < 0 \\ ax + b & 0 \le x \le 1 \\ \dfrac{5x+2}{x-1} - \dfrac{2x+5}{x^2-x} & x > 1 \end{cases}$$

(a) Calculate $\lim_{x\to 0^-} f(x)$.

(b) Calculate $\lim_{x\to 1^+} f(x)$.

(c) Find the values of a and b for which f is continuous for all x, or determine that no such values exist. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

D-17. Find all vertical asymptotes of the function $f(x) = \dfrac{x^3 - 36x}{x^3 - 12x^2 + 36x}$.

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

E-11. Find all horizontal asymptotes of the function $h(x) = \dfrac{6x+5}{\sqrt{4x^2 - 9}}$.

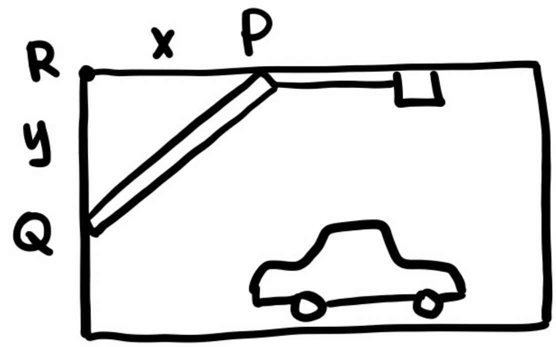
In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

Sample Midterm Exam #2A

I-9. For each part, calculate the indicated derivative. Do not simplify your answer.

(a) $\dfrac{d}{dx}\left(7x^{10} + \sqrt[3]{x} - \dfrac{8}{x^{20}} + \sec(8x)\right)$ (b) $\dfrac{d}{dx}\left(\dfrac{\ln\left(x^3+30\right)}{8x}\right)$ (c) $\dfrac{d}{dx}\left(\sin\left(xe^{-5x}\right)\right)$

K-17. A solid 14-foot tall garage door opens via a pulley mechanism. As the pulley opens the garage door, the top of the garage door (point P in the figure) moves to the right at 5 ft/s. At the same time, the bottom of the garage door (point Q in the figure) moves straight up.
As shown in the figure, the point R is the fixed point at the top of the garage door frame, x represents the distance between P and R, and y represents the distance between Q and R.



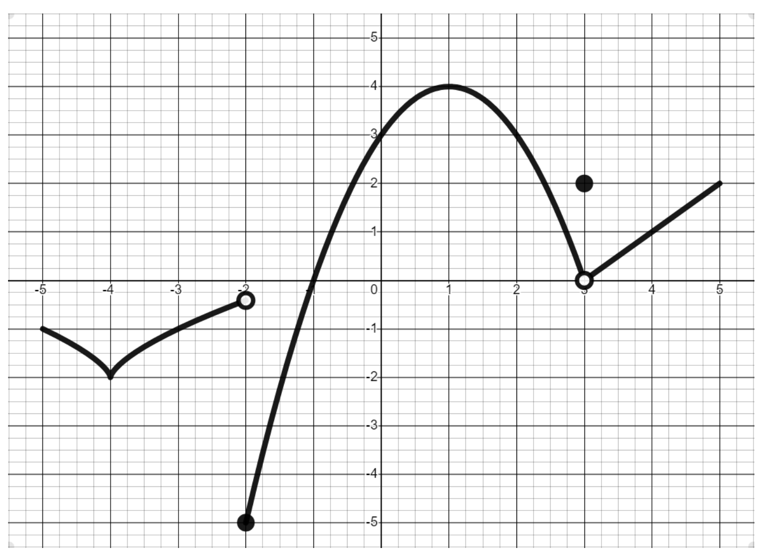
(a) What is the sign of $\dfrac{dx}{dt}$?

(b) What is the sign of $\dfrac{dy}{dt}$?

(c) What is the rate of change of the distance between the points Q and R when the distance between them is 9 feet? You must include correct units in your answer. You may leave unsimplified radicals in your answer.

G-22. For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

(a) $f'(-3)$
(b) $f'(-2)$
(c) $f'(-1)$
(d) $f'(1)$
(e) $f'(3)$



J-20. Consider the following curve.

$$\cos\left(5x + y - 5\right) = 8xe^y + y - 7$$

(a) Calculate $\dfrac{dy}{dx}$ for a general point on the curve.

(b) Find an equation of the line tangent to the curve at the point $(1, 0)$.

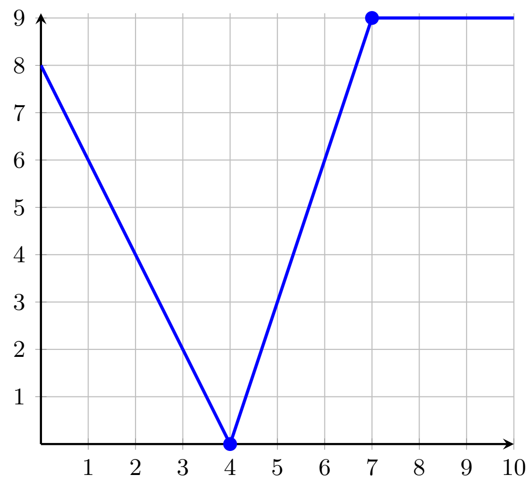
I-10. Find the coordinates of all points on the graph of $f(x) = x\sqrt{14 - x^2}$ where the tangent line is horizontal. You must give both the x- and y-coordinate of each such point.

G-23. Let $f(x) = \dfrac{8x}{x+5}$.

(a) Calculate $f'(x)$ by any method.

(b) Use the limit definition of derivative to calculate $f'(3)$. ***Hint:*** Use your answer from part (a) to check your final answer.

I-11. The graph of $y = f(x)$ is given below.



(a) Calculate $f'(6)$. Briefly explain how you found your answer.

(b) Let $g(x) = 9xf(2x)$. Find an equation of the line tangent to the graph of $y = g(x)$ at $x = 3$.

Sample Midterm Exam #3A

P-17. For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\displaystyle\lim_{x\to\pi}\left(\frac{\cos(6x)-1}{(x-\pi)^2}\right)$ (b) $\displaystyle\lim_{x\to 0}\left(e^{2x}+3x\right)^{1/x}$

M-24. Let $f(x)=4x^5-20x^4+7x+32$. Find where f is concave down and where f is concave up; write your answer using interval notation. Also find where inflection points of f occur.

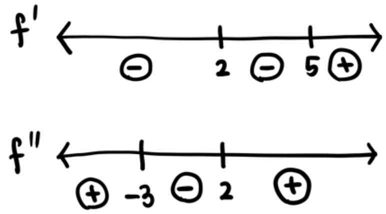
M-25. Suppose $f(x)$ satisfies all of the following properties. Sign charts for f' and f'' are also given below. Sketch a possible graph of $y=f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

(i) f is continuous and differentiable on $(-\infty,2)\cup(2,\infty)$

(ii) $\displaystyle\lim_{x\to-\infty}f(x)=\infty$; $\displaystyle\lim_{x\to\infty}f(x)=\infty$; $\displaystyle\lim_{x\to 2^-}f(x)=-\infty$; $\displaystyle\lim_{x\to 2^+}f(x)=\infty$

(iii) the only x-value for which $f'(x)=0$ is $x=5$

(iv) the only x-value for which $f''(x)=0$ is $x=-3$



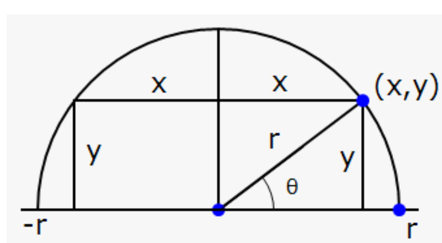
M-26. Let $f(x)=\dfrac{x^2+21}{x-2}$. Find where f is decreasing and where f is increasing; write your answer using interval notation. Also find where the local extrema of f occur.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

L-20. Find the absolute extreme values of $f(x)=x\,(x-8)^{5/3}$ on the interval $[0,9]$ and the x-values at which they occur.

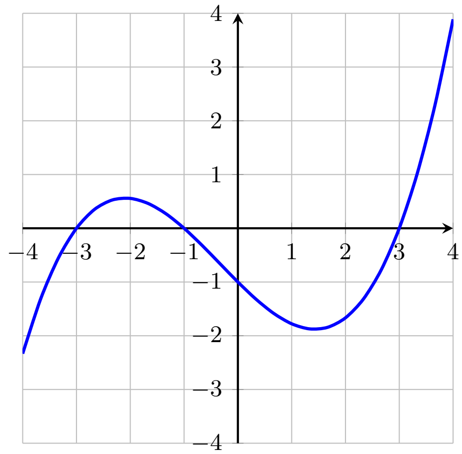
N-17. A rectangle (with base $2x$ and height y) is constructed with its base on the diameter of a semicircle with radius 5 and with its two other vertices on the semicircle. Find the dimensions of the rectangle with the maximum possible area. *As you work, fill in the answer boxes below. You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.*

constraint equation in terms of x and y:	
objective function in terms of x only:	
interval of interest:	
dimensions of rectangle:	______ $\times$ ______ $2x$ (base) y (height)



Sample Final Exam A

B-8. For each part, use the graph of $y = g(x)$.



(a) How many solutions does the equation $g'(x) = 0$ have?

(b) Order the following quantities from least to greatest: $g'(-2.5)$, $g'(-2)$, $g'(0)$, and $g'(4)$. In your answer, write these quantities symbolically; do not give a numerical estimate.

(c) What is the sign of $g''(-3)$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

(d) Let $h(x) = g(x)^2$. What is the sign of $h'(-4)$(negative, positive, or zero)? If there is not enough information to determine the value, explain why.

F-19. Let $f(x)$ be the following function, where k is an unspecified constant. Find the value of k that makes f continuous at $x = 2$ or determine that no such value of k exists.

$$f(x) = \begin{cases} 27x - kx^2 & x < 2 \\ -6 & x = 2 \\ 3x^3 + k & x > 2 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

1. Consider the curve described by the following equation: $2x^2 - 2xy + 3y^2 = 60$.

(a) Find $\dfrac{dy}{dx}$ for a general point on the curve.

(b) Find the x-coordinate of each point on the curve where the tangent line is horizontal.

S-3. The parts of this problem are not related.

(a) Calculate the integral $\displaystyle\int_2^4 \frac{18t - 3t^2}{t}\,dt$.

(b) Calculate the area of the region below the curve $y = 23\sin(x)\cos^2(x)$ and above the interval $\left[0, \frac{\pi}{2}\right]$ on the x-axis. (Note that $y \geq 0$ on this interval.)

D-13. For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\displaystyle\lim_{x\to 1}\left(\frac{x^4 - x}{\ln(77x - 76)}\right)$

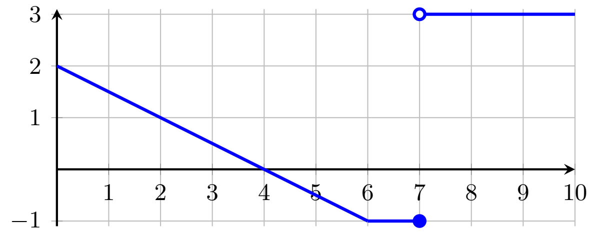
(b) $\displaystyle\lim_{x\to -\infty}\left(\frac{\sqrt{36x^2 + 63}}{31x}\right)$

(c) $\displaystyle\lim_{x\to 2^+} f(x)$, with $f(x) = \begin{cases} 1 + 4x & x \leq 2 \\ \dfrac{x^2 - 4}{x - 2} & x > 2 \end{cases}$

(d) $\displaystyle\lim_{x\to 5^-}\left(\frac{\cos(\pi x)}{x^2 - 25}\right)$

Q-4. For any time $t > 0$, the acceleration of a particle is given by $a(t) = 1 + \dfrac{3}{\sqrt{t}}$, and the particle has velocity $v = -20$ when $t = 1$. Find the velocity of the particle when $t = 16$.

R-8. Let $F(x) = \displaystyle\int_0^x f(t)\,dt$, where the graph of $y = f(t)$ is given below. For each part, use this information to calculate the indicated item.



(a) $F(10)$

(b) $F'(6)$

(c) $\displaystyle\int_0^6 |f(t)|\,dt$

(d) $\displaystyle\int_0^4 (f'(t)+5)\,dt$

O-18. Use linear approximation to estimate $\tan\left(\dfrac{\pi}{4} + 0.12\right) - \tan\left(\dfrac{\pi}{4}\right)$.

L-18. Let $f(x) = x^3(3x-4)$.

(a) Find where relative extrema of f occur (if any). Classify each as a local minimum or a local maximum.

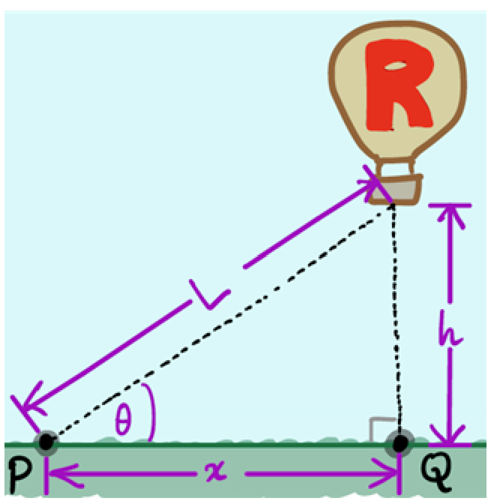
(b) Find the absolute extrema of f on $[-1, 2]$ and the x-values at which they occur.

D-14. For each part, find all vertical asymptotes of the given function.

(a) $f(x) = \dfrac{x^2 - 8x + 15}{x^2 - 9}$

(b) $g(x) = \dfrac{e^{x+3} - 1}{x^2 - 9}$

K-14. A hot-air balloon is floating directly above the point Q on the ground and is descending at a constant rate of 10 ft/sec. A camera is on the ground at point P, which is 500 feet from point Q. See the figure below.



(a) What is the sign of $\dfrac{dh}{dt}$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

(b) How is $\cos(\theta)$ changing over time? Circle your answer below.

(i) increasing over time

(ii) decreasing over time

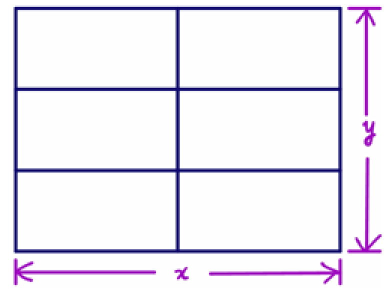
(iii) constant over time

(iv) sometimes increasing and sometimes decreasing

(v) not enough information to determine

(c) What is the rate of change of the distance between the camera and the balloon when the balloon is 600 feet above the ground? *You must give correct units as part of your answer.*

N-16. Farmer Green is building an enclosure that must have a total area of 48 m^2. The pen will also be subdivided into 6 pens of equal area, as shown on the right. Find the dimensions of the enclosure that will require the least amount of fencing. *As you work, fill in the answer boxes below. You must use calculus-based methods in your work. You must also justify that your answer really does give the least fencing.*



constraint equation in terms of x and y:	
objective function in terms of x only:	
interval of interest:	
dimensions of desired enclosure (in meters):	________ × ________ total length (x) total width (y)

M-23. Consider the function $g(x)$, whose first two derivatives are given below. ***Note:*** Do not attempt to calculate $g(x)$. Also assume that $g(x)$ has the same domain as $g'(x)$.

$$g'(x) = \frac{8x^{17}}{x-32} \qquad\qquad g''(x) = \frac{128x^{16}(x-34)}{(x-32)^2}$$

Fill in the table below with information about the graph of $y = g(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

where g is decreasing	
where g is increasing	
x-coordinate(s) of local minima of g	
x-coordinate(s) of local maxima of g	
where g is concave down	
where g is concave up	
x-coordinate(s) of inflection point(s) of g	

L-19. The parts of this problem are not related.

(a) Suppose that when x units are produced, the total cost is $C(x) = 2x^2 + 10x + 18$ and the selling price per unit is $p(x) = 46 - x$. Find the level of production that maximizes total profit.

(b) Suppose the total cost of producing q units is $C(q) = q^3 + 20q^2 + 200q + 2000$. Use marginal analysis to estimate the cost of the 3rd unit.

8 Chapter 8: Sample Exams (Set B)

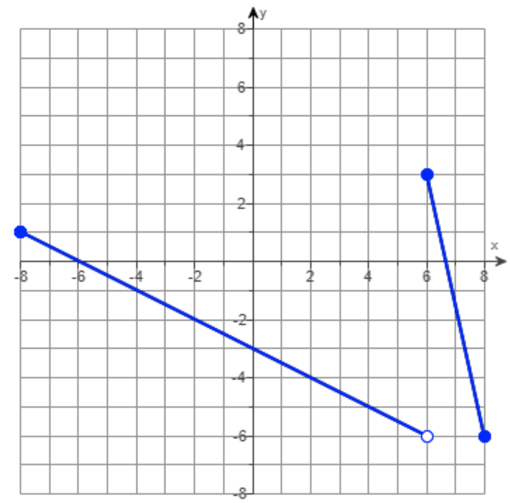
Sample Precalculus Exam B

A-16. The graph of $y = f(x)$ is given below.

Note that f is piecewise linear. An explicit formula for $f(x)$ can be written in the following form, where A and B are constants.

$$f(x) = \begin{cases} y_1(x) & \text{if } -8 \le x < A \\ y_2(x) & \text{if } B \le x \le 8 \end{cases}$$

Calculate each of A, B, $y_1(x)$, and $y_2(x)$.

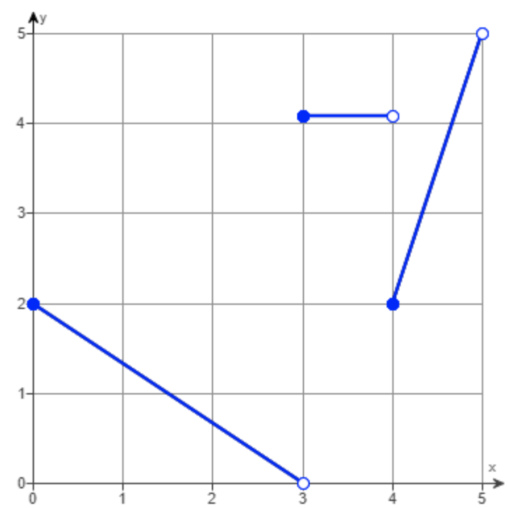


A-17. For each part, use the graph of $y = f(x)$.

(a) Calculate $f(f(2))$.

(b) State the domain of f in interval notation.

(c) State the range of f in interval notation.



A-18. Suppose $\log_3(x) = A$ and $\log_3(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_3\left(\frac{27\sqrt{x}}{y^4}\right)$$

A-19. Rewrite the expression below as a single logarithm. Assume x and y are positive.

$$\frac{1}{2}\left(\log_5(x) - 7\log_5(y)\right) + 3\log_5(x-1)$$

A-20. Suppose $\cos(\theta) = \frac{A}{7}$ with $0 < A < 7$ and $\sin(\theta) < 0$. Find $\sec(\theta)$, $\sin(\theta)$, and $\tan(\theta)$ in terms of A.

A-21. A bacteria colony has an initial population of 3500. The population grows exponentially and triples every 7 hours. Recall that this means the population P at time t satisfies $P(t) = P_0 e^{kt}$ for some constants P_0 and k.

(a) Find the exact value of the growth constant k.

(b) Find the population after 25 hours.

(c) Find the time (in hours) when the population will be 12,600.

A-22. A rectangular box is constructed according to the following rules.

- the length of the box is twice its width
- the height of the box is 5 feet more than three times the length

Let ℓ, w, and h denote the length, width, and height of the box, respectively, measured in feet.

(a) Write the height of the box in terms of w.

(b) Write an expression for $V(w)$, the volume of the box measured in cubic feet, as a function of its width.

(c) Suppose the rules also require that the sum of the box's width and height to be less than 26 feet. Under this condition, what is the domain of the function $V(w)$?

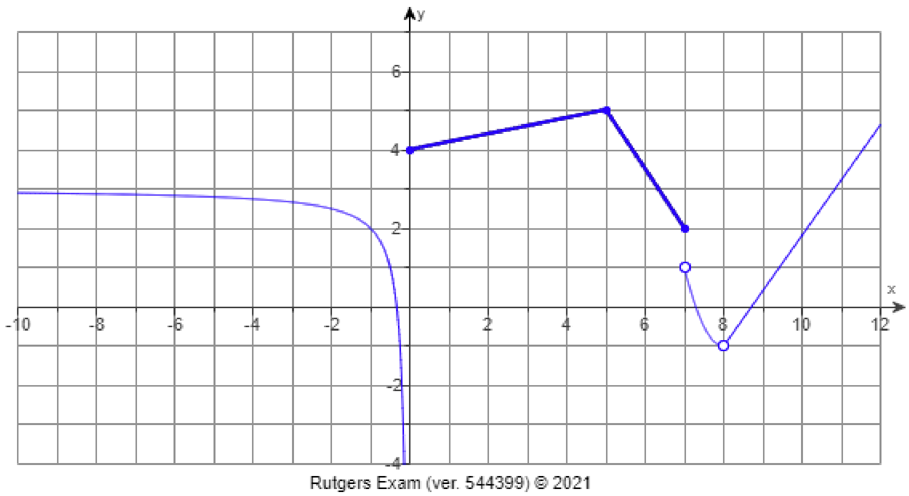
A-23. Let $f(x) = \dfrac{2}{3x}$ and assume $h \neq 0$. Fully simplify each of the following expressions:

(a) $f(x+h)$ (b) $f(x+h) - f(x)$ (c) $\dfrac{f(x+h) - f(x)}{h}$

A-24. Find the domain of the function $f(x) = \sqrt{x^2 + x - 6} + \ln(10 - x)$. Write your answer using interval notation.

Sample Midterm Exam #1B

B-6. For each part, use the graph of $y = f(x)$.



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

F-17. Consider the piecewise-defined function $f(x)$ below; A and B are unspecified constants and $g(x)$ is an unspecified function with domain $[94, \infty)$.

$$f(x) = \begin{cases} Ax^2 + 8 & x < 75 \\ \ln(B) + 6 & x = 75 \\ \dfrac{x - 75}{\sqrt{x+6} - 9} & 75 < x < 94 \\ 19 & x = 94 \\ g(x) & x > 94 \end{cases}$$

(a) Find $\lim_{x\to 75^-} f(x)$ in terms of A and B.

(b) Find $\lim_{x\to 75^+} f(x)$ in terms of A and B.

(c) Find the exact values of A and B for which f is continuous at $x = 75$.

(d) Suppose $g(94) = 19$. What does this imply about $\lim_{x\to 94} f(x)$? Select the best answer.

(i) $\lim_{x\to 94} f(x)$ exists.

(ii) $\lim_{x\to 94} f(x)$ does not exist.

(iii) It gives no information about $\lim_{x\to 94} f(x)$.

B-7. The position of a particle (measured in feet) after t seconds is modeled by the following function.

$$h(t) = -16t^2 + 96t + 100$$

(a) Calculate the average velocity of the particle (in feet per second) between $t = 4$ and $t = 5$.

(b) Find an equation of the secant line between $(4, h(4))$ and $(5, h(5))$.

C-20. Suppose $\lim_{x\to 6} |f(x)| = 2$. Which of the following statements must be true about $\lim_{x\to 6} f(x)$?

(i) $\lim_{x\to 6} f(x)$ does not exist.

(ii) $\lim_{x\to 6} f(x) = 2$.

(iii) $\lim_{x\to 6} f(x)$ exists and is equal to either 2 or -2, but there is not enough information to determine which of these possibilities must be true.

(iv) There is not enough information about $f(x)$ to determine whether $\lim_{x\to 6} f(x)$ exists.

(v) $\lim_{x\to 6} f(x) = -2$.

C-21. Consider the following function, where k is an unspecified constant.

$$f(x) = \frac{4x^2 - kx}{x^2 + 12x + 32}$$

(a) Find the value of k for which $\lim_{x\to -4} f(x)$ exists.

(b) For the value of k described in part (a), evaluate $\lim_{x\to -4} f(x)$.

C-22. Suppose $\lim_{x\to 0} \left(\frac{f(x)}{x}\right) = 8$. Calculate $\lim_{x\to 0} \left(\frac{f(x)}{\sin(6x)}\right)$ or show that the limit does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

F-18. Consider the following function.

$$f(x) = \frac{x^2 - x - 6}{x^3 - 2x^2 - 3x}$$

(a) Where is f discontinuous?

(b) At the leftmost x-value where f is discontinuous, what type of discontinuity does f have (removable, jump, infinite (vertical asymptote), or other)?

(c) At the rightmost x-value where f is discontinuous, what type of discontinuity does f have (removable, jump, infinite (vertical asymptote), or other)?

E-9. Let $f(x) = \dfrac{8 + 6e^x}{9e^x - \pi^6}$.

(a) Evaluate $\lim_{x\to\infty} f(x)$.

(b) Evaluate $\lim_{x\to -\infty} f(x)$.

(c) List all vertical asymptotes of f.

Sample Midterm Exam #2B

G-14. The following limit represents the derivative of a function f at a point a.

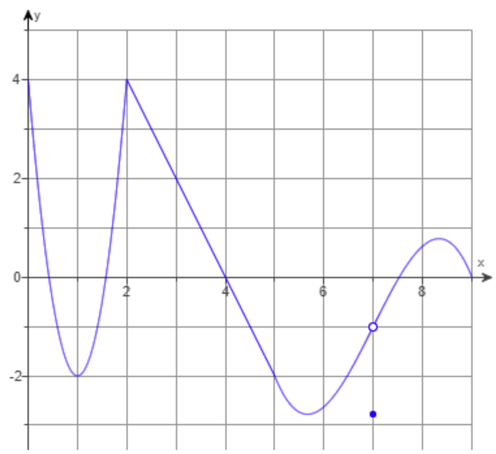
$$f'(a) = \lim_{h \to 0} \left(\frac{9\tan\left(\frac{\pi}{6} + h\right) - \frac{9}{\sqrt{3}}}{h} \right)$$

(a) Find a possible pair for f and a.

(b) Calculate the value of the limit.

G-15. For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

(a) $f'(1)$ (b) $f'(2)$ (c) $f'(3.5)$ (d) $f'(7)$



I-5. Let $f(x) = x^9 e^{4x}$.

(a) Find $f'(x)$.

(b) Explain how to find where the tangent line to the graph of f is horizontal.

(c) Find where the graph of f has a horizontal tangent line.

I-6. Selected values of the functions f and g and their derivatives are given in the table below. Use these values to complete the questions.

x	1	2	3	4
$f(x)$	4	3	2	1
$f'(x)$	-4	-1	-9	-3
$g(x)$	2	1	3	4
$g'(x)$	1	2	4	5

(a) Suppose $h(x) = 5f(x) - 8g(x)$. Find $h'(1)$.

(b) Suppose $p(x) = x^2 f(x)$. Find $p'(2)$.

(c) Suppose $q(x) = f(x^2)$. Find $q'(2)$.

G-16. Let $f(x)$ and $g(x)$ be functions such that $f'(-8) = g'(-8)$ and the line tangent to the graph of f at $x = -8$ is $y = -7x + 6$. For each part, compute the desired value, if possible.

(a) $f(-8)$ (b) $f'(-8)$ (c) $g(-8)$ (d) $g'(-8)$

J-17. Consider the curve defined by the following equation, where A and B are unspecified constants.

$$Ax^2 - 8xy = B\cos(y) + 3$$

(a) Find a formula for $\frac{dy}{dx}$.

(b) Suppose the point $(8, 0)$ is on the curve. Find an equation that A and B must satisfy.

(c) Suppose the tangent line to the curve at the point $(8, 0)$ is $y = 6x - 48$. Find the values of A and B.

K-13. The base of a right triangle is decreasing at a constant rate of 10 cm/sec and in such a way that the triangle always remains a right triangle. At the time when the base is 15 cm and the height is 22 cm, the area of the triangle is increasing by 25 cm^2/sec. Use this information to answer the questions below. Let B denote the base of the triangle.

(a) At the described time, what is the sign of $\frac{dB}{dt}$?

(b) At the described time, what is the sign of $\frac{d^2B}{dt^2}$?

(c) At the described time, at what rate is the height changing?

(d) What are the units of the answer to part (c)?

I-7. Suppose f is differentiable at x and $g(x) = \dfrac{16\ln(15x)}{6f(x) - \sqrt{x+17}}$. Find $g'(x)$.

Sample Midterm Exam #3B

L-17. Find the absolute extreme values of $f(x) = x^3 - 6x^2 + 9x + 20$ on $[-3, 2]$ and the x-value(s) at which they occur.

M-20. Consider the function f and its derivatives below.

$$f(x) = \frac{x-3}{x^2 - 6x - 16} \quad , \quad f'(x) = \frac{-(x-3)^2 - 25}{(x^2 - 6x - 16)^2} \quad , \quad f''(x) = \frac{2(x-3)\left((x-3)^2 + 75\right)}{(x^2 - 6x - 16)^3}$$

Find where f is concave down and where f is concave up; write your answers using interval notation. Also find the x-coordinate of each inflection point of f.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

M-21. Suppose f is differentiable on $(-\infty, 1) \cup (1, \infty)$ and satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

(i) $\lim_{x\to-\infty} f(x) = -3$; $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to 1^-} f(x) = -\infty$; $\lim_{x\to 1^+} f(x) = \infty$;

(ii) $f'(x) > 0$ on $(-\infty, -2)$ and $(5, \infty)$; $f'(x) < 0$ on $(-2, 1)$ and $(1, 5)$; $f'(-2) = f'(5) = 0$

(iii) $f''(x) > 0$ on $(-\infty, -7)$ and $(1, \infty)$; $f''(x) < 0$ on $(-7, 1)$; $f''(-7) = 0$

N-15. A storage shed with a volume of 1500 ft^3 is to be built in the shape of a rectangular box with a square base. The material for the base costs \$ 6/ft^2, the material for the roof costs \$ 9/ft^2, and the material for the sides costs \$2.50/ft^2. Find the dimensions of the cheapest shed. As you work, fill in the answer boxes below. Let x represent the length of the base of the shed.

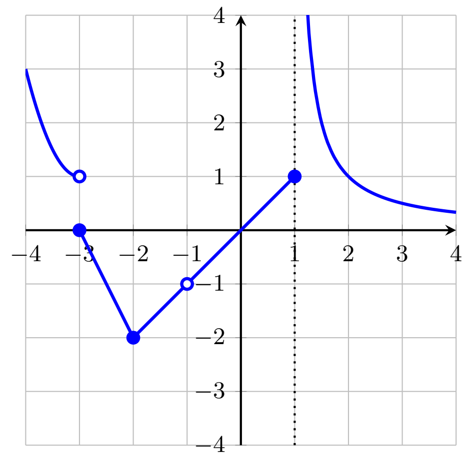
objective function in terms of x:	
interval of interest:	
dimensions of cheapest shed (in ft):	______ × ______ × ______ length of base, width of base, height of shed

M-22. Let $f(x) = -e^{-x}\left(x^2 - 5x - 23\right)$. Find all critical points of f. Then find where f is decreasing and where f is increasing; write your answers using interval notation. Also find where relative extrema of f occur.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

Sample Final Exam B

B-2. The graph of $y = f(x)$ is given below. Find all values of a in $(-4, 4)$ such that $\lim_{x\to a} f(x)$ does not exist.



G-5. Which statement is true about the graph of $f(x) = |x| + 91$ at the point $(0, 91)$?

(a) The graph has a tangent line at $y = 91$.
(b) The graph has infinitely many tangent lines.
(c) The graph has no tangent line.
(d) The graph has two tangent lines: $y = x + 91$ and $y = -x + 91$.
(e) None of the above statements is true.

O-11. Suppose the cost (in dollars) of manufacturing q units is given by

$$C(q) = 6q^2 + 34q + 112$$

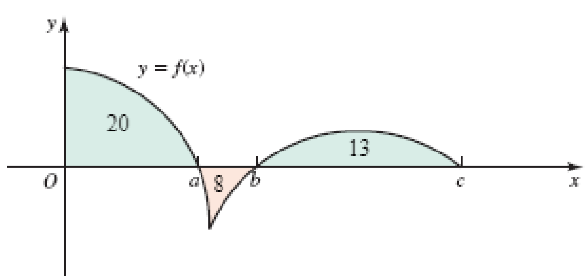
Use marginal analysis to estimate the cost of producing the 5th unit.

F-10. Consider the function $f(x)$, where k is an unspecified constant. Find the value of k for which f continuous for all x, or show that no such value of k exists.

$$f(x) = \begin{cases} 38 + kx & x < 3 \\ kx^2 + x - k & x \geq 3 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

R-3. The figure below shows the area of regions bounded by the graph of $y = f(x)$ and the x-axis, where $a = 4$, $b = 6$, and $c = 15$. Evaluate $\int_a^c (11f(x) - 6)\, dx$.



M-10. Consider the function f and its first two derivatives below.

$$f(x) = \frac{99e^x}{(x-25)^{47}} + 98 \quad , \quad f'(x) = \frac{99e^x(x-72)}{(x-25)^{48}} \quad , \quad f''(x) = \frac{99e^x\left((x-72)^2 + 47\right)}{(x-25)^{49}}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

P-11. A student is asked to calculate the following limit using l'Hospital's Rule and to show all their work.

$$L = \lim_{x\to 0}\left(\frac{\sin(2x) + 17x^2 + 2x}{4x^2 + \tan(x)}\right)$$

The student decides to cheat, so they find the solution online (shown below) and they submit the work as their own!

$$\begin{aligned}
L &= \lim_{x\to 0}\left(\frac{\sin(2x) + 17x^2 + 2x}{4x^2 + \tan(x)}\right) && (1)\\
&= \lim_{x\to 0}\left(\frac{2\cos(2x) + 34x + 2}{8x + \sec(x)^2}\right) && (2)\\
&= \lim_{x\to 0}\left(\frac{-4\sin(2x) + 34}{8 + 2\sec(x)^2\tan(x)}\right) && (3)\\
&= \frac{-4\sin(0) + 34}{8 + 2\sec(0)^2\tan(0)} && (4)\\
&= \frac{0 + 34}{8 + 0} && (5)\\
&= \frac{17}{4} && (6)
\end{aligned}$$

Unfortunately, this solution contains an error, and so the student lost all credit for the problem. The student was also later determined to be responsible for cheating, and so they earned a grade of 0 on the entire exam!

Your task is to find and correct the error(s). Answer the following questions.

(a) There may be several errors in this solution. Which line is the first incorrect line?

(b) Explain the error in the first incorrect line in your own words.

(c) Calculate the correct value of L (the original limit).

R-4. Consider the integral below.

$$\int_{-2}^{1} \sqrt{9 - (x-1)^2}\, dx$$

(a) Explain in your own words how you can calculate this integral without using Riemann sums or the fundamental theorem of calculus. ***Hint:*** Try graphing the integrand!

(b) Find the exact value of the integral.

J-12. Consider the curve described by the following equation.

$$e^{12x+2y} = 6y - 3xy + 1$$

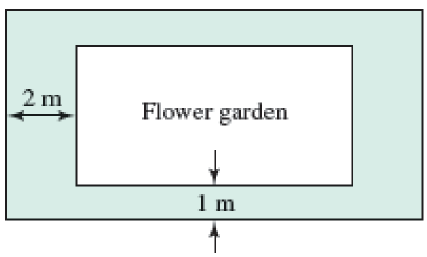
(a) Find $\dfrac{dy}{dx}$ at a general point on this curve.
(b) Calculate the slope of the line tangent to the curve at $(2, -12)$.
(c) There is a point on the curve close to the origin with coordinates $(0.07, b)$, and the line tangent to the curve at the origin is $y = 3x$. Use linear approximation to estimate the value of b.

G-6. Suppose the derivative of f is $f'(x) = 3x^2 - 6x - 9$ and that $f(1) = 10$.

(a) Find an equation of the line tangent to the graph of $y = f(x)$ at $x = 1$.
(b) Find the critical points of f.
(c) Where does f have a local minimum value? local maximum value?
(d) Calculate $f(0)$.
(e) Calculate the absolute maximum value of f on the interval $[0, 6]$. At what x-value does it occur?

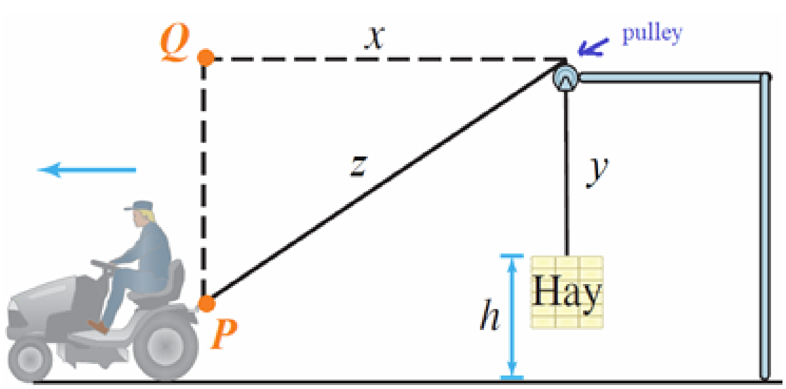
N-10. A local park has hired you to construct a rectangular flower garden surrounded by a grass border that is 1 m wide on two sides and 2 m wide on the other two sides. (See the figure below.) The area of the garden only (the small rectangle) must be 126 m^2.

Your primary task is to find the dimensions of the garden that give the smallest possible combined area of the garden and the grass border. For this problem, let W be the horizontal width of the garden and let H be the vertical height of the garden.



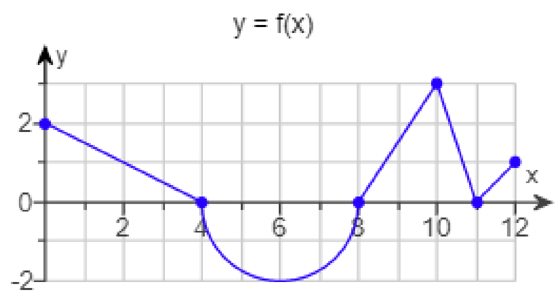
(a) What is the objective function for this problem in terms of W and H?
(b) What is the constraint equation for this problem in terms of W and H?
(c) Find the objective function in terms of W only.
(d) What is the interval of interest for the objective function?
(e) Find the values of W and H that minimize the total combined area.
(f) What horizontal width W of the garden will *maximize* the total area?

K-9. A farmer's tractor pulls a rope of length 12 m attached to a bale of hay through a pulley is 8 m above the ground. The vertical distance between the tractor and the pulley (the distance from P to Q) is 7 m. The tractor is moving to the left at rate of 2 m/sec, which causes the bale of hay to rise off the ground.



(a) The rate of change (with respect to time) of which variable is equal to the speed of the tractor?
(b) Use the Pythagorean theorem to find an equation that holds for all time and involves only the variables x and z.
(c) Use the fact that the length of the rope is constant to find an equation that holds for all time and involves only the variables z and y.
(d) Use the fact that the height of the pulley is constant to find an equation that holds for all time and involves only the variables h and y.
(e) Combine the equations from parts (b), (c), and (d) to find an equation that holds for all time and involves only the variables x and h.
(f) The rate of change (with respect to time) of which variable is equal to the rate at which the bale of hay is rising?
(g) Find the rate at which the bale of hay is rising off the ground when the horizontal distance between the tractor and the bale of hay is 8 m.

R-5. Define the function g by $g(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(x)$ is given below. The graph consists of four line segments and one semicricle. ***Note:*** f and g are different functions!



(a) Calculate $f'(9)$.
(b) Calculate $f'(6)$.
(c) Calculate $g'(6)$.
(d) Calculate $g(11) - g(8)$.
(e) Is the statement "$g(4) > g(0)$" true or false?
(f) Find the critical numbers of g in the interval $(0, 12)$.

T-1. ***Note:*** The parts of this problem are not related.

(a) Suppose we use the fundamental theorem of calculus to calculate an integral as follows:

$$\int_a^b g(u)\,du = G(b) - G(a)$$

What is the relationship between the functions g and G?
(b) Calculate the following definite integral:

$$\int_{e^{-3}}^{e^2} \frac{2\ln(x) - 3}{5x}\,dx$$

(c) Consider the following indefinite integral:

$$J = \int \frac{\ln(x)}{3x^2}\,dx$$

Use the substitution $u = \ln(x)$ to write J as an equivalent indefinite integral in terms of u. *Do not attempt to calculate J.*