All Practice Exercises for Math 135

Contains Midterm Exams Fall 2017 to Fall 2022
(Arranged by Topic)

Contents

1 Midterm Exams (Arranged by Topic)

1.1 Chapter 1: Algebra and Precalculus Review

§1.1, 1.2, 1.3, 1.4, 7.2, Appendix B

10 p **A1.** Find all solutions to the following equation.

$$2\ln(x) = \ln\left(\frac{x^5}{5-x}\right) - \ln\left(\frac{x^3}{2+x}\right)$$

10 p **A2.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F $\ln(3) - \ln(11) = \dfrac{\ln(3)}{\ln(11)}$

(b) T F The domain of $f(x) = \sqrt[9]{x-4}$ is all real numbers.

(c) T F The lines $9x + y = 1$ and $x - 9y = 4$ are perpendicular to each other.

(d) T F The equations $2\ln(x) = 0$ and $\ln(x^2) = 0$ have the same solutions.

(e) T F $\cos\left(\dfrac{5\pi}{6}\right) = -\dfrac{\sqrt{3}}{2}$

5 p **A3.** The number N of bacteria at time t grows exponentially, so that $N(t) = N_0 e^{kt}$. Suppose an initial population of 100 bacteria grows to 500 after 2 hours. How many hours does it take for an initial population of 150 bacteria to grow to 300?

5 p **A4.** Solve the inequality $\dfrac{3x-6}{x+4} > 0$. Write your answer using interval notation.

5 p **A5.** Solve the inequality $\dfrac{3x+6}{x-4} < 0$. *Write your answer using interval notation.*

5 p **A6.** Find the domain of the function $f(x) = \dfrac{\ln(80-x)}{\sqrt{x}-5}$. *Write your answer using interval notation.*

13 p **A7.** Let $f(x) = 8 - \dfrac{1}{5x}$. Fully simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ with $h \neq 0$. In your work, make clear where you use the assumption $h \neq 0$.

14 p **A8.** For both parts of this problem, consider the following inequality.

$$\frac{(x-3)(x-6)}{x-5} < 0$$

Your goal is to identify an error in a false solution of this inequality, and then to solve the inequality yourself.

(a) A student submits the following work for solving this equality.

> "First we multiply both sides by $(x-5)$. On the left side, this factor cancels, and on the right side we get 0. So we have $(x-3)(x-6) < 0$. The graph of $y = (x-3)(x-6)$ is a parabola that opens upward and crosses the x-axis at $x = 3$ and $x = 6$. This means that the graph is below the x-axis between these two x-values. So the solution to $(x-3)(x-6) < 0$ is the interval $(3, 6)$. But since the original inequality was undefined at $x = 5$, we also have to exclude 5. So the final answer is $(3, 5) \cup (5, 6)$."

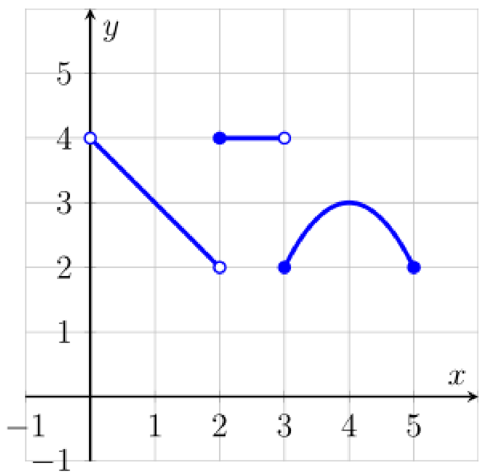
The student's teacher does not give full credit for this solution, simply noting that $x = 4$ is included in the student's answer, but $x = 4$ does not satisfy the original inequality. So the final answer must be wrong.

What is the student's error? Be as specific as possible and explain why this is an error. ***To explain why the given solution is wrong, it is not enough to simply write the correct solution and observe that the two solutions are different.***

(b) Solve the original inequality. Write your answer using interval notation.

12 p **A9.** Fully simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = \sqrt{x+2}$ and $h \neq 0$. Write your answer without square roots or fractional exponents in the numerator.

12 p **A10.** For each part, use the graph of $y = g(x)$ below.



(a) Find the domain of $g(x)$. Write your answer in interval notation.

(b) Calculate $g(g(4))$.

(c) As $x \to 2$, which of the left-sided and right-sided limits of $g(x)$ exist?

18 p **A11.** While solving the logarithmic equation

$$\log_2(3x+1) = 3$$

a student wrote the following steps (this work contains two distinct errors):

$$\log_2(3x) + \log_2(1) = 3 \quad (1)$$
$$\log_2(3x) + 0 = 3 \quad (2)$$
$$3x = 3^2 \quad (3)$$
$$x = 3 \quad (4)$$

(a) Identify the lines in which the two errors occur and describe each error.

(b) What is the correct solution to the original equation?

14 p **A12.** Fully simplify the difference quotient $\dfrac{f(3+h)-f(3)}{h}$ for $f(x) = \dfrac{6}{9-2x}$ and $h \neq 0$. Your answer cannot contain a complex fraction (fraction within a fraction).

16 p **A13.** Suppose we have all of the following:

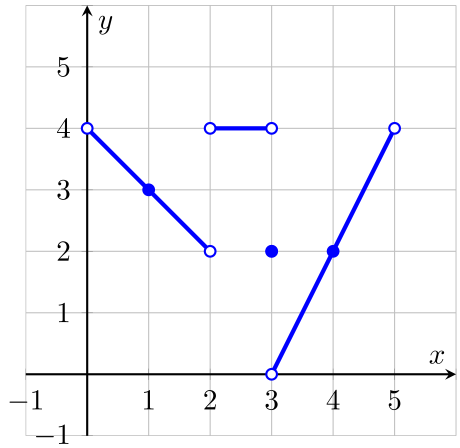
$$\log_3(x) = A \quad , \quad \log_3(y) = B \quad , \quad \log_{b^5}(z) = C$$

Write each of the following in terms of A, B, and C. Your final answer cannot contain any "log" symbol.

(a) $\log_3\left(\dfrac{\sqrt{x}}{9y^4}\right)$

(b) $\log_b(z)$

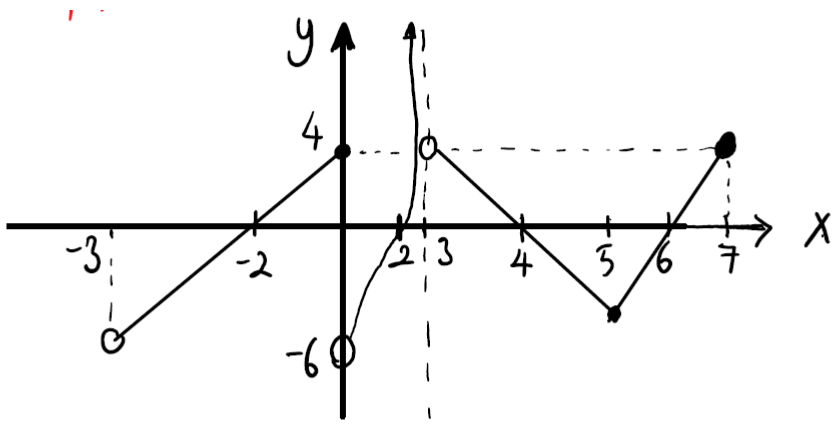
16 p **A14.** For each part, use the graph of $y = g(x)$ below.



(a) Calculate $g(g(1.5))$.

(b) Find the range of $g(x)$. Write your answer in interval notation.

18 p **A15.** For each part, use the graph of $y = f(x)$.



(a) Calculate $f(f(2))$.

(b) Find where $f(x) = 0$.

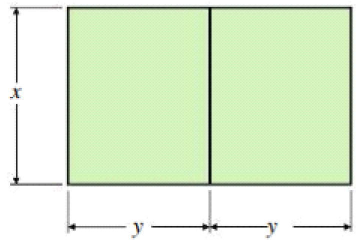
(c) State the domain of f in interval notation.

(d) State the range of f in interval notation.

(e) For each part below, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(i) $\lim\limits_{x\to 0^-} f(x)$ (ii) $\lim\limits_{x\to 0^+} f(x)$ (iii) $\lim\limits_{x\to 0} f(x)$ (iv) $\lim\limits_{x\to 3^-} f(x)$ (v) $\lim\limits_{x\to 3^+} f(x)$

12 p **A16.** Suppose you have exactly 840 ft of fencing that will be used to build an enclosure that consists of two identical rectangular pens that share a common fence. Let x be the (vertical) length of each pen and let y be the (horizontal) width of each pen. See the figure below.



(a) Find an expression for $F(x)$, the area of one individual pen, as a function of x.

(b) Now suppose that, for each of the two pens, the sum of the length and width must not exceed 250 ft. In the context of this problem, what is the domain of F? Write your answer in interval notation.

8 p **A17.** Suppose $\log_3(x) = A$ and $\log_3(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

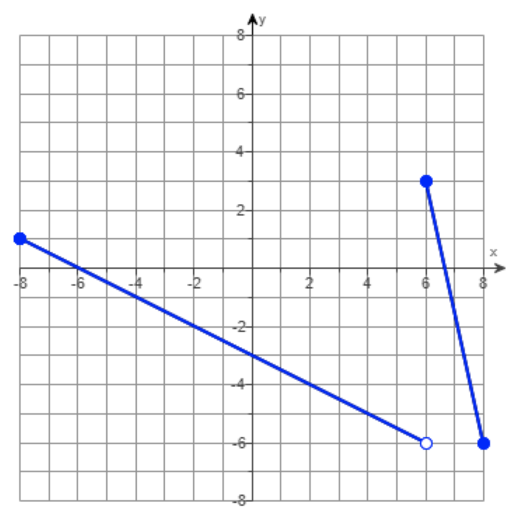
$$\log_3\left(\frac{27\sqrt{x}}{y^4}\right)$$

14 p **A18.** The graph of $y = f(x)$ is given below.

Note that f is piecewise linear. An explicit formula for $f(x)$ can be written in the following form, where A and B are constants.

$$f(x) = \begin{cases} y_1(x) & \text{if } -8 \le x < A \\ y_2(x) & \text{if } B \le x \le 8 \end{cases}$$

Calculate each of A, B, $y_1(x)$, and $y_2(x)$.

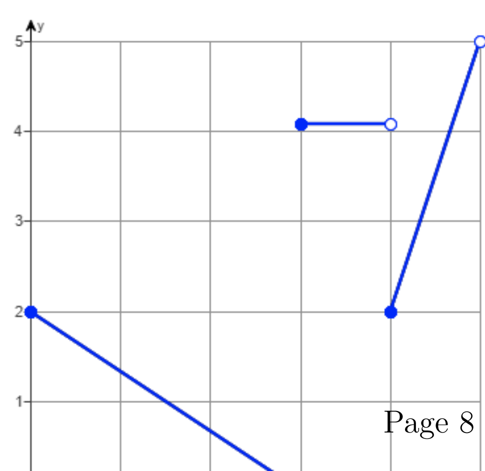


10 p **A19.** For each part, use the graph of $y = f(x)$.

(a) Calculate $f(f(2))$.

(b) State the domain of f in interval notation.

(c) State the range of f in interval notation.



9 p **A20.** Suppose $\log_3(x) = A$ and $\log_3(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_3\left(\frac{27\sqrt{x}}{y^4}\right)$$

9 p **A21.** Rewrite the expression below as a single logarithm. Assume x and y are positive.

$$\frac{1}{2}\left(\log_5(x) - 7\log_5(y)\right) + 3\log_5(x-1)$$

11 p **A22.** Suppose $\cos(\theta) = \frac{A}{7}$ with $0 < A < 7$ and $\sin(\theta) < 0$. Find $\sec(\theta)$, $\sin(\theta)$, and $\tan(\theta)$ in terms of A.

11 p **A23.** A bacteria colony has an initial population of 3500. The population grows exponentially and triples every 7 hours. Recall that this means the population P at time t satisfies $P(t) = P_0e^{kt}$ for some constants P_0 and k.

(a) Find the exact value of the growth constant k.

(b) Find the population after 25 hours.

(c) Find the time (in hours) when the population will be 12,600.

14 p **A24.** A rectangular box is constructed according to the following rules.

- the length of the box is twice its width
- the height of the box is 5 feet more than three times the length

Let ℓ, w, and h denote the length, width, and height of the box, respectively, measured in feet.

(a) Write the height of the box in terms of w.

(b) Write an expression for $V(w)$, the volume of the box measured in cubic feet, as a function of its width.

(c) Suppose the rules also require that the sum of the box's width and height to be less than 26 feet. Under this condition, what is the domain of the function $V(w)$?

10 p **A25.** Let $f(x) = \dfrac{2}{3x}$ and assume $h \neq 0$. Fully simplify each of the following expressions:

(a) $f(x+h)$ (b) $f(x+h) - f(x)$ (c) $\dfrac{f(x+h) - f(x)}{h}$

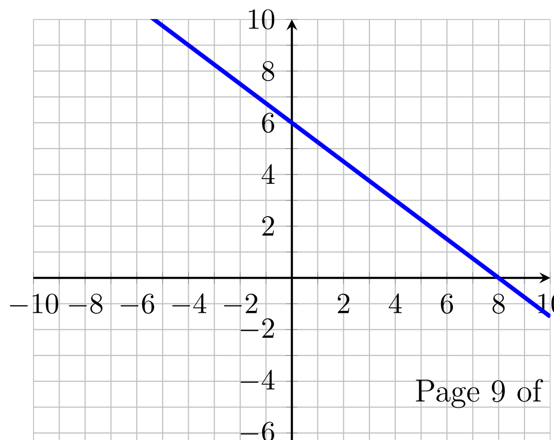
12 p **A26.** Find the domain of the function $f(x) = \sqrt{x^2 + x - 6} + \ln(10 - x)$. Write your answer using interval notation.

12 p **A27.** For each part, use the graph of $y = g(x)$ given below and let $f(x) = 8x^2 - 4x + 15$.

(a) Find an expression for $g(x)$.

(b) Calculate the y-intercept of the graph of $y = f(g(x))$.

(c) Calculate $g(f(x))$.



12 p **A28.** A 100-gram sample of a radioactive substance decays to 65% of its initial mass in 15 hours. Recall that the mass of the sample M at time t satisfies $M(t) = M_0 e^{kt}$ for some constants M_0 and k.

(a) Find the growth constant k.

(b) Find the mass of the sample after 22 hours.

(c) Find the time in hours when the sample will have a mass of 41 grams.

13 p **A29.** A rectangular box is constructed according to the following rules.

- The length of the box is 5 times its width.
- The volume of the box is 110 cubic feet.

Let L, W, and H be the length, width, and height of the box (measured in feet), respectively.

(a) Write an equation in terms of L, W, and H that expresses the first constraint.

(b) Write an equation in terms of L, W, and H that expresses the second constraint.

(c) Write an expression for $S(W)$, the total surface area of the box as a function of W.

(d) Suppose the rules also require that the sum of the box's length and width be less than 78 feet. What is the domain of $S(W)$ in this context?

12 p **A30.** Suppose $\log_{16}(x) = A$ and $\log_{16}(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_{16}\left(\frac{4x^7}{\sqrt[9]{y}}\right)$$

12 p **A31.** Let $f(x) = \sqrt{3x}$ and assume $h \neq 0$. Fully simplify each of the following expressions:

(a) $f(x+h)$ (b) $f(x+h) - f(x)$ (c) $\dfrac{f(x+h) - f(x)}{h}$

12 p **A32.** Consider the function $f(x) = \dfrac{x-6}{x^2 - 9x + 20}$.

(a) Solve the equation $f(x) = 0$.

(b) List all numbers that are not in the domain of $f(x)$.

(c) Solve the inequality $f(x) > 0$ and write your answer using interval notation.

12 p **A33.** Find all solutions to the following equation in the interval $[0, 2\pi)$.

$$2\sin(\theta)\cos(\theta) - \cos(\theta) = 0$$

15 p **A34.** Complete each of the following algebra exercises.

(a) Fully factor the polynomial $5x^4 + 25x^3 - 180x^2$.

(b) Solve the rational equation below.

$$\frac{4}{x+5} + \frac{9x}{x^2 - 25} = \frac{6}{x-5}$$

(c) Simplify the complex fraction below by writing it as a simple fraction.

$$\frac{\dfrac{4}{x} - \dfrac{2}{xy}}{8 + \dfrac{7}{y}}$$

28 p **A35.** Complete each of the following algebra exercises.

(a) Simplify $\left(\dfrac{27x^{3/5}}{x^{-3}z^{15}}\right)^{-1/3}$, leaving positive exponents and integer coefficients.

(b) Simplify $\dfrac{x^2-9}{3-\sqrt{6-x}}$ for $x \neq -3$. (All common factors must be canceled.)

(c) Factor the expression completely: $5x^9 - 14x^8 - 3x^7$.

(d) Fully simplify the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ for $f(x) = \dfrac{2}{x} - 3$ and $h \neq 0$.

24 p **A36.** For each part, find all solutions to the given equation.

(a) $\sqrt{2x+1} + 1 = x$

(b) $(10-x^2)^{1/2} - x^2(10-x^2)^{-1/2} = 0$

(c) $2 + \sin(\theta) = 2\cos(\theta)^2$ (find solutions in $[0, 2\pi)$ only)

8 p **A37.** Find the domain of the function $f(x) = \ln(x^2 - 20)$. Write your answer using interval notation.

8 p **A38.** The length of a rectangular box is three times its width, and the total surface area of the box is 200 in^2. Let W be the width of the box in inches. Find the volume of the box in terms of W.

12 p **A39.** For each part, write an equation for the line in the xy-plane that satisfies the given description.

(a) The line through the point $(-2, 10)$ with slope -3.

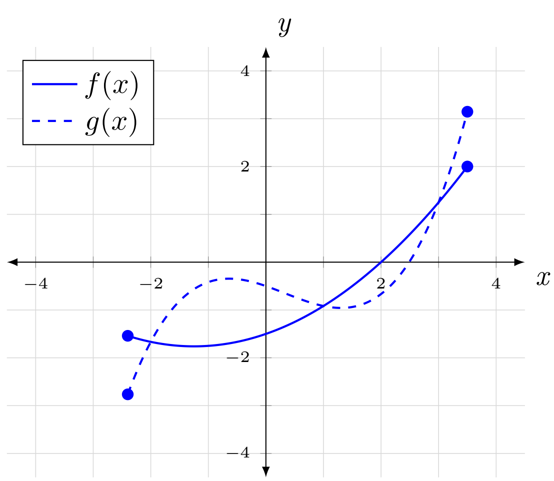
(b) The line through the points $(3, 5)$ and $(-1, 4)$.

(c) The line through the point $(5, 1)$ and perpendicular to the line $x + 3y = 10$.

(d) The horizontal line through the point $(-2, 15)$.

8 p **A40.** The number of bacteria in a certain colony grows exponentially. Recall that this means the number of bacteria N at time t is $N(t) = N_0 e^{kt}$, where N_0 and k are constants. Suppose there are initially 500 bacteria, and the number of bacteria triples every 2 hours. How much time must pass before the number of bacteria increases from 500 to 5000?

12 p **A41.** For each part, use the graphs of $y = f(x)$ and $y = g(x)$ below.



(a) Calculate $f(2)$.

(b) Estimate the value of $g(0) - f(0)$.

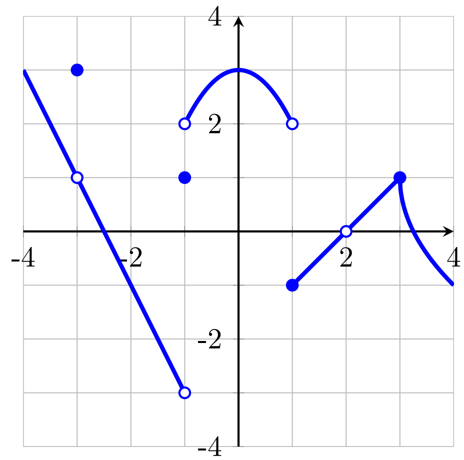
(c) Find all solutions to the equation $f(x) = g(x)$.

(d) Solve the inequality $g(x) > f(x)$. Write your answer using interval notation.

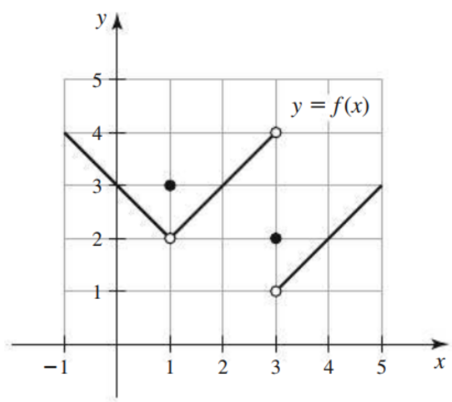
1.2 Chapter 2: Limits

§2.1, 2.2: Introduction to Limits

5 p **B1.** The graph of $y = f(x)$ is given below. Find all values of a in the interval $(-4, 4)$ for which $\lim_{x \to a} f(x)$ does not exist. If there are no such values of a, write "does not exist".



12 p **B2.** For each part, use the graph of $f(x)$ below.



(a) Calculate $\lim_{x \to 3} f(x)$ or determine that the limit does not exist.

(b) Find all values of a such that both $\lim_{x \to a} f(x)$ exists and this limit is not equal to $f(a)$.

12 p **B3.** Consider the function below.

$$f(x) = \begin{cases} x^2 + 4x - 1 & x < 2 \\ 11 & x = 2 \\ 19 - x^3 & x > 2 \end{cases}$$

A student correctly calculates that $\lim_{x \to 2} f(x) = 11$ and enters this as their final answer on an online exam, initially getting full credit. However, after inspecting the student's work, the teacher overrides this score and gives no credit. The teacher writes the comment "you have not correctly justified your answer." The student wrote the following:

"Since $f(x)$ is defined for all x and $f(2) = 11$, the answer is $\lim_{x \to 2} f(x) = 11$."

(a) Why is the student's justification incorrect?

(b) Write a complete and correct justification for the statement $\lim_{x \to 2} f(x) = 11$.

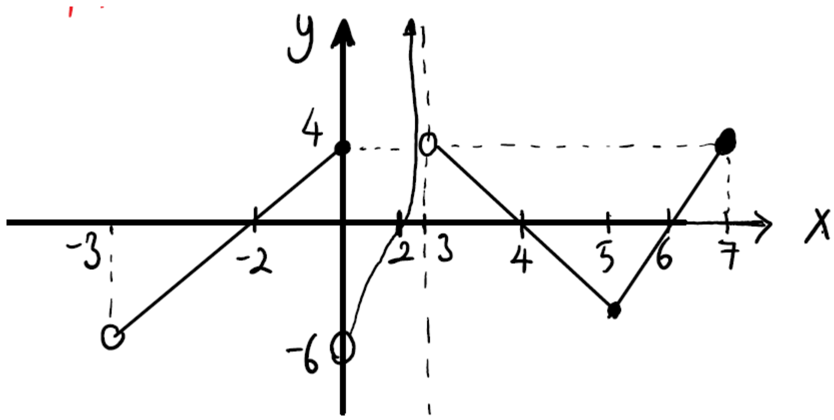
14 p **B4.** Sketch the graph of a function $f(x)$ with all of the given properties. Do not attempt to find a formula for the function.

(i) $f(1) = 0$ and $\lim_{x\to 1} f(x)$ does not exist

(ii) $f(2) = 1$ and $\lim_{x\to 2} f(x) = 0$

(iii) $f(3) = -2$ and $\lim_{x\to 3^-} f(x) = -1$ and $\lim_{x\to 3^+} f(x) = -2$

18 p **B5.** For each part, use the graph of $y = f(x)$.



(a) Calculate $f(f(2))$.

(b) Find where $f(x) = 0$.

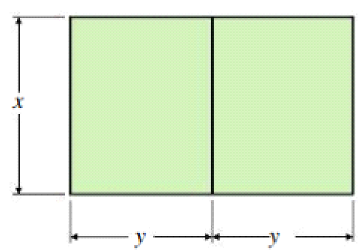
(c) State the domain of f in interval notation.

(d) State the range of f in interval notation.

(e) For each part below, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(i) $\lim_{x\to 0^-} f(x)$ (ii) $\lim_{x\to 0^+} f(x)$ (iii) $\lim_{x\to 0} f(x)$ (iv) $\lim_{x\to 3^-} f(x)$ (v) $\lim_{x\to 3^+} f(x)$

12 p **B6.** Suppose you have exactly 840 ft of fencing that will be used to build an enclosure that consists of two identical rectangular pens that share a common fence. Let x be the (vertical) length of each pen and let y be the (horizontal) width of each pen. See the figure below.



(a) Find an expression for $F(x)$, the area of one individual pen, as a function of x.

(b) Now suppose that, for each of the two pens, the sum of the length and width must not exceed 250 ft. In the context of this problem, what is the domain of F? Write your answer in interval notation.

8 p **B7.** Suppose $\log_3(x) = A$ and $\log_3(y) = B$. Rewrite the expression below in terms of A and B. Your final answer may not contain any logarithm symbol.

$$\log_3\left(\frac{27\sqrt{x}}{y^4}\right)$$

10 p **B8.** Determine whether the following statement is true or false. Explain your answer in 1 or 2 sentences. Your answer should contain English with few mathematical symbols.

"Suppose f and g are functions with $g(3) = 1$. Put $H(x) = \dfrac{f(x)}{g(x)-1}$. Then H must have a vertical asymptote at $x = 3$."

8 p **B9.** Suppose $\lim_{x\to 0} f(x) = 4$. Calculate $\lim_{x\to 0}\left(\dfrac{xf(x)}{\sin(5x)}\right)$ or show that the limit does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

12 p **B10.** Consider the following limit, where a is an unspecified constant.

$$\lim_{x\to -3}\left(\frac{x^2-a}{x^3+x^2-6x}\right)$$

(a) Find the value of a for which this limit exists.

(b) For this value of a, calculate the value of the limit.

16 p **B11.** Consider the following function, where k is an unspecified constant.

$$g(x) = \begin{cases} xe^{x+4} - 7\ln(x+5) & x < -4 \\ -4\cos(\pi x) & -4 < x < 5 \\ 10 & x = 5 \\ \sqrt{2x-5} + k & 5 < x \end{cases}$$

Note that $g(-4)$ is undefined.

(a) Does $\lim_{x\to -4} g(x)$ exist? Choose the best answer below.

(i) Yes, $\lim_{x\to -4} g(x)$ exists and is equal to ______.

(ii) Yes, $\lim_{x\to -4} g(x)$ exists but we cannot determine its value with the given information.

(iii) No, $\lim_{x\to -4} g(x)$ does not exist because the corresponding one-sided limits are not equal.

(iv) No, $\lim_{x\to -4} g(x)$ does not exist because $g(-4)$ does not exist.

(v) No, $\lim_{x\to -4} g(x)$ does not exist because the limit is infinite.

(b) Calculate the following limits. Your answer may contain k.

(i) $\lim_{x\to 5^-} g(x)$ (ii) $\lim_{x\to 5^+} g(x)$

(c) Is it possible to choose a value of k so that $\lim_{x\to 5} g(x)$ exists? If so, what is that value of k?

16 p **B12.** Let $f(x) = \dfrac{(x+a)(x-3)}{(x-2)(x+1)}$, where a is an unspecified, **positive** constant. For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0} f(x)$ (b) $\lim_{x\to 2^-} f(x)$ (c) $\lim_{x\to 2^+} f(x)$ (d) $\lim_{x\to 2} f(x)$

12 p **B13.** The following limit represents the derivative of a function f at a point a.

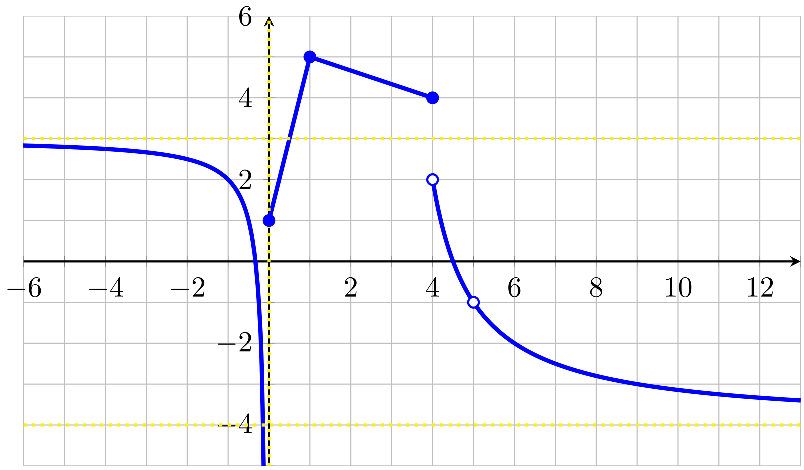
$$f'(a) = \lim_{h\to 0}\left(\frac{5\ln(e^4+h)-20}{h}\right)$$

(a) Find a possible function $f(x)$.

(b) For your choice of f in part (a), find a possible value of a.

(c) Calculate the value of the limit. Explain your calculation briefly in one sentence.

12 p **B14.** Use the graph of f below to answer the following questions. Dashed lines indicate the location of asymptotes.



(a) Calculate $\lim_{x\to\infty} f(x)$.

(b) Calculate $\lim_{x\to-\infty} f(x)$.

(c) List the values of x where f is not continuous.

(d) List the values of x where f is not differentiable.

(e) What is the sign of $f'(-1)$? (choices: positive, negative, zero, does not exist)

(f) What is the sign of $f'(0.5)$? (choices: positive, negative, zero, does not exist)

16 p **B15.** Consider the function g below, where a and b are unspecified constants. Assume that g is continuous for all x.

$$g(x) = \begin{cases} be^x + a + 1 & x \le 0 \\ ax^2 + b(x+3) & 0 < x \le 1 \\ a\cos(\pi x) + 7bx & 1 < x \end{cases}$$

(a) What relation must hold between a and b for g to be continuous at $x = 0$? Your answer should be an equation involving a and b.

(b) What relation must hold between a and b for g to be continuous at $x = 1$? Your answer should be an equation involving a and b.

(c) Calculate the values of a and b.

12 p **B16.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim_{x\to a} f(x)$ can be evaluated by direct substitution, then f is continuous at $x = a$.

(b) T F The value of $\lim_{x\to a} f(x)$, if it exists, is found by calculating $f(a)$.

(c) T F If f is not differentiable at $x = a$, then f is also not continuous at $x = a$.

20 p **B17.** Suppose that an equation of the tangent line to f at $x = 5$ is $y = 3x - 8$. Let $g(x) = \dfrac{f(x)}{x^2 + 10}$.

(a) Calculate $f(5)$ and $f'(5)$.

(b) Calculate $g(5)$ and $g'(5)$.

(c) Write down an equation of the tangent line to g at $x = 5$.

12 p **B18.** Suppose $f(2) = -7$ and $f'(2) = 3$.

(a) Let $g(x) = \cos(x)f(x)$. Calculate $g'(2)$.

(b) Let $h(x) = e^{2f(x)+3}$. Calculate $h'(2)$.

16 p **B19.** Let $f(x) = x^2 + bx + c$, where b and c are unspecified constants. An equation of the tangent line to f at $x = 3$ is $12x + y = 10$.

(a) Calculate $f(3)$ and $f'(3)$. Your answers must not contain the letters b or c.

(b) Calculate the value of b.

(c) Calculate the value of c.

20 p **B20.** A local gym has two cylindrical swimming pools. The larger pool has radius 20 meters and is filled with water. The smaller pool has radius 12 meters and is empty. Water is drained from the large pool and immediately emptied into the small pool. The height of the water in the small pool increases at a rate of 0.2 m/min.
Let V_L, V_S, h_L, and h_S refer to the volume of the large pool, volume of the small pool, height of the large pool, and height of the small pool, respectively.

(a) How are $\dfrac{dV_L}{dt}$ and $\dfrac{dV_S}{dt}$ related?

(b) What is the sign of $\dfrac{dh_L}{dt}$?

(c) Find $\dfrac{dV_S}{dt}$.

(d) Find $\dfrac{dh_L}{dt}$.

10 p **B21.** Use the identity $4^2 + \sqrt{4} = 18$ and linear approximation to estimate $(3.81)^2 + \sqrt{3.81}$.

15 p **B22.** The total cost (in dollars) of producing x items is modeled by the function $C(x) = x^2 + 4x + 3$, and the price per item (in dollars) is $p(x) = \dfrac{98x + 49}{x + 3}$.

(a) Calculate the exact cost of producing the 5th item.

(b) Using marginal analysis, estimate the revenue derived from producing the 5th item.

20 p **B23.** Consider the curve defined by the equation below, where a and b are unspecified constants.

$$\sqrt{xy} = ay^3 + b$$

Suppose the equation of the tangent line to the curve at the point $(3,3)$ is $y = 3 + 4(x-3)$.

(a) What is the value of $\frac{dy}{dx}$ at $(3,3)$?

(b) Calculate a and b.

15 p **B24.** Suppose $f''(x)$ is continuous. You are also given the following values:

$$f\left(\frac{1}{8}\right) = 20 \quad , \quad f'\left(\frac{1}{8}\right) = -22$$

Calculate the following limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 8}\left(\frac{20 - f\left(\frac{1}{x}\right)}{x^2 + x - 72}\right)$$

20 p **B25.** The length L (measured in meters) of a certain fish depends on time t (measured in years since birth) and is modeled by the function f.

$$L = f(t) = 4t^{2.99}$$

The mass m (measured in kilograms) of the fish depends on the length L and is modeled by the function g.

$$m = g(L)$$

The function g is not explicitly given.

(a) Describe in one English sentence, as precisely and specifically as you can, what the quantity $Q = f(4) - f(0)$ represents in the context of this problem.

(b) Describe in one English sentence, as precisely and specifically as you can, what the quantity $f'(1)$ represents in the context of this problem.

(c) What are the units of $g'(4.87)$?

(d) Suppose that $\frac{dm}{dL} = 7$ when $L = 4$. (Note that $L = 4$ when $t = 1$.) At what rate (measured in kg/yr) is the mass of the fish changing with respect to time exactly 1 year after its birth?

16 p **B26.** Consider the function below, where A is an unspecified, **positive** constant.

$$f(x) = \frac{A}{x - 8\sqrt{x} + 60}$$

For parts (c) and (d) only, assume the absolute minimum of f on $[0,21]$ is 8.

(a) List all x-values that must be tested to find the absolute extrema of f on $[0,21]$.

(b) At which x-value does the absolute minimum of f occur on $[0,21]$?

(c) Find the value of A.

(d) Find the absolute maximum of f on $[0,21]$ and all x-values at which it occurs.

18 p **B27.** An airline policy states that all baggage must be shaped like a rectangular box with the sum of the length, width, and height not exceeding 122 inches. You plan to purchase a bag from a company that makes customized bagged whose height must be 3 times its width. Find the dimensions of the baggage with the largest volume. (Let L, W, and H be the length, width, and height of the baggage, respectively.)

(a) Before considering any constraints particular to this problem, find the objective function in terms of L, W, and H.

(b) There are two constraints for this problem. One constraint is from the airline and the other is from the baggage company. Find these constraints.

(c) Write the objective function in terms of W only.

(d) Find the interval of interest for the objective function in part (c).

(e) Find the dimensions of the baggage with the largest volume.

14 p **B28.** Consider the function $f(x)$ whose second derivative is given.

$$f''(x) = \frac{(x-2)^2(x-5)^3}{(x-9)^5}$$

You may assume the domain of $f(x)$ is $(-\infty, 9) \cup (9, \infty)$.

Find where $f(x)$ is concave down, where $f(x)$ is concave up, and where $f(x)$ has an inflection point. Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

14 p **B29.** A particle travels along the x-axis in such a way that its velocity (measured in ft/sec) at any time t (measures in sec) is

$$v(t) = 4t^3 - 2t + 2$$

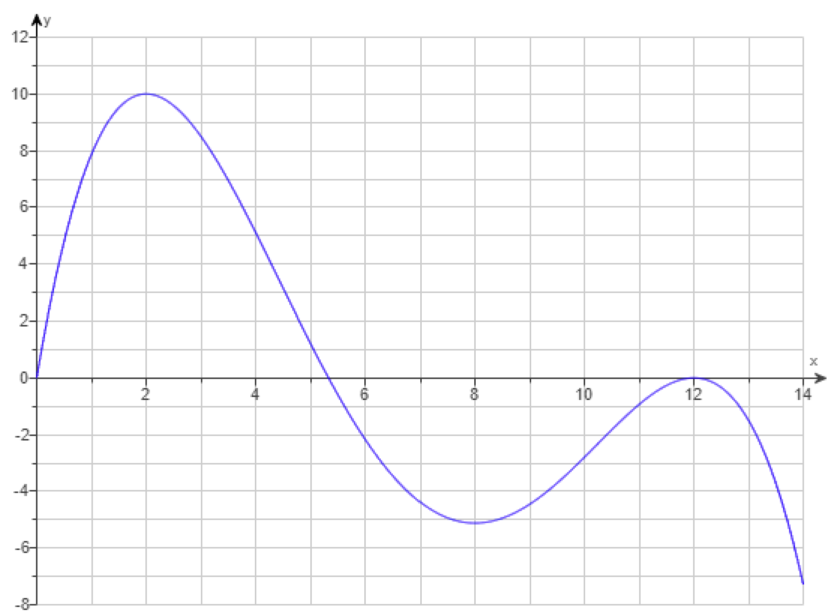
The particle is at $x = 3$ when $t = 2$.

(a) Find the position of the particle at any time t.

(b) Find the position of the particle at time $t = 4$.

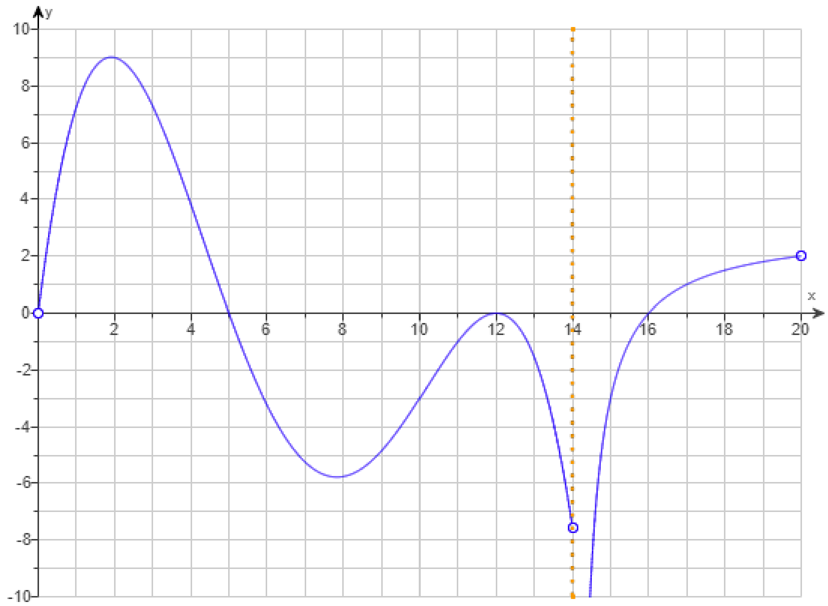
(c) Find the acceleration of the particle when $t = 4$.

8 p **B30.** Use the graph of $y = f(x)$ on $[0, 14]$ below to answer the questions.



(a) List the critical points of f in $(0, 14)$.
(b) How many local extrema does f have on $(0, 14)$?
(c) Find the absolute maximum of f and the x-value at which it occurs.
(d) Find the absolute minimum of f and the x-value at which it occurs.

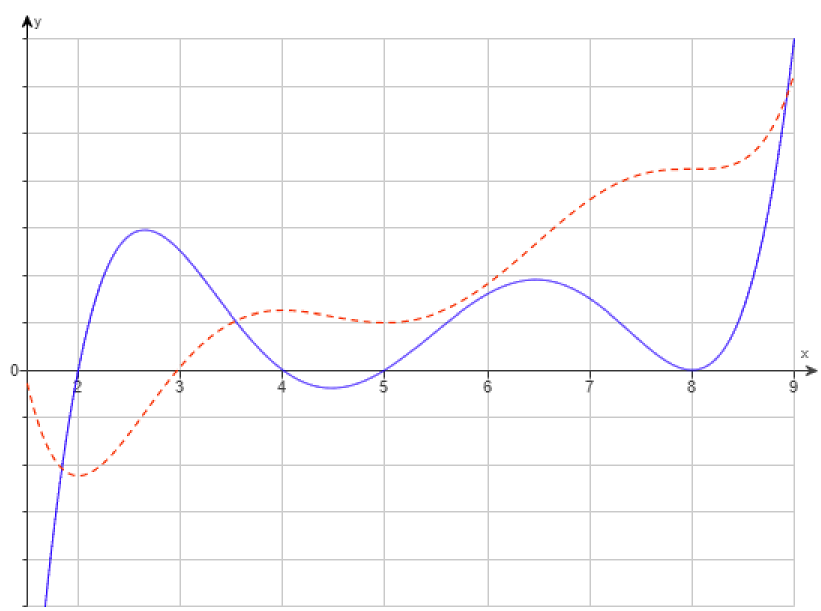
18 p **B31.** Use the graph of $y = f'(x)$ below to answer the questions. You may assume that $f'(x)$ has a vertical asymptote at $x = 14$ and that the domain of f is $(0, 14) \cup (14, 20)$.



Note: You are given a graph of the first derivative of f, not a graph of f.

(a) Find the critical points of f.
(b) Find where f is decreasing, where f is increasing, where f has a local minimum, and where f has a local maximum. Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

12 p **B32.** The figure below shows the graphs of two functions. One function is $f(x)$ and the other is $f'(x)$, but you are not told which is which.

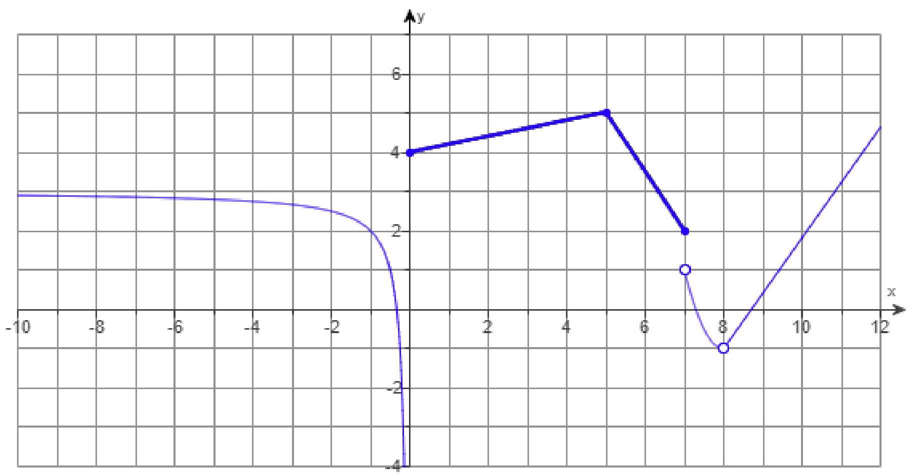


(a) Which graph is that of $y = f(x)$?

(b) Explain your answer to part (a) based on the behavior of the graphs at $x = 4$ only.

(c) Explain your answer to part (a) based on the behavior of the graphs near $x = 3.5$ only.

10 p **B33.** For each part, use the graph of $y = f(x)$.



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

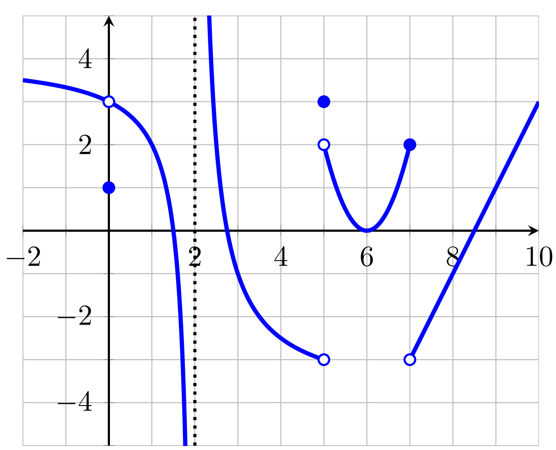
8 p **B34.** The position of a particle (measured in feet) after t seconds is modeled by the following function.

$$h(t) = -16t^2 + 96t + 100$$

(a) Calculate the average velocity of the particle (in feet per second) between $t = 4$ and $t = 5$.

(b) Find an equation of the secant line between $(4, h(4))$ and $(5, h(5))$.

10 p **B35.** For each part, use the graph of $y = g(x)$ below to calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*



(a) $\lim_{x\to 0} g(x)$ (b) $\lim_{x\to 2^-} g(x)$ (c) $\lim_{x\to 5^-} g(x)$ (d) $\lim_{x\to 5^+} g(x)$ (e) $\lim_{x\to 7} g(x)$

24 p **B36.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim_{x\to 1} f(x)$ and $\lim_{x\to 1} g(x)$ both exist, then $\lim_{x\to 1} (f(x)g(x))$ exists.

(b) T F If $f(9)$ is undefined, then $\lim_{x\to 9} f(x)$ does not exist.

(c) T F If $\lim_{x\to 1^+} f(x) = 10$ and $\lim_{x\to 1} f(x)$ exists, then $\lim_{x\to 1} f(x) = 10$.

(d) T F A function is continuous for all x if its domain is $(-\infty, \infty)$.

(e) T F If $f(x)$ is continuous at $x = 3$, then $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$.

(f) T F If $\lim_{x\to 2} f(x)$ exists, then f is continuous at $x = 2$.

(g) T F If $\lim_{x\to 5^-} f(x) = -\infty$, then $\lim_{x\to 5^+} f(x) = +\infty$.

(h) T F A function can have two different horizontal asymptotes.

§2.3: Techniques for Computing Limits

18 p **C1.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 7}\left(\frac{\frac{1}{7}-\frac{1}{x}}{x-7}\right)$ (b) $\lim_{x\to 0}\left(\frac{\sin(7x)}{\tan(2x)}\right)$ (c) $\lim_{x\to -1}\left(\frac{|x+1|}{x+1}\right)$

15 p **C2.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0}\left(\frac{(2x+9)^2-81}{x}\right)$ (b) $\lim_{x\to 3^-}\left(\frac{|x-3|}{x-3}\right)$ (c) $\lim_{x\to 1}\left(\frac{5-\sqrt{32-7x}}{x-1}\right)$

20 p **C3.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{u\to 4}\left(\frac{(u+6)^2-25u}{u-4}\right)$

(b) $\lim_{s\to 1} g(s)$ where $g(s)=\begin{cases}\sqrt{1-s} & s\le 1\\ \frac{s^2-s}{s-1} & s>1\end{cases}$

(c) $\lim_{h\to 0}\left(\frac{\sin(7+h)-\sin(7)}{h}\right)$

Hint: Use the definition of the derivative.

(d) $\lim_{x\to 6}\left(\frac{\frac{1}{36}-x^{-2}}{x^2-36}\right)$

10 p **C4.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 5}\left(\frac{x-5}{x^2-2x-15}\right)$ (b) $\lim_{x\to 0}\left(\frac{\sin(9x)}{\sin(16x)}\right)$

15 p **C5.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 5}\left(\frac{x^2-3x-10}{x^2-x-20}\right)$ (b) $\lim_{x\to 0}\left(\frac{\sin^2(4x)}{x^2}\right)$ (c) $\lim_{x\to 4}\left(\frac{3-\sqrt{2x+1}}{x-4}\right)$

30 p **C6.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0}\left(\frac{(4x+1)^2-1}{x}\right)$

(b) $\lim_{x\to 0}\left(\frac{9x\cos(2x)}{\sin(4x)}\right)$

(c) $\lim_{x\to -1}\left(\frac{4-\sqrt{16x+32}}{x+1}\right)$

(d) $\lim_{x\to 4^-}\left(\frac{|x^2-16|}{4-x}\right)$

(e) $\lim_{x\to 3} g(x)$, where $g(x)=\begin{cases}\frac{x-3}{x^3-9x} & x<3\\ 18 & x=3\\ \frac{x-2}{x^2+9} & x>3\end{cases}$

24 p **C7.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 3}\left(\frac{x^3+2x^2-15x}{x^2-9}\right)$ (b) $\lim_{x\to 0}\left(\frac{\sin(6x)^2}{x^2\cos(2x)}\right)$

12 p **C8.** The parts of this problem are related.

(a) Suppose $x<3$. Write an algebraic expression that is equivalent to $|x-3|$ but without absolute value symbol.

(b) Calculate $\lim_{x\to 2}\left(\frac{|x-3|-1}{x-2}\right)$. Explain why your work to part (a) is relevant here and precisely where you use it.

13 p **C9.** The parts of this problem are related.

(a) Consider the function below.

$$f(x)=\begin{cases}\dfrac{x-1}{3-\sqrt{10-x}} & x\neq 1\\ -6 & x=1\end{cases}$$

Show that $\lim_{x\to 1} f(x)\neq f(1)$.

(b) Now consider the similar function below.

$$g(x)=\begin{cases}\dfrac{x-1}{3-\sqrt{10-x}} & x\neq 1\\ b & x=1\end{cases}$$

where b is an unspecified constant. Explain how to determine whether the following statement is true: $\lim_{x\to 1} g(x)\neq g(1)$. How does your work for part (a) change, if at all, to determine the truth of the statement? Explain your answer.

24 p **C10.** For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 5}\left(\frac{25-x^2}{x-5}\right)$ (b) $\lim_{x\to 4}\left(\frac{\frac{1}{x}-\frac{1}{4}}{4-x}\right)$

10 p **C11.** A student is asked to solve a certain limit and determines the limit does not exist. (This may or may not be the correct answer.) They write the following for their justification:

> "I used the direct substitution property to evaluate the limit. I noticed the denominator gives me a zero, therefore the limit does not exist."

Explain why the student's justification is incorrect.

Note: To solve this problem, it is not necessary to be given the actual limit the student was asked to compute.

12 p **C12.** Determine whether $\lim_{x\to 0} f(x)$ exists, where $f(x)=\begin{cases}3e^x-7 & x<0\\ 4+\sin(x) & x\geq 0\end{cases}$.

12 p **C13.** A student is asked to calculate the following limit:

$$L = \lim_{x\to 0}\left(\frac{x\cos x}{\sin(3x)}\right)$$

Analyze their work below, which contains two distinct errors. ***Note:*** The correct answer is $\frac{1}{3}$, not 0.

$$L = \lim_{x\to 0}\left(\frac{x\cos(x)}{3\sin(x)}\right) \quad (1)$$

$$= \left[\lim_{x\to 0}\left(\frac{1}{3}\right)\right]\left[\lim_{x\to 0}\left(\frac{x}{\sin(x)}\right)\right]\left[\lim_{x\to 0}\left(\cos(x)\right)\right] \quad (2)$$

$$= \left(\frac{1}{3}\right)(1)(0) \quad (3)$$

$$= 0 \quad (4)$$

Identify the lines in which the two errors occur and describe each error.

C14. Consider the function $f(x)$ below, where $g(x)$ is an unspecified function with domain $[4, \infty)$.

$$f(x) = \begin{cases} 4 & x \le 0 \\ \dfrac{x-4}{\frac{1}{4}-\frac{1}{x}} & 0 < x < 4 \\ 16 & x = 4 \\ g(x) & x > 4 \end{cases}$$

12 p (a) Show that $\lim_{x\to 4^-} f(x) = f(4)$.

4 p (b) Suppose $g(4) = 16$. Is it necessarily true that $\lim_{x\to 4} f(x)$ exists? Justify your response.

8 p **C15.** A student is asked to solve a certain limit and determines the limit does not exist. (This may or may not be the correct answer.) They write the following for their justification:

> "I used the direct substitution property to evaluate the limit. I obtained the expression "$\frac{0}{0}$", which is undefined. Therefore the limit does not exist."

Is the student's justification correct? Explain.

Note: To solve this problem, it is not necessary to be given the actual limit the student was asked to compute.

C16. Consider the limit $\lim_{x\to 3}\left(\frac{(5x-c)(x+4)}{x-3}\right)$, where c is an unspecified constant.

10 p (a) For what value(s) of c does this limit exist? Explain.

6 p (b) Suppose the limit exists. What is its value? Show all work.

8 p **C17.** Suppose $\lim_{x\to 0} f(x) = 4$. Calculate $\lim_{x\to 0}\left(\frac{xf(x)}{\sin(5x)}\right)$ or show that the limit does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

12 p **C18.** Consider the following limit, where a is an unspecified constant.

$$\lim_{x\to -3}\left(\frac{x^2-a}{x^3+x^2-6x}\right)$$

(a) Find the value of a for which this limit exists.

(b) For this value of a, calculate the value of the limit.

16 p **C19.** Consider the following function, where k is an unspecified constant.

$$g(x)=\begin{cases} xe^{x+4}-7\ln(x+5) & x<-4 \\ -4\cos(\pi x) & -4<x<5 \\ 10 & x=5 \\ \sqrt{2x-5}+k & 5<x \end{cases}$$

Note that $g(-4)$ is undefined.

(a) Does $\lim_{x\to -4} g(x)$ exist? Choose the best answer below.

(i) Yes, $\lim_{x\to -4} g(x)$ exists and is equal to ______.

(ii) Yes, $\lim_{x\to -4} g(x)$ exists but we cannot determine its value with the given information.

(iii) No, $\lim_{x\to -4} g(x)$ does not exist because the corresponding one-sided limits are not equal.

(iv) No, $\lim_{x\to -4} g(x)$ does not exist because $g(-4)$ does not exist.

(v) No, $\lim_{x\to -4} g(x)$ does not exist because the limit is infinite.

(b) Calculate the following limits. Your answer may contain k.

(i) $\lim_{x\to 5^-} g(x)$ (ii) $\lim_{x\to 5^+} g(x)$

(c) Is it possible to choose a value of k so that $\lim_{x\to 5} g(x)$ exists? If so, what is that value of k?

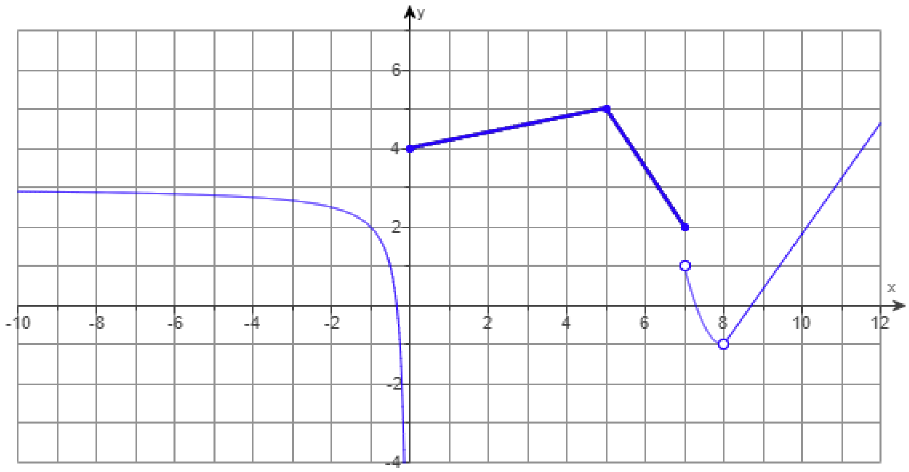
12 p **C20.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) [T] [F] If $\lim_{x\to a} f(x)$ can be evaluated by direct substitution, then f is continuous at $x=a$.

(b) [T] [F] The value of $\lim_{x\to a} f(x)$, if it exists, is found by calculating $f(a)$.

(c) [T] [F] If f is not differentiable at $x=a$, then f is also not continuous at $x=a$.

10 p **C21.** For each part, use the graph of $y = f(x)$.



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

20 p **C22.** Consider the piecewise-defined function $f(x)$ below; A and B are unspecified constants and $g(x)$ is an unspecified function with domain $[94, \infty)$.

$$f(x) = \begin{cases} Ax^2 + 8 & x < 75 \\ \ln(B) + 6 & x = 75 \\ \dfrac{x - 75}{\sqrt{x+6} - 9} & 75 < x < 94 \\ 19 & x = 94 \\ g(x) & x > 94 \end{cases}$$

(a) Find $\lim_{x\to 75^-} f(x)$ in terms of A and B.

(b) Find $\lim_{x\to 75^+} f(x)$ in terms of A and B.

(c) Find the exact values of A and B for which f is continuous at $x = 75$.

(d) Suppose $g(94) = 19$. What does this imply about $\lim_{x\to 94} f(x)$? Select the best answer.

(i) $\lim_{x\to 94} f(x)$ exists.

(ii) $\lim_{x\to 94} f(x)$ does not exist.

(iii) It gives no information about $\lim_{x\to 94} f(x)$.

4 p **C23.** Suppose $\lim_{x\to 6} |f(x)| = 2$. Which of the following statements must be true about $\lim_{x\to 6} f(x)$?

(i) $\lim_{x\to 6} f(x)$ does not exist.

(ii) $\lim_{x\to 6} f(x) = 2$.

(iii) $\lim_{x \to 6} f(x)$ exists and is equal to either 2 or -2, but there is not enough information to determine which of these possibilities must be true.

(iv) There is not enough information about $f(x)$ to determine whether $\lim_{x \to 6} f(x)$ exists.

(v) $\lim_{x \to 6} f(x) = -2$.

8 p **C24.** Consider the following function, where k is an unspecified constant.

$$f(x) = \frac{4x^2 - kx}{x^2 + 12x + 32}$$

(a) Find the value of k for which $\lim_{x \to -4} f(x)$ exists.

(b) For the value of k described in part (a), evaluate $\lim_{x \to -4} f(x)$.

4 p **C25.** Suppose $\lim_{x \to 0} \left(\frac{f(x)}{x} \right) = 8$. Calculate $\lim_{x \to 0} \left(\frac{f(x)}{\sin(6x)} \right)$ or show that the limit does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

44 p **C26.** For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x \to 3} \left(\frac{x - 3}{10 - \sqrt{x + 97}} \right)$

(b) $\lim_{x \to 6} \left(\frac{36 - x^2}{\frac{1}{x} - \frac{1}{6}} \right)$

(c) $\lim_{x \to 0} \left(\frac{x^2 \csc(3x)}{\cos(7x) \sin(4x)} \right)$

(d) $\lim_{x \to 2^-} \left(\frac{6x^2 - 7x}{x^2 - 4} \right)$

27 p **C27.** For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x \to 5} \left(\frac{6x + 10}{x^2 - 25} - \frac{4}{x - 5} \right)$

(b) $\lim_{x \to 6} \left(\frac{x - \sqrt{5x + 6}}{6 - x} \right)$

(c) $\lim_{x \to \infty} \left(\frac{5e^{2x} - 3e^x}{9e^{3x} - 12} \right)$

30 p **C28.** For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim_{x \to 8} \left(\frac{(x - 2)^2 - 36}{x - 8} \right)$

(b) $\lim_{x \to 5} \left(\frac{40 - 8x}{\sqrt{19 - 3x} - 2} \right)$

(c) $\lim_{x \to 2^-} \left(\frac{4 + x}{x^2 + x - 6} \right)$

C29. Consider the function below, where a and b are unspecified constants.

$$f(x) = \begin{cases} \dfrac{\sin(4x)\sin(6x)}{x^2} & x < 0 \\ ax + b & 0 \le x \le 1 \\ \dfrac{5x + 2}{x - 1} - \dfrac{2x + 5}{x^2 - x} & x > 1 \end{cases}$$

10 p (a) Calculate $\lim_{x \to 0^-} f(x)$.

10 p (b) Calculate $\lim_{x \to 1^+} f(x)$.

5 p (c) Find the values of a and b for which f is continuous for all x, or determine that no such values exist. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

§2.4: Infinite Limits

D1. Consider the function $f(x)$.

$$f(x) = \frac{4 - 3e^{-2x}}{6 - 5e^{-2x}}$$

6 p (a) Find all horizontal asymptotes of $f(x)$.

6 p (b) Find all vertical asymptotes of $f(x)$. Then, at each vertical asymptote, calculate both one-sided limits of $f(x)$.

D2. Consider the function f and its derivatives below.

$$f(x) = \frac{x^2}{x^2 - 1} \quad , \quad f'(x) = \frac{-2x}{(x^2 - 1)^2} \quad , \quad f''(x) = \frac{6x^2 + 2}{(x^2 - 1)^3}$$

6 p (a) Find all horizontal asymptotes of f.

6 p (b) Find all vertical asymptotes of f. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of f.

6 p (c) Find where f is decreasing and find where f is increasing. Then calculate all points of local extrema, classifying each as either a local minimum, a local maximum, or neither.

6 p (d) Find where f is concave down and find where f is concave up. Then calculate all points of inflection.

D3. Consider the function f and its derivatives below.

$$f(x) = \frac{2x^3 + 3x^2 - 1}{x^3} \quad , \quad f'(x) = \frac{3 - 3x^2}{x^4} \quad , \quad f''(x) = \frac{6x^2 - 12}{x^5}$$

For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

4 p (a) Find all horizontal asymptotes of f.

3 p (b) Find all vertical asymptotes of f. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of f.

7 p (c) Find where f is decreasing and find where f is increasing. Then calculate the x-coordinates of all points of local extrema.

7 p (d) Find where f is concave down and find where f is concave up. Then calculate the x-coordinates of all points of inflection.

4 p **D4.** Consider the following function, where a and b are unspecified constants.

$$f(x) = \frac{x^2 + ax + b}{x - 2}$$

Is the line $x = 2$ necessarily a vertical asymptote of $f(x)$? Explain your answer. *Your answer may contain either English, mathematical symbols, or both.*

5 p **D5.** Which of the following limits are equal to $+\infty$? Select all that apply.

(a) $\lim_{x \to 5^-} \left(\frac{x^2 + 25}{5 - x} \right)$

(b) $\lim_{x \to 5^+} \left(\frac{x^2 + 25}{5 - x} \right)$

(c) $\lim_{x \to -3^-} \left(\frac{x^3}{|x + 3|} \right)$

(d) $\lim_{x \to 0^-} \left(\frac{x^4 - 2x - 5}{\sin(x)} \right)$

(e) $\lim_{x \to 1^+} \left(\frac{x^6 - x^2}{x - 1} \right)$

10 p **D6.** Consider the function below.

$$f(x) = \frac{x^3 + 2x^2 - 13x + 10}{x^2 - 1}$$

Show that $x = -1$ is a vertical asymptote of f, but $x = 1$ is *not* a vertical asymptote of f.

4 p **D7.** Determine which of the following limits are equal to $-\infty$. Select all that apply.

(a) $\lim_{x\to 6^-} \left(\frac{x^2 - 5x - 6}{x - 6}\right)$

(b) $\lim_{x\to 6^-} \left(\frac{x^2 - 5x - 6}{x^2 - 12x + 36}\right)$

(c) $\lim_{x\to \infty} \left(\frac{x^2 - 5x - 6}{x - 6}\right)$

(d) $\lim_{x\to \infty} \left(\frac{x^2 - 5x - 6}{x^2 - 12x + 36}\right)$

10 p **D8.** Let $h(x) = \frac{f(x)}{g(x)}$, where f and g are continuous and $\lim_{x\to a} g(x) = 0$. Is the following true or false?

"The line $x = a$ is necessarily a vertical asymptote of $h(x)$."

You must justify your answer. This means that if your answer is "true", you should explain why the above statement is always true. If your answer is "false", you should give an example to show that the above statement is sometimes false.

10 p **D9.** Suppose that as x increases to 1, the values of $f(x)$ get larger and larger, and the values stay positive. Is the following true or false?

"Therefore, $\lim_{x\to 1^-} f(x) = +\infty$."

You must justify your answer. This means that if your answer is "true", you should explain why the above statement is always true. If your answer is "false", you should give an example to show that the above statement is sometimes false.

18 p **D10.** Let $f(x) = \frac{9x - x^3}{x^2 + x - 6}$.

(a) Calculate all vertical asymptotes of f. Justify your answer.

(b) Where is f discontinuous?

(c) For each point at which f is discontinuous, determine what value should be reassigned to f, if possible, to guarantee that f will be continuous there.

24 p **D11.** Let $f(x) = \frac{3 + 7e^{2x}}{1 - e^x}$. Calculate each of the following limits.

(a) $\lim_{x\to -\infty} f(x)$ (b) $\lim_{x\to +\infty} f(x)$ (c) $\lim_{x\to 0^-} f(x)$

24 p **D12.** Consider the function $f(x) = \frac{(ax - 6)(x + 1)}{x - 2}$, where a is an unspecified constant.

(a) For which value(s) of a does f have a vertical asymptote? What is the equation of this vertical asymptote?

(b) For which value(s) of a does f have a horizontal asymptote? What is the equation of this horizontal asymptote?

10 p **D13.** For which value(s) of n, if any, is the following statement true: $\lim_{x\to 2^-} (2 - x)^n = +\infty$? Explain your answer.

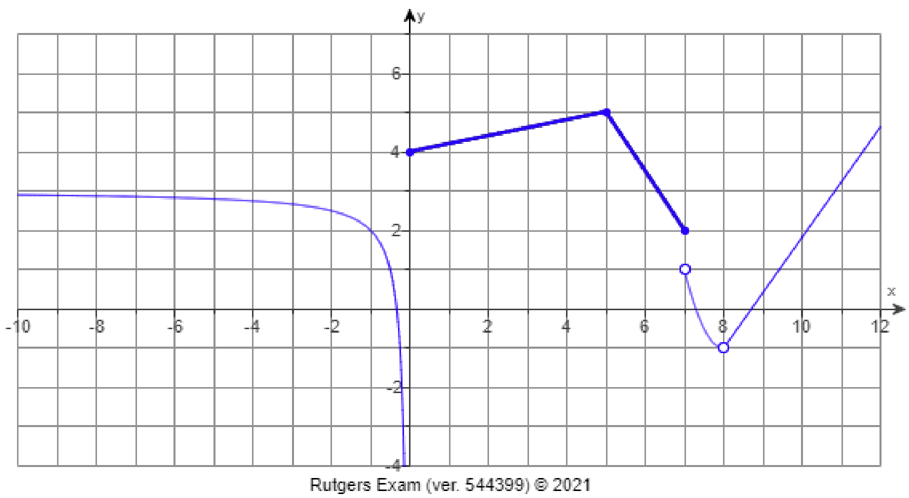
10 p **D14.** Determine whether the following statement is true or false. Explain your answer in 1 or 2 sentences. Your answer should contain English with few mathematical symbols.

"Suppose f and g are functions with $g(3) = 1$. Put $H(x) = \dfrac{f(x)}{g(x)-1}$. Then H must have a vertical asymptote at $x = 3$."

16 p **D15.** Let $f(x) = \dfrac{(x+a)(x-3)}{(x-2)(x+1)}$, where a is an unspecified, **positive** constant. For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 0} f(x)$ (b) $\lim\limits_{x\to 2^-} f(x)$ (c) $\lim\limits_{x\to 2^+} f(x)$ (d) $\lim\limits_{x\to 2} f(x)$

10 p **D16.** For each part, use the graph of $y = f(x)$.



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim\limits_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

12 p **D17.** Let $f(x) = \dfrac{8+6e^x}{9e^x - \pi^6}$.

(a) Evaluate $\lim\limits_{x\to\infty} f(x)$. (b) Evaluate $\lim\limits_{x\to-\infty} f(x)$. (c) List all vertical asymptotes of f.

44 p **D18.** For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\lim\limits_{x\to 3} \left(\dfrac{x-3}{10-\sqrt{x+97}} \right)$

(b) $\lim\limits_{x\to 6} \left(\dfrac{36-x^2}{\frac{1}{x}-\frac{1}{6}} \right)$

(c) $\lim\limits_{x\to 0} \left(\dfrac{x^2\csc(3x)}{\cos(7x)\sin(4x)} \right)$

(d) $\lim\limits_{x\to 2^-} \left(\dfrac{6x^2-7x}{x^2-4} \right)$

12 p **D19.** For the function f below, find its domain and all vertical and horizontal asymptotes.

$$f(x) = \frac{x^2 - 8x + 12}{3x^2 - 8x + 4}$$

13 p **D20.** Consider the function $f(x) = \dfrac{x^3 - 3x + 1}{x^2 - 2x + 1}$.

(a) Find all horizontal asymptotes of f, if any.

(b) Find all vertical asymptotes of f. Then calculate $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$, where $x = a$ is the rightmost vertical asymptote of f.

24 p **D21.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim_{x \to 1} f(x)$ and $\lim_{x \to 1} g(x)$ both exist, then $\lim_{x \to 1} (f(x)g(x))$ exists.

(b) T F If $f(9)$ is undefined, then $\lim_{x \to 9} f(x)$ does not exist.

(c) T F If $\lim_{x \to 1^+} f(x) = 10$ and $\lim_{x \to 1} f(x)$ exists, then $\lim_{x \to 1} f(x) = 10$.

(d) T F A function is continuous for all x if its domain is $(-\infty, \infty)$.

(e) T F If $f(x)$ is continuous at $x = 3$, then $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$.

(f) T F If $\lim_{x \to 2} f(x)$ exists, then f is continuous at $x = 2$.

(g) T F If $\lim_{x \to 5^-} f(x) = -\infty$, then $\lim_{x \to 5^+} f(x) = +\infty$.

(h) T F A function can have two different horizontal asymptotes.

15 p **D22.** Find all vertical asymptotes of the function $f(x) = \dfrac{x^3 - 36x}{x^3 - 12x^2 + 36x}$.

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

§2.5: Limits at Infinity

E1. Consider the function $f(x)$.

$$f(x) = \frac{4 - 3e^{-2x}}{6 - 5e^{-2x}}$$

6 p (a) Find all horizontal asymptotes of $f(x)$.

6 p (b) Find all vertical asymptotes of $f(x)$. Then, at each vertical asymptote, calculate both one-sided limits of $f(x)$.

E2. Consider the function f and its derivatives below.

$$f(x) = \frac{x^2}{x^2 - 1} \quad , \quad f'(x) = \frac{-2x}{(x^2-1)^2} \quad , \quad f''(x) = \frac{6x^2 + 2}{(x^2-1)^3}$$

6 p (a) Find all horizontal asymptotes of f.

6 p (b) Find all vertical asymptotes of f. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of f.

6 p (c) Find where f is decreasing and find where f is increasing. Then calculate all points of local extrema, classifying each as either a local minimum, a local maximum, or neither.

6 p (d) Find where f is concave down and find where f is concave up. Then calculate all points of inflection.

E3. Consider the function f and its derivatives below.

$$f(x) = \frac{2x^3 + 3x^2 - 1}{x^3} \quad , \quad f'(x) = \frac{3 - 3x^2}{x^4} \quad , \quad f''(x) = \frac{6x^2 - 12}{x^5}$$

For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

4 p (a) Find all horizontal asymptotes of f.

3 p (b) Find all vertical asymptotes of f. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of f.

7 p (c) Find where f is decreasing and find where f is increasing. Then calculate the x-coordinates of all points of local extrema.

7 p (d) Find where f is concave down and find where f is concave up. Then calculate the x-coordinates of all points of inflection.

11 p **E4.** Find the equation of each horizontal asymptote, if any, of $f(x) = \dfrac{4x^3 - 3x^2}{2x^3 + 9x + 1}$.

E5. The parts of this problem *are* related!

3 p (a) Show that $\displaystyle\lim_{x \to \infty} \left(\frac{x}{x-3} \right) = 1$.

8 p (b) Calculate the following limit or show it does not exist.

$$\lim_{x \to \infty} \left(\frac{x}{x-3} \right)^x$$

Hint: First use part (a) to identify the appropriate indeterminate form.

10 p **E6.** Find all horizontal asymptotes of

$$f(x) = \frac{12x+5}{\sqrt{16x^2+x+1}}$$

or determine that there are no horizontal asymptotes.

5 p **E7.** Suppose the function f has domain $(-\infty, \infty)$. Give a brief explanation of how you would find all horizontal asymptotes of f. Note that for this problem, f is unspecified; you should not assume it has any particular form. *Your answer may contain either English, mathematical symbols, or both.*

12 p **E8.** Let $f(x) = \dfrac{(x-3)(2x+1)}{(5x+2)(3x-10)}$. Calculate all horizontal asymptotes of f.

24 p **E9.** Let $f(x) = \dfrac{3+7e^{2x}}{1-e^x}$. Calculate each of the following limits.

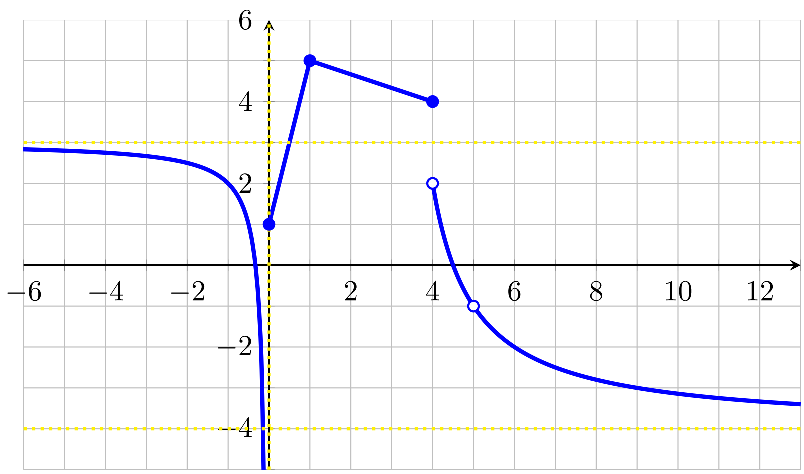
(a) $\lim\limits_{x\to-\infty} f(x)$ (b) $\lim\limits_{x\to+\infty} f(x)$ (c) $\lim\limits_{x\to 0^-} f(x)$

16 p **E10.** Calculate all horizontal asymptotes of the function $h(x) = \dfrac{\sqrt{3x^2+x+10}}{2-5x}$.

10 p **E11.** Suppose the line $y = 3$ is a horizontal asymptote for f. Which of the following statements MUST be true? Select all that apply.

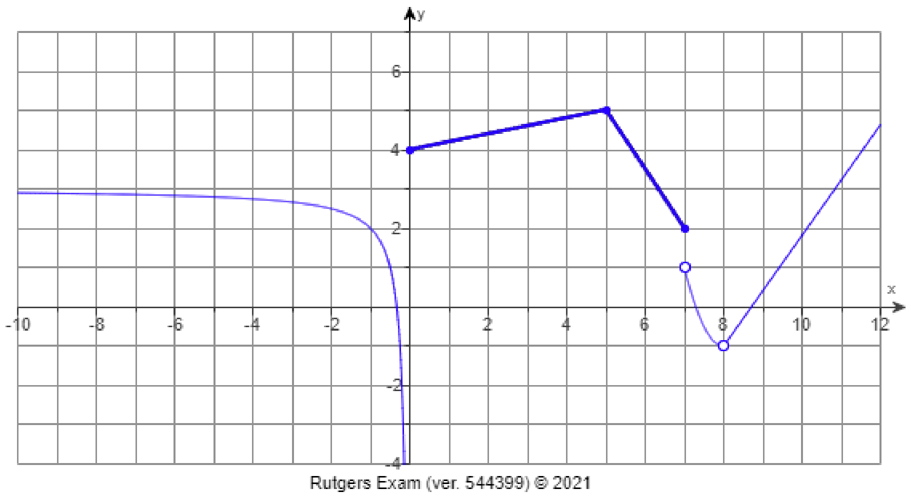
(a) $f(x) \neq 3$ for all x in the domain of f
(b) $f(3)$ is undefined
(c) $\lim\limits_{x\to3} f(x) = \infty$
(d) $\lim\limits_{x\to\infty} f(x) = 3$
(e) none of the above

12 p **E12.** Use the graph of f below to answer the following questions. Dashed lines indicate the location of asymptotes.



(a) Calculate $\lim\limits_{x\to\infty} f(x)$.
(b) Calculate $\lim\limits_{x\to-\infty} f(x)$.
(c) List the values of x where f is not continuous.
(d) List the values of x where f is not differentiable.
(e) What is the sign of $f'(-1)$? (choices: positive, negative, zero, does not exist)
(f) What is the sign of $f'(0.5)$? (choices: positive, negative, zero, does not exist)

10 p **E13.** For each part, use the graph of $y = f(x)$.



(a) List the x-values where f is not continuous or determine that f is continuous for all x.

(b) List all vertical asymptotes of f.

(c) List all horizontal asymptotes of f.

(d) Calculate $\lim_{x\to 8} f(x)$ or determine that the limit does not exist.

(e) At $x = 7$, which of the one-sided limits of f exist?

12 p **E14.** Let $f(x) = \dfrac{8+6e^x}{9e^x - \pi^6}$.

(a) Evaluate $\lim_{x\to\infty} f(x)$.

(b) Evaluate $\lim_{x\to-\infty} f(x)$.

(c) List all vertical asymptotes of f.

11 p **E15.** Find all horizontal asymptotes of the function $g(x) = \dfrac{2e^x - 15}{5e^{3x}+8}$.

12 p **E16.** For the function f below, find its domain and all vertical and horizontal asymptotes.

$$f(x) = \frac{x^2 - 8x + 12}{3x^2 - 8x + 4}$$

13 p **E17.** Consider the function $f(x) = \dfrac{x^3 - 3x + 1}{x^2 - 2x + 1}$.

(a) Find all horizontal asymptotes of f, if any.

(b) Find all vertical asymptotes of f. Then calculate $\lim_{x\to a^-} f(x)$ and $\lim_{x\to a^+} f(x)$, where $x = a$ is the rightmost vertical asymptote of f.

24 p **E18.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim_{x\to 1} f(x)$ and $\lim_{x\to 1} g(x)$ both exist, then $\lim_{x\to 1} (f(x)g(x))$ exists.

(b) T F If $f(9)$ is undefined, then $\lim_{x\to 9} f(x)$ does not exist.

(c) T F If $\lim_{x\to 1^+} f(x) = 10$ and $\lim_{x\to 1} f(x)$ exists, then $\lim_{x\to 1} f(x) = 10$.

(d) T F A function is continuous for all x if its domain is $(-\infty, \infty)$.

(e) T F If $f(x)$ is continuous at $x = 3$, then $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$.

(f) T F If $\lim_{x\to 2} f(x)$ exists, then f is continuous at $x = 2$.

(g) T F If $\lim_{x\to 5^-} f(x) = -\infty$, then $\lim_{x\to 5^+} f(x) = +\infty$.

(h) T F A function can have two different horizontal asymptotes.

15 p **E19.** Find all horizontal asymptotes of the function $h(x) = \dfrac{6x+5}{\sqrt{4x^2-9}}$.

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

§2.6: Continuity

10 p

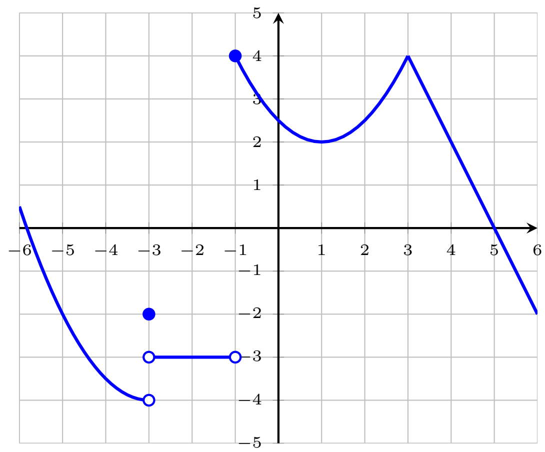
F1. Find the values of the constants a and b so that the following function is continuous for all x. If this is not possible, explain why.

$$f(x) = \begin{cases} ax + b & x < 1 \\ -2 & x = 1 \\ 3\sqrt{x} + b & x > 1 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

10 p

F2. The graph of a function $f(x)$ is shown below.



(a) State where $f(x)$ is *not* continuous in the interval $(-5, 5)$.

(b) State where $f(x)$ is *not* differentiable in the interval $(-5, 5)$.

(c) State where $f'(x) = 0$ in the interval $(-5, 5)$.

(d) State where $f'(x) < 0$ in the interval $(-5, 5)$.

F3. Each part of this question refers to the function $f(x)$ below, where a and b are unspecified constants.

$$f(x) = \begin{cases} \dfrac{\sin(ax)}{x} & x < 0 \\ 2x + 3 & 0 \le x < 1 \\ b & x = 1 \\ \dfrac{x^2 - 1}{x - 1} & 1 < x \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

7 p (a) Find the value of a so that f is continuous at $x = 0$. If this is not possible, explain why.

7 p (b) Find the value of b so that f is continuous at $x = 1$. If this is not possible, explain why.

14 p **F4.** Find the values of the constants a and b so that the following function is continuous at $x = 0$. If this is not possible, explain why.

$$f(x) = \begin{cases} \dfrac{4 - \sqrt{16 + 49x^2}}{ax^2} & x < 0 \\ -23 & x = 0 \\ \dfrac{\tan(2bx)}{x} & x > 0 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

5 p **F5.** Find the value of k that makes $f(x)$ continuous at $x = 1$. If no such value of k exists, write "does not exist".

$$f(x) = \begin{cases} k\cos(\pi x) - 3x^2 & x \leq 1 \\ 8e^x - k\ln(x) & x > 1 \end{cases}$$

10 p **F6.** Consider the function $f(x)$ below.

$$f(x) = \begin{cases} \dfrac{4 - \sqrt{2x + 10}}{x - 3} & x \neq 3 \\ 1 & x = 3 \end{cases}$$

Is $f(x)$ continuous at $x = 3$? Explain your answer. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

12 p **F7.** Find the values of a and b that make f continuous at $x = 1$ or determine that no such values exist.

$$f(x) = \begin{cases} -3x + ax^2 & x < 1 \\ b & x = 1 \\ 4ax - 1 & x > 1 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

5 p **F8.** Determine where f is continuous. *Write your answer using interval notation.*

$$f(x) = \begin{cases} 9 - 16x & x < 0 \\ 3x^2 - x^3 & 0 \leq x \leq 3 \\ 1 - e^{x-3} & x > 3 \end{cases}$$

10 p **F9.** Find the value of k that makes f continuous at $x = -2$ or determine that no such value of k exists.

$$f(x) = \begin{cases} 3x^2 + k & x < -2 \\ -10 & x = -2 \\ kx^3 - 6 & x > -2 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

10 p **F10.** In a certain parking garage, the cost of parking is \$20 per hour or any fraction thereof. For example, if you are in the garage for two hours and fifteen minutes, you pay \$60 (\$20 for the first hour, \$20 for the second hour, and \$20 for the fifteen-minute portion of the third hour). Let $P(t)$ be the cost of parking for t hours, where t is any non-negative real number. For example, $P(2.25) = 60$. Is the following true or false?

"$P(t)$ is a continuous function of t."

You must justify your answer.

16 p **F11.** Consider the following function, where a and b are unspecified constants.

$$f(x) = \begin{cases} 3 & x \leq -1 \\ ax^2 + 2x + b & -1 < x \leq 2 \\ 14 - ax & x > 2 \end{cases}$$

Find the values of a and b for which f is continuous for all x, or determine that no such values exist. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

18 p **F12.** Let $f(x) = \dfrac{9x - x^3}{x^2 + x - 6}$.

(a) Calculate all vertical asymptotes of f. Justify your answer.

(b) Where is f discontinuous?

(c) For each point at which f is discontinuous, determine what value should be reassigned to f, if possible, to guarantee that f will be continuous there.

16 p **F13.** Determine where the following function is continuous. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

$$f(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & x < 3 \\ 0 & x = 3 \\ 5x - 9 & 3 < x < 4 \\ 11 & x = 4 \\ 27 - x^2 & x > 4 \end{cases}$$

F14. Consider the function f below, where A, B, and C are unspecified constants.

$$f(x) = \begin{cases} 2x^3 + Ax & x < -1 \\ C & x = -1 \\ Bx^2 + 4 & x > -1 \end{cases}$$

2 p (a) Calculate $\lim\limits_{x \to -1^-} f(x)$.

2 p (b) Calculate $\lim\limits_{x \to -1^+} f(x)$.

2 p (c) How must A and B be related if $\lim\limits_{x \to -1} f(x)$ exists?

8 p (d) Suppose $C = 10$ and f is continuous for all x. Find the values of A and B.

10 p **F15.** Which of the following equations expresses the fact that $f(x)$ is continuous at $x = 6$. (There is only one correct choice.)

(a) $\lim_{x\to 6} f(6) = f(6)$

(b) $\lim_{x\to 6} f(6) = 6$

(c) $\lim_{x\to 6} f(x) = f(6)$

(d) $\lim_{x\to 6} f(x) = 6$

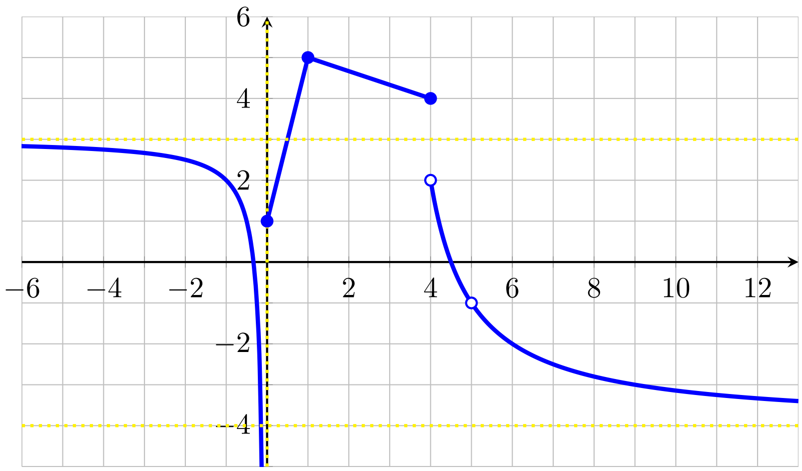
(e) $\lim_{x\to 6} f(x) = 0$

(f) $\lim_{x\to 6} f(x) = \infty$

(g) $\lim_{x\to\infty} f(x) = f(6)$

(h) $\lim_{x\to\infty} f(x) = \infty$

12 p **F16.** Use the graph of f below to answer the following questions. Dashed lines indicate the location of asymptotes.



(a) Calculate $\lim_{x\to\infty} f(x)$.

(b) Calculate $\lim_{x\to-\infty} f(x)$.

(c) List the values of x where f is not continuous.

(d) List the values of x where f is not differentiable.

(e) What is the sign of $f'(-1)$? (choices: positive, negative, zero, does not exist)

(f) What is the sign of $f'(0.5)$? (choices: positive, negative, zero, does not exist)

16 p **F17.** Consider the function g below, where a and b are unspecified constants. Assume that g is continuous for all x.

$$g(x) = \begin{cases} be^x + a + 1 & x \le 0 \\ ax^2 + b(x+3) & 0 < x \le 1 \\ a\cos(\pi x) + 7bx & 1 < x \end{cases}$$

(a) What relation must hold between a and b for g to be continuous at $x = 0$? Your answer should be an equation involving a and b.

(b) What relation must hold between a and b for g to be continuous at $x = 1$? Your answer should be an equation involving a and b.

(c) Calculate the values of a and b.

12 p **F18.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim_{x\to a} f(x)$ can be evaluated by direct substitution, then f is continuous at $x = a$.

(b) T F The value of $\lim_{x\to a} f(x)$, if it exists, is found by calculating $f(a)$.

(c) T F If f is not differentiable at $x = a$, then f is also not continuous at $x = a$.

20 p **F19.** Consider the piecewise-defined function $f(x)$ below; A and B are unspecified constants and $g(x)$ is an unspecified function with domain $[94, \infty)$.

$$f(x) = \begin{cases} Ax^2 + 8 & x < 75 \\ \ln(B) + 6 & x = 75 \\ \dfrac{x - 75}{\sqrt{x+6} - 9} & 75 < x < 94 \\ 19 & x = 94 \\ g(x) & x > 94 \end{cases}$$

(a) Find $\lim\limits_{x \to 75^-} f(x)$ in terms of A and B.

(b) Find $\lim\limits_{x \to 75^+} f(x)$ in terms of A and B.

(c) Find the exact values of A and B for which f is continuous at $x = 75$.

(d) Suppose $g(94) = 19$. What does this imply about $\lim\limits_{x \to 94} f(x)$? Select the best answer.

(i) $\lim\limits_{x \to 94} f(x)$ exists.

(ii) $\lim\limits_{x \to 94} f(x)$ does not exist.

(iii) It gives no information about $\lim\limits_{x \to 94} f(x)$.

12 p **F20.** Consider the following function.

$$f(x) = \frac{x^2 - x - 6}{x^3 - 2x^2 - 3x}$$

(a) Where is f discontinuous?

(b) At the leftmost x-value where f is discontinuous, what type of discontinuity does f have (removable, jump, infinite (vertical asymptote), or other)?

(c) At the rightmost x-value where f is discontinuous, what type of discontinuity does f have (removable, jump, infinite (vertical asymptote), or other)?

11 p **F21.** Determine where $f(x)$ is continuous. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

$$f(x) = \begin{cases} \dfrac{(x+1)^2 - 16}{2x - 6} & \text{if } x < 3 \\ 3 - \ln(x - 2) & \text{if } x \geq 3 \end{cases}$$

12 p **F22.** Consider the function $f(x)$ defined below, where A and B are unspecified constants. Find the values of A and B for which f is continuous at $x = 2$, or determine that no such values exist.

$$f(x) = \begin{cases} Ax + B - 4 & \text{if } x < 2 \\ 9 & \text{if } x = 2 \\ Ax^2 - 5 & \text{if } x > 2 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

12 p **F23.** Consider the function $f(x) = \dfrac{\sin(7x)}{x^2 - 5x}$.

(a) Find the domain of f. Write your answer using interval notation.

(b) Find the x-values where f is discontinuous.

(c) For each value of x where f is discontinuous, classify the type of discontinuity as "removable", "jump", "infinite", or "essential". Clearly label your work and justify your answers.

12 p **F24.** Consider the limit $\displaystyle\lim_{x\to 3}\left(\frac{x^3 - 4x^2 + ax}{x^2 - 9}\right)$, where a is an unspecified constant.

(a) For what values of a does this limit exist? Explain your answer.

(b) Given that the limit does exist, what is its value?

12 p **F25.** Consider the function below, where a and b are unspecified constants. Find the values of a and b for which f is continuous for all x, or determine that no such values exist.

$$f(x) = \begin{cases} ax^2 + 3x + b & x < -1 \\ 2 + ax + \sin\left(\dfrac{\pi x}{2}\right) & -1 \le x < 4 \\ b(x-3)^2 + 1 & x \ge 4 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

24 p **F26.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) [T] [F] If $\lim_{x\to 1} f(x)$ and $\lim_{x\to 1} g(x)$ both exist, then $\lim_{x\to 1}(f(x)g(x))$ exists.

(b) [T] [F] If $f(9)$ is undefined, then $\lim_{x\to 9} f(x)$ does not exist.

(c) [T] [F] If $\lim_{x\to 1^+} f(x) = 10$ and $\lim_{x\to 1} f(x)$ exists, then $\lim_{x\to 1} f(x) = 10$.

(d) [T] [F] A function is continuous for all x if its domain is $(-\infty, \infty)$.

(e) [T] [F] If $f(x)$ is continuous at $x = 3$, then $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$.

(f) [T] [F] If $\lim_{x\to 2} f(x)$ exists, then f is continuous at $x = 2$.

(g) [T] [F] If $\lim_{x\to 5^-} f(x) = -\infty$, then $\lim_{x\to 5^+} f(x) = +\infty$.

(h) [T] [F] A function can have two different horizontal asymptotes.

15 p **F27.** On the axes provided, sketch the graph of a function $f(x)$ that satisfies all of the following properties. ***Note:*** Make sure to read these properties carefully!

- the domain of $f(x)$ is $[-10, 7) \cup (7, 10]$
- $\lim_{x\to -8} f(x)$ exists but f is discontinuous at $x = -8$
- $\lim_{x\to -5^+} f(x) = f(-5)$ but $\lim_{x\to -5} f(x)$ does not exist
- $\lim_{x\to 2^-} f(x) = 4$ and f is continuous at $x = 2$

- the line $x = 5$ is a vertical asymptote for f (***Note:*** $x = 5$ is in the domain of f.)
- $\lim_{x\to 7} f(x) = +\infty$ (***Note:*** $x = 7$ is not in the domain of f.)

F28. Consider the function below, where a and b are unspecified constants.

$$f(x) = \begin{cases} \dfrac{\sin(4x)\sin(6x)}{x^2} & x < 0 \\ ax + b & 0 \leq x \leq 1 \\ \dfrac{5x+2}{x-1} - \dfrac{2x+5}{x^2-x} & x > 1 \end{cases}$$

10 p (a) Calculate $\lim_{x\to 0^-} f(x)$.

10 p (b) Calculate $\lim_{x\to 1^+} f(x)$.

5 p (c) Find the values of a and b for which f is continuous for all x, or determine that no such values exist. *In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.*

1.3 Chapter 3: Derivatives

§3.1, 3.2: Introduction to the Derivative

10 p **G1.** Find an equation of the line tangent to the graph of $f(x) = 2x^2 - 3x + 1$ at $x = 1$.

G2. The parts of this question are independent of each other.

2 p (a) Given the function $g(x)$, state the definition of $g'(x)$.

12 p (b) Let $f(x) = \sqrt{6x+1}$. Calculate $f'(1)$ directly from the definition. Show all work.

If you simply quote a rule, you will receive no credit. You must use the definition of derivative.

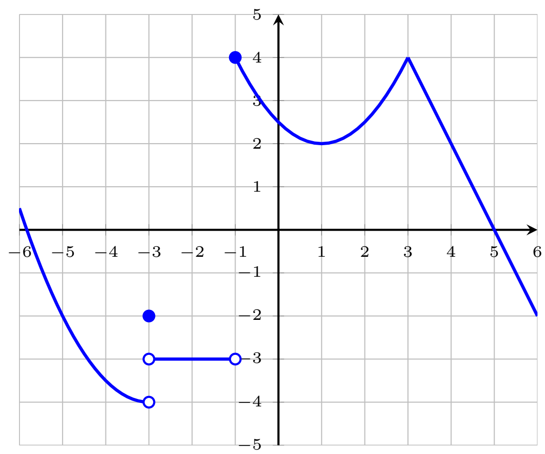
G3. The parts of this question are independent of each other.

2 p (a) Given the function $g(x)$, state the definition of $g'(4)$.

12 p (b) Let $F(x) = \dfrac{1}{3x-5}$. Calculate $F'(2)$ directly from the definition. Show all work.

If you simply quote a rule, you will receive no credit. You must use the definition of derivative.

10 p **G4.** The graph of a function $f(x)$ is shown below.



(a) State where $f(x)$ is *not* continuous in the interval $(-5, 5)$.

(b) State where $f(x)$ is *not* differentiable in the interval $(-5, 5)$.

(c) State where $f'(x) = 0$ in the interval $(-5, 5)$.

(d) State where $f'(x) < 0$ in the interval $(-5, 5)$.

10 p **G5.** Find an equation of each line that is both tangent to the graph of $f(x) = 4x^2 - 3x - 1$ and parallel to the line $y = 13x - 5$.

20 p **G6.** For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\displaystyle\lim_{u\to 4}\left(\frac{(u+6)^2 - 25u}{u-4}\right)$

(b) $\displaystyle\lim_{s\to 1} g(s)$ where $g(s) = \begin{cases} \sqrt{1-s} & s \le 1 \\ \dfrac{s^2-s}{s-1} & s > 1 \end{cases}$

(c) $\displaystyle\lim_{h\to 0}\left(\frac{\sin(7+h) - \sin(7)}{h}\right)$

Hint: Use the definition of the derivative.

(d) $\displaystyle\lim_{x\to 6}\left(\frac{\frac{1}{36} - x^{-2}}{x^2 - 36}\right)$

12 p **G7.** Let $f(x) = \frac{1}{3}x^3$ and let $g(x) = x^2 + 15x - 3$. Find all values of a for which the tangent lines to $y = f(x)$ and $y = g(x)$ at $x = a$ are parallel.

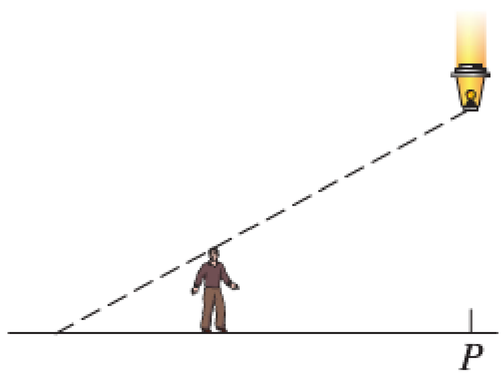
10 p **G8.** Let $g(x) = 6 - \dfrac{9}{x}$. Calculate $g'(3)$ directly from the limit definition of the derivative. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

12 p **G9.** Let $f(x) = 2x^2 - 5x + 7$. Use the limit definition of the derivative to calculate $f'(x)$. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

10 p **G10.** A spherical snowball melts in such a way that it always remains a sphere, and its volume decreases at 8 cm^3/sec. At what rate is the surface area of the snowball changing when its surface area is 40π cm^2? *You must give correct units as part of your answer.*

10 p **G11.** Let $f(x) = \dfrac{x+8}{x-3}$. Use the limit definition of derivative to calculate $f'(2)$. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

8 p **G12.** A 6-ft tall person is initially standing 12 ft from point P directly beneath a lantern hanging 42 ft above the ground, as shown in the diagram below. The person then begins to walk towards point P at 5 ft/sec. Let D denote the distance between the person's feet and the point P. Let S denote the length of the person's shadow.



(a) Write an equation that relates D and S.

(b) Write an equation that expresses the English sentence "*The person then begins to walk towards point P at 5 ft/sec.*"

(c) Is the length of the person's shadow increasing, decreasing or remaining constant?

(d) At what rate is the length of the person's shadow changing when the person is 8 ft from point P? Include correct units as part of your answer.

4 p **G13.** Each of the following statements describes a scenario in which a certain rectangle is changing over time. For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If two opposite sides of the rectangle increase in length and if the area remains constant, then the other two opposite sides must decrease in length.

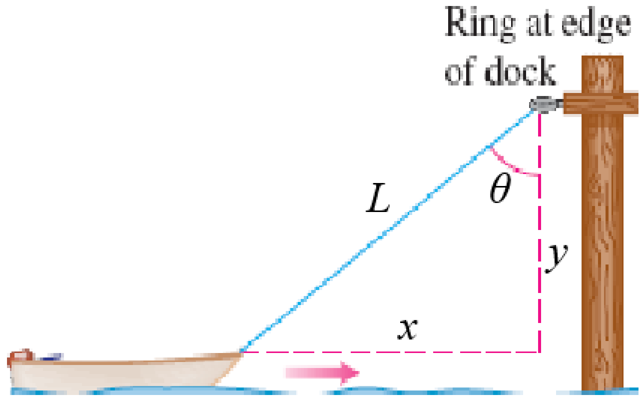
(b) T F If the area of the rectangle increases, then all sides of the rectangle must also increase in length.

(c) T F If the length of the rectangle remains the same, then the area and the width of the rectangle cannot change in opposite ways (i.e., one cannot increase while the other decreases).

(d) T F If two opposite sides of the rectangle increase in length and the other two opposite sides decrease in length, then the area of the rectangle must remain constant.

10 p **G14.** The volume of a cube is decreasing at the rate of 300 cm^3/sec at the moment its total surface area is 150 cm^2. What is the rate of change of the length of one edge of the cube at this moment?

10 p **G15.** A boat is pulled toward a dock by a rope through a ring on the dock 4 ft above the front of the boat. The rope is hauled in at the rate of 12 ft/sec.



(a) Which of the marked variables (x, y, L, and θ) are changing over time?

(b) Write a mathematical equation that expresses the English sentence *"The rope is hauled in at the rate of 12 ft/sec"*.

(c) Is $\cos(\theta)$ increasing, decreasing, or constant?

(d) Write a mathematical expression for *"the rate at which the boat approaches the dock"*.

(e) How fast in ft/sec is the boat approaching the dock when the rope is 5 ft long?

4 p **G16.** The numbers a, b, and c (which are not necessarily positive) satisfy the formula $a = \frac{b}{c}$. The choices below describe scenarios in which the numbers a, b, and c are changing over time. For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

Hint: There is at most one true statement.

(a) T F Suppose a, b, and c are all positive numbers. If a and b are both increasing, then c must also be increasing.

(b) T F Suppose b is a positive number and c is a negative number. If b and c are both increasing, then a must be decreasing.

(c) T F Suppose a, b, and c are all positive numbers. If a is constant, then it is possible for b and c to change in opposite ways (i.e., one can increase while the other decreases).

(d) T F Suppose c is a positive number. If b is constant and c is increasing, then a must be decreasing.

10 p **G17.** Explain the relationship between $f'(3)$ and the line tangent to the graph of $y = f(x)$ at $x = 3$.

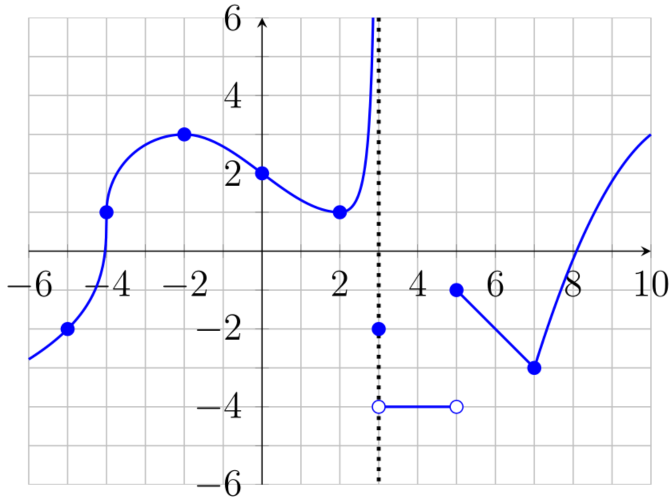
10 p **G18.** Suppose $f'(7)$ exists. What can be said about the limit $\lim_{x\to 7} f(x)$?

15 p **G19.** Consider the following limit.

$$\lim_{h\to 0}\left(\frac{(4+h)^{3/2}-8}{h}\right)$$

Use the limit definition of derivative to identify this limit as the derivative of some function $f(x)$ at the point $x = a$. Then calculate the value of the limit.

18 p **G20.** Use the graph of $y = f(x)$ below to answer the following questions.



(a) In the interval $(-6, 10)$, where is f not differentiable?

(b) Calculate a reasonable estimate of $f'(0)$. Explain your reasoning.

(c) In the interval $(-6, 10)$, where is $f'(x) = 0$?

(d) In the interval $(-6, 10)$, where is $f'(x) < 0$?

(e) In the interval $(-6, 10)$, where is $f'(x) > 0$?

28 p **G21.** In a right triangle, the base is decreasing in length by 3 cm/sec and the area is increasing by 15 cm^2/sec. (The triangle always remains a right triangle.) At the time when the base is 15 cm in length and the height is 20 cm in length...

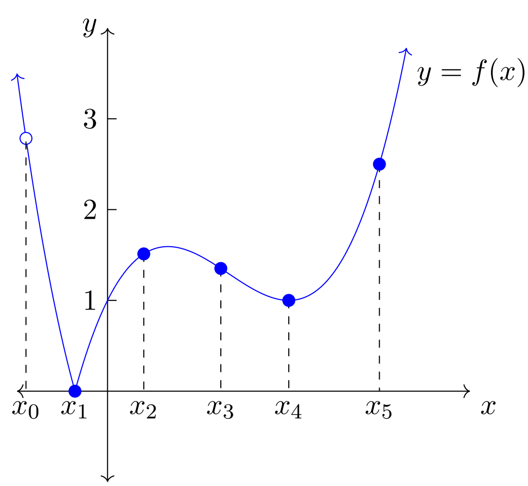
(a) ...at what rate is the height changing? (Give a number only.)

(b) ...at what rate is the length of the hypotenuse changing? (Give a number only.)

(c) What are the units of your answer in part (a)?

(d) In part (b), is the length of the hypotenuse increasing, decreasing, or staying constant?

G22. Consider the graph of $y = f(x)$ below.



8 p (a) For which values of x is $f'(x) \geq 0$? Choose from x_0, x_1, x_2, x_3, x_4, and x_5. Select all that apply.

4 p (b) For which values of x does $f'(x)$ not exist? Choose from x_0, x_1, x_2, x_3, x_4, and x_5. Select all that apply.

8 p (c) Give a brief, one-sentence explanation of your answer to part (b).

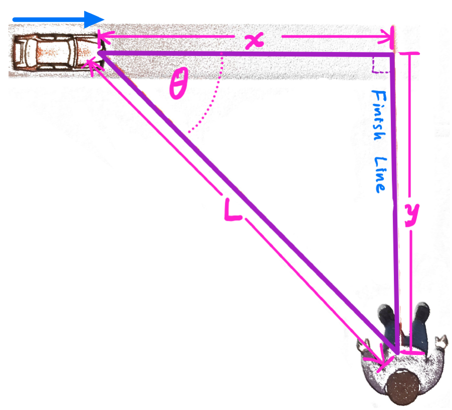
20 p **G23.** Consider the following limit.

$$\lim_{x \to \frac{\pi}{8}} \left(\frac{\tan(2x) - 1}{x - \frac{\pi}{8}} \right)$$

(a) Use the limit definition of derivative to identify this limit as the derivative of some function $f(x)$ at the point $x = a$. You must explicitly identify f and a.

(b) Use your identifications in part (a) to calculate the given limit. Show all work.

38 p **G24.** At a certain moment, a race official is watching a race car approach the finish line along a straight track at some constant, positive speed. Suppose the official is sitting still at the finish line, 20 m from the point where the car will cross.



For parts (a)–(e), the allowed answers are "positive", "negative", "zero", or "not enough information".

(a) At the moment described, what is the sign of $\frac{dx}{dt}$?

(b) At the moment described, what is the sign of $\frac{dy}{dt}$?

(c) At the moment described, what is the sign of $\frac{dL}{dt}$?

(d) At the moment described, what is the sign of $\frac{d(\cos(\theta))}{dt}$?

(e) At the moment described, what is the sign of $\frac{d^2x}{dt^2}$?

(f) Suppose the speed of the car is 70 m/sec. At what rate is the distance between the car and the race official changing when the car is 60 m from the finish line? *Your answer must have the correct units. Your answer must be exact. No decimal approximations.*

12 p **G25.** The following limit represents the derivative of a function f at a point a.

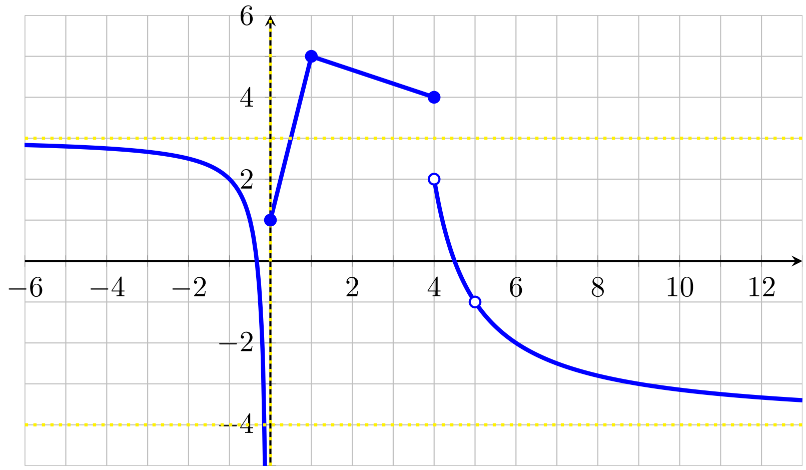
$$f'(a) = \lim_{h \to 0} \left(\frac{5\ln(e^4 + h) - 20}{h} \right)$$

(a) Find a possible function $f(x)$.

(b) For your choice of f in part (a), find a possible value of a.

(c) Calculate the value of the limit. Explain your calculation briefly in one sentence.

12 p **G26.** Use the graph of f below to answer the following questions. Dashed lines indicate the location of asymptotes.



(a) Calculate $\lim_{x\to\infty} f(x)$.

(b) Calculate $\lim_{x\to-\infty} f(x)$.

(c) List the values of x where f is not continuous.

(d) List the values of x where f is not differentiable.

(e) What is the sign of $f'(-1)$? (choices: positive, negative, zero, does not exist)

(f) What is the sign of $f'(0.5)$? (choices: positive, negative, zero, does not exist)

12 p **G27.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If $\lim_{x\to a} f(x)$ can be evaluated by direct substitution, then f is continuous at $x = a$.

(b) T F The value of $\lim_{x\to a} f(x)$, if it exists, is found by calculating $f(a)$.

(c) T F If f is not differentiable at $x = a$, then f is also not continuous at $x = a$.

20 p **G28.** A local gym has two cylindrical swimming pools. The larger pool has radius 20 meters and is filled with water. The smaller pool has radius 12 meters and is empty. Water is drained from the large pool and immediately emptied into the small pool. The height of the water in the small pool increases at a rate of 0.2 m/min.
Let V_L, V_S, h_L, and h_S refer to the volume of the large pool, volume of the small pool, height of the large pool, and height of the small pool, respectively.

(a) How are $\frac{dV_L}{dt}$ and $\frac{dV_S}{dt}$ related?

(b) What is the sign of $\frac{dh_L}{dt}$?

(c) Find $\frac{dV_S}{dt}$.

(d) Find $\frac{dh_L}{dt}$.

9 p **G29.** The following limit represents the derivative of a function f at a point a.

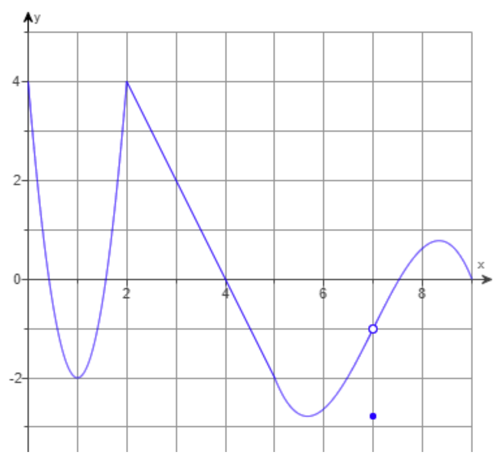
$$f'(a) = \lim_{h \to 0} \left(\frac{9\tan\left(\frac{\pi}{6} + h\right) - \frac{9}{\sqrt{3}}}{h} \right)$$

(a) Find a possible pair for f and a.

(b) Calculate the value of the limit.

12 p **G30.** For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

(a) $f'(1)$ (b) $f'(2)$ (c) $f'(3.5)$ (d) $f'(7)$



12 p **G31.** Let $f(x)$ and $g(x)$ be functions such that $f'(-8) = g'(-8)$ and the line tangent to the graph of f at $x = -8$ is $y = -7x + 6$. For each part, compute the desired value, if possible.

(a) $f(-8)$ (b) $f'(-8)$ (c) $g(-8)$ (d) $g'(-8)$

12 p **G32.** The base of a right triangle is decreasing at a constant rate of 10 cm/sec and in such a way that the triangle always remains a right triangle. At the time when the base is 15 cm and the height is 22 cm, the area of the triangle is increasing by 25 cm^2/sec. Use this information to answer the questions below. Let B denote the base of the triangle.

(a) At the described time, what is the sign of $\frac{dB}{dt}$?

(b) At the described time, what is the sign of $\frac{d^2B}{dt^2}$?

(c) At the described time, at what rate is the height changing?

(d) What are the units of the answer to part (c)?

10 p **G33.** For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

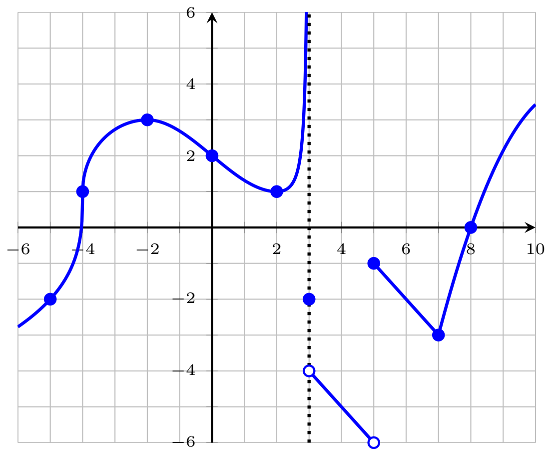
(a) $f'(-4)$

(b) $f'(-2)$

(c) $f'(0)$

(d) $f'(5)$

(e) $f'(8)$

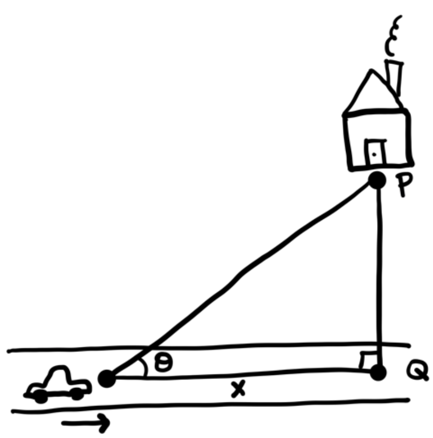


12 p **G34.** For both parts below, $f(x) = \sqrt{2x+1}$.

(a) Use the limit definition of the derivative to calculate $f'(4)$.

(b) Find an equation for the line tangent to the graph of $y = f(x)$ at $x = 4$.

12 p **G35.** A house sits at point P, which is 20 m from point Q on a straight road. A car travels along the road toward the point Q at 19 m/s. Let x be the distance between the car and point Q, and let θ be the angle between the road and the line of sight from the car to the house. See the figure below.



(a) What is the sign of $\dfrac{dx}{dt}$?

(b) What is the sign of $\dfrac{d\theta}{dt}$?

(c) Find the rate of change of the distance between the car and the house when the car is 45 m from point Q. You must include correct units in your answer. You may leave unsimplified radicals in your answer.

12 p **G36.** Find the x-coordinate of each point on the graph of $y = 6x^3 - 9x^2 - 16x + 5$ at which the tangent line is perpendicular to the line $x + 20y = 10$.

12 p **G37.** Suppose that an equation to the tangent line to $y = f(x)$ at $x = 9$ is $y = 3x - 20$. Let $g(x) = xf(x^2)$.

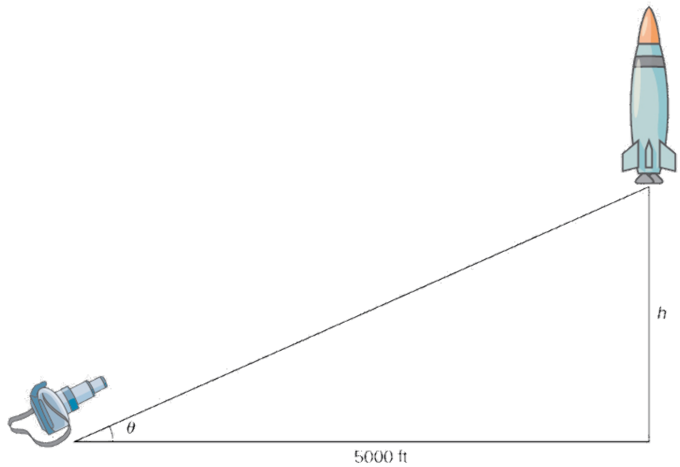
(a) Calculate $f(9)$ and $f'(9)$. Explain.

(b) Calculate $g'(x)$.

(c) Find the tangent line to $y = g(x)$ at $x = -3$.

12 p **G38.** Let $f(x) = \dfrac{4}{x-6} + 3$. Use the limit definition of derivative to calculate $f'(8)$. *If you simply quote a rule, you will receive no credit. You must use the definition of derivative.*

12 p **G39.** A rocket is launched so that it rises vertically. A camera is positioned 5000 feet from the launch pad and turns so that it stays forces on the rocket. At the moment when the rocket is 12,000 feet above the launch pad, its velocity is 600 feet/sec. Let h be the height of the rocket above the launch pad and let θ be the viewing angle of the camera. See the figure below.



(a) Determine the sign of $\dfrac{d}{dt}(\cos(\theta))$ at the moment described or determine that there is not enough information to do so.

(b) Determine the sign of $\dfrac{d^2h}{dt^2}$ at the moment described or determine that there is not enough information to do so.

(c) At the moment described, what is the rate at which the camera is turning? That is, what is the rate at which θ is changing over time? *You must include proper units as part of your answer.*

16 p **G40.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If f is continuous at $x = 3$, then f is differentiable at $x = 3$.

(b) T F If f is differentiable at $x = 3$, then f is continuous at $x = 3$.

(c) T F If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$ for all x.

(d) T F The function $f(x) = |x|$ has two tangent lines at $x = 0$: the lines $y = x$ and $y = -x$.

(e) T F If $f(x) = x^{1/3}$, then $f'(0)$ does not exist.

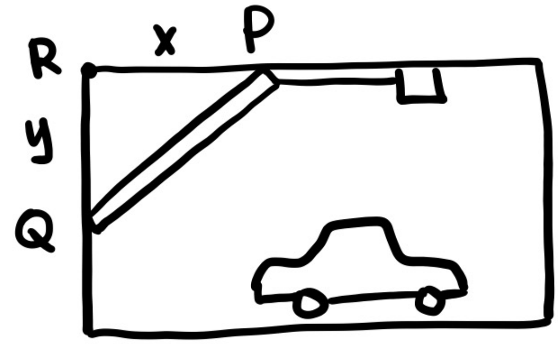
(f) T F If $f(x) = x^{1/3}$, then there is no tangent line to f at $x = 0$.

(g) T F $\dfrac{d}{dx}(e^{2x}) = 2xe^{2x-1}$

(h) T F A certain cylindrical tank has a radius of 5 ft. If the height of the water in the tank increases at a constant rate, then the volume of the water in the tank also increases at a constant rate.

14 p **G41.** A solid 14-foot tall garage door opens via a pulley mechanism. As the pulley opens the garage door, the top of the garage door (point P in the figure) moves to the right at 5 ft/s. At the same time, the bottom of the garage door (point Q in the figure) moves straight up.

As shown in the figure, the point R is the fixed point at the top of the garage door frame, x represents the distance between P and R, and y represents the distance between Q and R.



(a) What is the sign of $\dfrac{dx}{dt}$?

(b) What is the sign of $\dfrac{dy}{dt}$?

(c) What is the rate of change of the distance between the points Q and R when the distance between them is 9 feet? You must include correct units in your answer. You may leave unsimplified radicals in your answer.

10 p **G42.** For each part, use the graph of $y = f(x)$ to determine whether the value exists. If the value exists, state its sign (negative, positive, or zero).

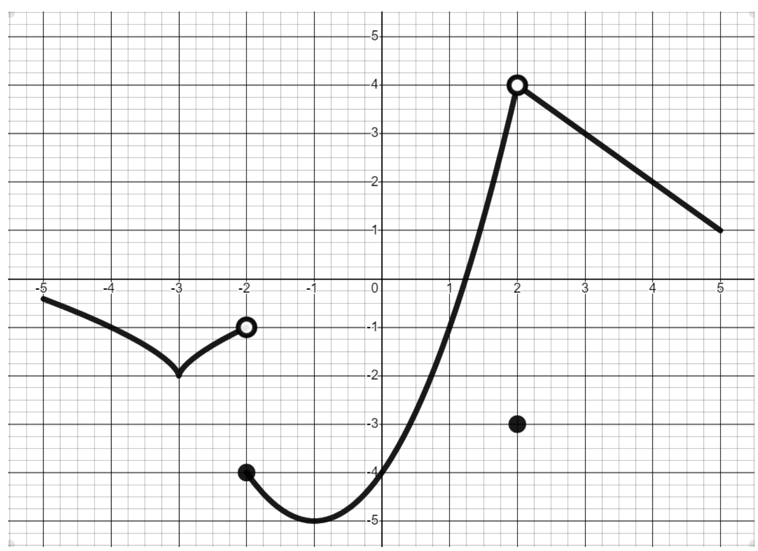
(a) $f'(-3)$

(b) $f'(-2)$

(c) $f'(-1)$

(d) $f'(1)$

(e) $f'(3)$

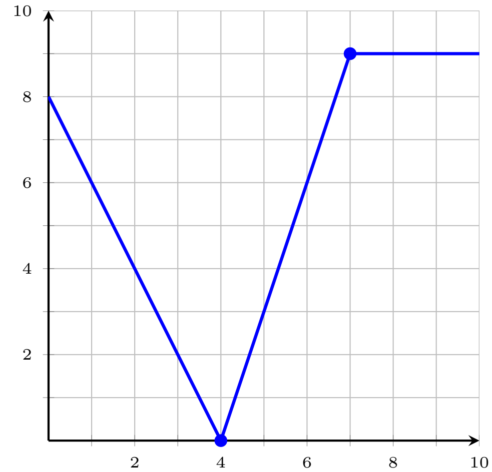


14 p **G43.** Let $f(x) = \dfrac{8x}{x+5}$.

(a) Calculate $f'(x)$ by any method.

(b) Use the limit definition of derivative to calculate $f'(3)$. ***Hint:*** Use your answer from part (a) to check your final answer.

15 p **G44.** The graph of $y = f(x)$ is given below.



(a) Calculate $f'(6)$. Briefly explain how you found your answer.

(b) Let $g(x) = 9xf(2x)$. Find an equation of the line tangent to the graph of $y = g(x)$ at $x = 3$.

§3.3, 3.4, 3.5, 3.9: Rules for Computing Derivatives

18 p **H1.** For each part, calculate $f'(x)$. *After calculating the derivative, do not simplify your answer.*

(a) $f(x) = \dfrac{7x^3}{3x^{1/2}x^5}$ (b) $f(x) = -\cos(x)\ln(x)$ (c) $f(x) = \dfrac{\csc(x) + 4x^3}{e^x - e^5}$

15 p **H2.** For each part, calculate $f'(x)$. *After calculating the derivative, do not simplify your answer.*

(a) $f(x) = \dfrac{x^{-1}x^{8/3}}{4\sqrt[3]{x^2}}$ (b) $f(x) = \left(x + \sqrt{5x-6}\right)^{1/4}$ (c) $f(x) = \dfrac{x^2e^x}{\ln(x) - \cos(x)}$

8 p **H3.** Calculate $f'(x)$ where f is the function below.

$$f(x) = \left(\frac{x^8\sin(3x)}{\ln(x) - \ln(11)}\right)^{2/3}$$

After calculating the derivative, do not simplify your answer.

H4. Suppose f and g are differentiable for all x. For each part, use the table below or explain why there is not enough information.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	−1	−4	4	2
1	−1	−3	2	−4
2	−4	3	1	−1

8 p (a) Let $F(x) = \dfrac{f(x)}{g(x)}$. Calculate $F'(0)$.

8 p (b) Let $G(x) = f\Big(xg(x)\Big)$. Calculate $G'(1)$.

15 p **H5.** For each part, calculate $f'(x)$. *Do not simplify your answers.*

(a) $f(x) = e^x\sin(x)$ (b) $f(x) = \dfrac{\ln\left(e^{4x}+6\right)}{9\tan(x) - \pi^9}$

5 p **H6.** Find the slope of the line tangent to the graph of $y = 3\ln(x) - 6\sqrt{x}$ at $x = 3$.

10 p **H7.** For each part, calculate $f'(x)$. *Do not simplify your answers.*

(a) $f(x) = \dfrac{\ln(x)}{10 - x^3}$ (b) $f(x) = \sqrt{\cos(3 + x^5)}$

12 p **H8.** Find all points on the graph of $f(x) = x\ln(x)$ where the tangent line is horizontal.

20 p **H9.** For each part, calculate $f'(x)$. *Do not simplify your answers.*

(a) $f(x) = 2x^2 - \dfrac{1}{5x} - 8\sqrt{x} + 14\pi^{3/2}$ (c) $f(x) = \sin\left(12x - x^9\right)\ln(x)$

(b) $f(x) = \left(\dfrac{x^4 - 20x}{x^3 + 20}\right)^{2/3}$ (d) $f(x) = \dfrac{e^{5\sec(6x)+1}}{7}$

10 p **H10.** Find the x-coordinate of each point on the graph of $f(x) = 3x + \dfrac{10}{x}$ where the tangent line is parallel to the line $y = 20 - 2x$.

16 p **H11.** Let $f(x) = x^{15}e^{2-5x}$. Find the x-coordinate of each point where the tangent line to f is horizontal.

15 p **H12.** Let $f(x) = 3x^5 - 2x^3 + 7x - 16$. Find an equation of the tangent line to f at $x = -1$.

20 p **H13.** Consider the function $f(x) = x^3 - 6x + c$, where c is an unspecified constant. Suppose the line $102x - y = 609$ is tangent to the graph of $y = f(x)$ at the point P in the first quadrant.

(a) What is the value of $f'(x)$ at the point P? Give a brief, one-sentence explanation.
(b) Find the x-coordinate of P.
(c) Find the y-coordinate of P.
(d) Find the value of c.

20 p **H14.** Let $f(x) = \dfrac{8e^x}{x-3}$. Find the equation of each horizontal tangent line of f.

20 p **H15.** Suppose $f(1) = -8$ and $f'(1) = 12$. Let $F(x) = x^3 f(x) + 10$. Find an equation of the tangent line to F at $x = 1$.

20 p **H16.** Suppose that an equation of the tangent line to f at $x = 5$ is $y = 3x - 8$. Let $g(x) = \dfrac{f(x)}{x^2 + 10}$.

(a) Calculate $f(5)$ and $f'(5)$.
(b) Calculate $g(5)$ and $g'(5)$.
(c) Write down an equation of the tangent line to g at $x = 5$.

12 p **H17.** Suppose $f(2) = -7$ and $f'(2) = 3$.

(a) Let $g(x) = \cos(x)f(x)$. Calculate $g'(2)$.
(b) Let $h(x) = e^{2f(x)+3}$. Calculate $h'(2)$.

16 p **H18.** Let $f(x) = x^2 + bx + c$, where b and c are unspecified constants. An equation of the tangent line to f at $x = 3$ is $12x + y = 10$.

(a) Calculate $f(3)$ and $f'(3)$. Your answers must not contain the letters b or c.
(b) Calculate the value of b.
(c) Calculate the value of c.

9 p **H19.** The following limit represents the derivative of a function f at a point a.

$$f'(a) = \lim_{h \to 0} \left(\frac{9\tan\left(\frac{\pi}{6} + h\right) - \frac{9}{\sqrt{3}}}{h} \right)$$

(a) Find a possible pair for f and a.
(b) Calculate the value of the limit.

9 p **H20.** Selected values of the functions f and g and their derivatives are given in the table below. Use these values to complete the questions.

x	1	2	3	4
$f(x)$	4	3	2	1
$f'(x)$	-4	-1	-9	-3
$g(x)$	2	1	3	4
$g'(x)$	1	2	4	5

(a) Suppose $h(x) = 5f(x) - 8g(x)$. Find $h'(1)$.

(b) Suppose $p(x) = x^2 f(x)$. Find $p'(2)$.

(c) Suppose $q(x) = f(x^2)$. Find $q'(2)$.

30 p **H21.** For each part, calculate $f'(x)$. After calculating the derivative, do not simplify your answer.

(a) $f(x) = 3x^{13} + 7\sqrt{x} - \dfrac{5}{x^3} + 12$

(b) $f(x) = \dfrac{e^x - 2\sin(x)}{\ln(x) + x^3}$

(c) $f(x) = 2x^4 \cos(3e^x)$

12 p **H22.** For both parts below, suppose the line tangent to the graph of $y = f(x)$ at $x = 5$ is $y = 2x - 3$.

(a) Calculate $f(5)$ and $f'(5)$.

(b) Let $g(x) = xf(x) + 14$. Find an equation of the line tangent to the graph of $y = g(x)$ at $x = 5$.

24 p **H23.** For each part, calculate the derivative. After calculating the derivative, do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\tan\left(\dfrac{\ln(x)}{2x-5}\right)\right)$ (b) $\dfrac{d}{dx}\left(3x^7\cos(x) - 8e^{3x}\right)$ (c) $\dfrac{d}{dx}\left(10x^{12} - \dfrac{3}{x^3} + \sqrt[4]{x}\right)$

12 p **H24.** Suppose that an equation to the tangent line to $y = f(x)$ at $x = 9$ is $y = 3x - 20$. Let $g(x) = xf(x^2)$.

(a) Calculate $f(9)$ and $f'(9)$. Explain.

(b) Calculate $g'(x)$.

(c) Find the tangent line to $y = g(x)$ at $x = -3$.

§3.7: The Chain Rule

15 p **I1.** For each part, calculate $f'(x)$. *After calculating the derivative, do not simplify your answer.*

(a) $f(x) = \dfrac{x^{-1}x^{8/3}}{4\sqrt[3]{x^2}}$ (b) $f(x) = \left(x + \sqrt{5x-6}\right)^{1/4}$ (c) $f(x) = \dfrac{x^2 e^x}{\ln(x) - \cos(x)}$

I2. Suppose f and g are differentiable for all x. For each part, use the table below or explain why there is not enough information.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	−1	−4	4	2
1	−1	−3	2	−4
2	−4	3	1	−1

8 p (a) Let $F(x) = \dfrac{f(x)}{g(x)}$. Calculate $F'(0)$.

8 p (b) Let $G(x) = f\Big(xg(x)\Big)$. Calculate $G'(1)$.

5 p **I3.** Suppose $f(4) = -8$ and $f'(4) = 3$. Let $g(x) = f\left(\frac{1}{4}x^2\right)$. Find $g'(4)$ or explain why it is impossible to do so with the given information.

5 p **I4.** Find an equation of the line tangent to the graph of $y = \tan(2x)$ at $x = \dfrac{\pi}{8}$.

5 p **I5.** Find an equation of the line tangent to the graph of $f(x) = 5e^{2\cos(x)}$ at $x = 3\pi/2$.

16 p **I6.** For each part, calculate the derivative by any valid method.

(a) $f(x) = x^2\cos(3x) + \dfrac{1}{5x}$ (b) $f(x) = \sqrt{\sin\left(\dfrac{e^x}{x+1}\right)}$

12 p **I7.** Suppose $f(2) = -7$ and $f'(2) = 3$.

(a) Let $g(x) = \cos(x)f(x)$. Calculate $g'(2)$.
(b) Let $h(x) = e^{2f(x)+3}$. Calculate $h'(2)$.

9 p **I8.** Let $f(x) = x^9 e^{4x}$.

(a) Find $f'(x)$.
(b) Explain how to find where the tangent line to the graph of f is horizontal.
(c) Find where the graph of f has a horizontal tangent line.

9 p **I9.** Selected values of the functions f and g and their derivatives are given in the table below. Use these values to complete the questions.

x	1	2	3	4
$f(x)$	4	3	2	1
$f'(x)$	−4	−1	−9	−3
$g(x)$	2	1	3	4
$g'(x)$	1	2	4	5

(a) Suppose $h(x) = 5f(x) - 8g(x)$. Find $h'(1)$.

(b) Suppose $p(x) = x^2 f(x)$. Find $p'(2)$.
(c) Suppose $q(x) = f(x^2)$. Find $q'(2)$.

12 p **I10.** Suppose f is differentiable at x and $g(x) = \dfrac{16\ln(15x)}{6f(x) - \sqrt{x+17}}$. Find $g'(x)$.

30 p **I11.** For each part, calculate $f'(x)$. After calculating the derivative, do not simplify your answer.

(a) $f(x) = 3x^{13} + 7\sqrt{x} - \dfrac{5}{x^3} + 12$

(b) $f(x) = \dfrac{e^x - 2\sin(x)}{\ln(x) + x^3}$

(c) $f(x) = 2x^4 \cos(3e^x)$

12 p **I12.** Let $h(x) = \dfrac{f(x^2)}{g(x)}$. Use the table of values below to calculate $h'(1)$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	−4	6	2	3
2	5	−2	−1	9

24 p **I13.** For each part, calculate the derivative. After calculating the derivative, do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\tan\left(\dfrac{\ln(x)}{2x-5}\right)\right)$ (b) $\dfrac{d}{dx}\left(3x^7\cos(x) - 8e^{3x}\right)$ (c) $\dfrac{d}{dx}\left(10x^{12} - \dfrac{3}{x^3} + \sqrt[4]{x}\right)$

16 p **I14.** For each part, mark "T" if the statement is true or mark "F" if the statement is false. You do not have to explain your answers or show any work.

(a) T F If f is continuous at $x = 3$, then f is differentiable at $x = 3$.
(b) T F If f is differentiable at $x = 3$, then f is continuous at $x = 3$.
(c) T F If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$ for all x.
(d) T F The function $f(x) = |x|$ has two tangent lines at $x = 0$: the lines $y = x$ and $y = -x$.
(e) T F If $f(x) = x^{1/3}$, then $f'(0)$ does not exist.
(f) T F If $f(x) = x^{1/3}$, then there is no tangent line to f at $x = 0$.
(g) T F $\dfrac{d}{dx}(e^{2x}) = 2xe^{2x-1}$
(h) T F A certain cylindrical tank has a radius of 5 ft. If the height of the water in the tank increases at a constant rate, then the volume of the water in the tank also increases at a constant rate.

I15. For each part, calculate the indicated derivative. Do not simplify your answer.

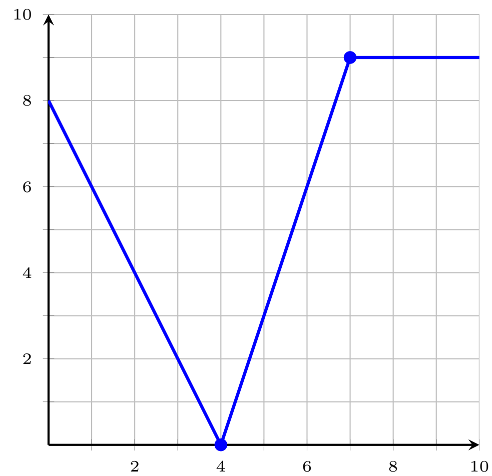
6 p (a) $\dfrac{d}{dx}\left(7x^{10} + \sqrt[3]{x} - \dfrac{8}{x^{20}} + \sec(8x)\right)$

6 p (b) $\dfrac{d}{dx}\left(\dfrac{\ln\left(x^3+30\right)}{8x}\right)$

6 p (c) $\dfrac{d}{dx}\left(\sin\left(xe^{-5x}\right)\right)$

15 p **I16.** Find the coordinates of all points on the graph of $f(x) = x\sqrt{14 - x^2}$ where the tangent line is horizontal. You must give both the x- and y-coordinate of each such point.

15 p **I17.** The graph of $y = f(x)$ is given below.



(a) Calculate $f'(6)$. Briefly explain how you found your answer.

(b) Let $g(x) = 9xf(2x)$. Find an equation of the line tangent to the graph of $y = g(x)$ at $x = 3$.

§3.8: Implicit Differentiation

12 p **J1.** Find all points on the graph of the equation

$$2x^2 - 4xy + 7y^2 = 45$$

at which the tangent line is horizontal. ***Hint:*** Find a second equation that such points must satisfy. Then solve a system of two equations in the two unknowns x and y.

8 p **J2.** Find an equation of the line tangent to the following curve at the point $(2, 0)$.

$$x^3 + e^{xy} = 3y + 9$$

12 p **J3.** Find an equation of the line tangent to the following curve at $(8, 1)$.

$$\sin\left(\frac{\pi x}{y}\right) = x - 8y$$

10 p **J4.** Find an equation of the line tangent to the following curve at the point $(1, 1)$.

$$\frac{5x}{y} = 4x + y^3$$

5 p **J5.** Find an equation of the line tangent to the following curve at the origin.

$$\sin(x + 2y) + 9x + 1 = e^y$$

10 p **J6.** Suppose y is defined implicitly as a function of x by the following equation.

$$x^3y^2 + (x + y)^2 = 100$$

Find $\frac{dy}{dx}$. *Do not simplify your answer.*

10 p **J7.** A particle in the fourth quadrant is moving along a path described by the equation

$$x^2 + xy + 2y^2 = 16$$

such that at the moment its x-coordinate is 2, its y-coordinate is decreasing at a rate of 5 cm/sec. At what rate is its x-coordinate changing at that time?

10 p **J8.** Find an equation of line tangent to the following curve at the origin.

$$\sin(x + 3y) + 9x + 1 = e^y$$

10 p **J9.** Consider the curve described by the equation

$$3x^2 + 2xy + 4y^2 = 132$$

At any point on this curve, we have
$$\frac{dy}{dx} = \frac{-3x - y}{x + 4y}$$

(a) Describe in two or three sentences the steps you should take to find the points on the curve where the tangent line is horizontal. *Your answer may contain either English, mathematical symbols, or both.*

(b) What is the rightmost (i.e., greatest x-coordinate) point on the curve where the tangent line is horizontal?

(c) Describe in one or two sentences how parts (a) and (b) would change if instead you wanted to find the points where the tangent line is vertical. You do not have to solve the problem again, but only describe generally what you would do differently. *Your answer may contain either English, mathematical symbols, or both.*

10 p **J10.** Find an equation of the line tangent to the following curve at $(1, 7)$.

$$\ln(xy + x - 7) = 2x + 4y - 30$$

10 p **J11.** Consider the curve described by the equation

$$5x^2 - 4xy + y^2 = 8$$

At any point on this curve, we have

$$\frac{dy}{dx} = \frac{-5x + 2y}{-2x + y}$$

(a) Describe in two or three sentences the steps you should take to find each point on the curve where the tangent line is parallel to the line $y = x$. *Your answer may contain either English, mathematical symbols, or both.*

(b) What is the leftmost (i.e., least x-coordinate) point on the curve where the tangent line is parallel to $y = x$?

(c) Describe in one or two sentences how parts (a) and (b) would change if instead you wanted to find the points where the tangent line is perpendicular to the line $y = 4$. You do not have to solve the problem again, but only describe generally what you would do differently. *Your answer may contain either English, mathematical symbols, or both.*

20 p **J12.** Consider the curve described by the equation

$$x^4 - x^2y + y^4 = 1$$

(a) Find $\frac{dy}{dx}$ at a general point on the curve.

(b) Find an equation of the line tangent to the curve at the point $(-1, 1)$.

18 p **J13.** On an online exam, a student uses logarithmic differentiation to find the first derivative of

$$f(x) = (3 + \sin(x))^{2+x^2}$$

They type the following two lines for their work.

$$y = (3 + \sin(x))^{2+x^2}$$
$$\ln(y) = \ln(\cdots$$

Unfortunately, the student runs out of time and is unable to submit the rest of their work. Oh no! Find $f'(x)$ by completing the student's work.

32 p **J14.** Consider the following curve, where a and b are unspecified constants.

$$ax^2y - 3xy^2 + 4x = b$$

(a) Show that $\dfrac{dy}{dx} = \dfrac{3y^2 - 2axy - 4}{ax^2 - 6xy}$.

(b) Suppose the tangent line to the curve at the point $(1, 1)$ is $y = 1 + 5(x - 1)$. Use part (a) to find the value of a.

(c) Use your answer to part (b) to find the value of b.

20 p **J15.** Consider the curve defined by the equation below, where a and b are unspecified constants.

$$\sqrt{xy} = ay^3 + b$$

Suppose the equation of the tangent line to the curve at the point $(3, 3)$ is $y = 3 + 4(x - 3)$.

(a) What is the value of $\frac{dy}{dx}$ at $(3, 3)$?

(b) Calculate a and b.

15 p **J16.** Consider the curve defined by the following equation, where A and B are unspecified constants.

$$Ax^2 - 8xy = B\cos(y) + 3$$

(a) Find a formula for $\frac{dy}{dx}$.

(b) Suppose the point $(8, 0)$ is on the curve. Find an equation that A and B must satisfy.

(c) Suppose the tangent line to the curve at the point $(8, 0)$ is $y = 6x - 48$. Find the values of A and B.

12 p **J17.** Consider the curve described by the following equation:

$$12x^2 + 6xy + y^2 = 20$$

Find all points on the curve where the tangent line is horizontal. Write your answer as a comma-separated list of coordinate pairs.

Hint: Find a second equation that such points must satisfy.

12 p **J18.** Find all points on the graph of the following equation where the tangent line is vertical.

$$x^2 - 2xy + 10y^2 = 450$$

14 p **J19.** Consider the following curve.

$$\cos(5x + y - 5) = 8xe^y + y - 7$$

(a) Calculate $\dfrac{dy}{dx}$ for a general point on the curve.

(b) Find an equation of the line tangent to the curve at the point $(1, 0)$.

§3.11: Related Rates

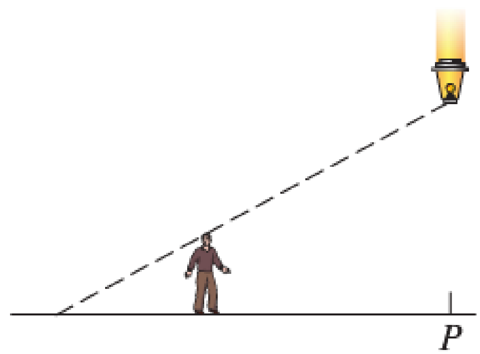
12 p **K1.** A camera is located 5 feet from a straight wire along which a bead is moving at 6 feet per second. The camera automatically turns so that it is pointed at the bead at all times. How fast is the camera turning 2 seconds after the bead passes closest to the camera?

You must give correct units as part of your answer.

10 p **K2.** The total surface area of a cube is changing at a rate of 12 in^2/s when the length of one of the sides is 10 in. At what rate is the volume of the cube changing at that time?

14 p **K3.** A person 5 feet tall stands stationary 8 feet from the point P, which is directly beneath a lantern that falls toward the ground. At the moment when the lantern is 15 feet above the ground, the lantern is falling at a speed of 4 feet per second. At what rate is the length of the person's shadow changing at this moment?

You must give correct units as part of your answer.



10 p **K4.** A child flies a kite at a constant height of 30 feet and the wind is carrying the kite horizontally away from the child at a rate of 5 ft./sec. At what rate must the child let out the string when the kite is 50 feet away from the child?

You must give correct units as part of your answer.

1.4 Chapter 4: Applications of the Derivative

§4.1: Maxima and Minima

12 p **L1.** Find the minimum and maximum values of $f(x) = 2x^3 - 3x^2 - 12x + 18$ on the interval $[-3, 3]$.

Hint: You may use the factorization $f(x) = (x^2 - 6)(2x - 3)$ to make any required arithmetic easier.

L2. Let $f(x) = 4(x-3)^{1/3} - \frac{1}{3}x + 1$. *Note:* The domain of f is $(-\infty, \infty)$.

11 p (a) Calculate all critical points of f. *For each number you find, you must clearly indicate in your work why it is a critical point.*

4 p (b) What are the absolute extreme values of f on the interval $[2, 30]$?

11 p **L3.** Find all critical points of $f(x) = x - \dfrac{3}{2}(x-8)^{2/3}$ or explain why f has no critical points.

11 p **L4.** Find the absolute extreme values of $f(x) = \dfrac{20x}{x^2+4}$ on $[-4, 0]$.

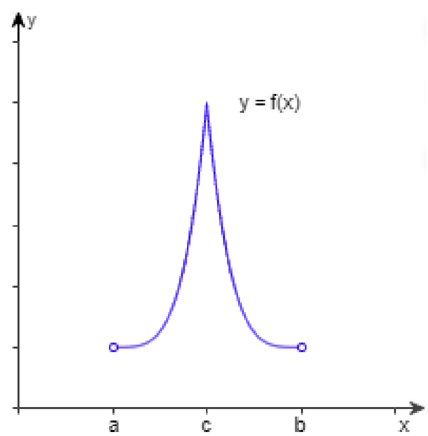
5 p **L5.** Find all critical points of $f(x) = 2 - \left(x^2 - 2x\right)^{1/3}$ or explain why f has no critical points. ***Note:*** The domain of f is $(-\infty, \infty)$.

24 p **L6.** For each part, find the absolute extreme values of $f(x)$ on the given interval.

(a) $f(x) = x + \dfrac{9}{x}$ on $[1, 18]$.

(b) $f(x) = (6-x)e^x$ on $[0, 6]$.
(***Hint:*** $2 < e < 3$.)

8 p **L7.** Determine from the given graph whether the function has any absolute extreme values on (a, b).



8 p **L8.** Consider the following function

$$g(x) = \frac{3}{2}x^4 + 8x^3 - 36x^2$$

(a) Where does g have a local minimum on $(-7, 3)$? local maximum?

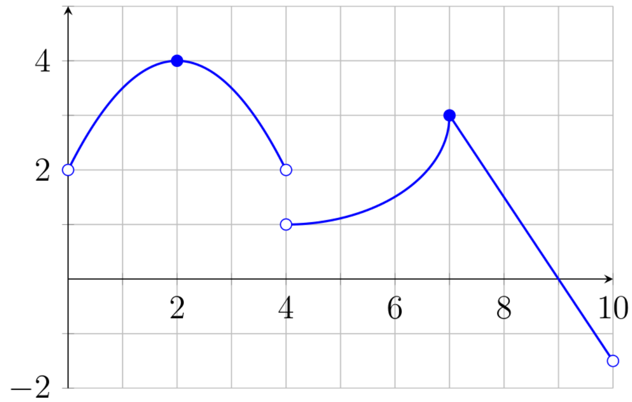
(b) Where does g have a global minimum on $[-7, 3]$? global maximum?

8 p **L9.** Find all critical points of the function

$$f(x) = 2x^{4/3} - 16x^{2/3} + 24$$

Note: The function f is continuous on the interval $(-\infty, \infty)$.

16 p **L10.** Suppose $f(x)$ is continuous on $[0, 10]$. The figure below shows the graph of $y = f'(x)$ on $[0, 10]$. ***Note:*** The figure does not show a graph of $f(x)$ but rather its derivative.)



Use the graph to answer the following questions. Read each question carefully. Some questions ask about f and others ask about the derivative f'.

(a) Find the absolute maximum of $f'(x)$ on $(0, 10)$ or determine that it does not exist.

(b) Find the absolute minimum of $f'(x)$ on $(0, 10)$ or determine that it does not exist.

(c) Find all critical points of $f(x)$ in $(0, 10)$.

18 p **L11.** Let $f(x) = \dfrac{1-2x}{6+x^2}$. Find the absolute extrema of f on $[-3, 2]$ and where they occur.

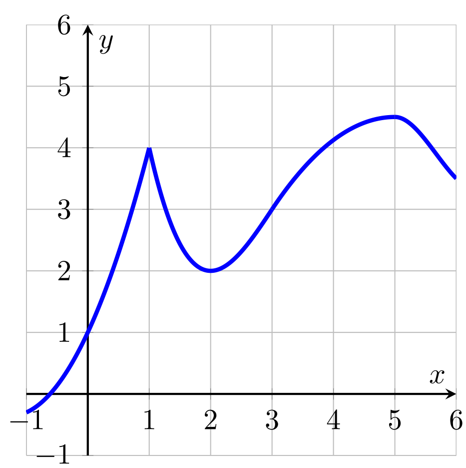
16 p **L12.** Let $f(x) = x^{1/3}(x-16)^{1/5}$. Find all critical points of f. You must be clear why each of your answers really is a critical point. ***Note:*** The domain of f is $(-\infty, \infty)$.

18 p **L13.** For each part, use the graph of $y = f(x)$. Assume that the domain of f is $(-\infty, \infty)$.

(a) Where does f have a local minimum?

(b) List all of the critical points of f.

(c) Estimate the absolute maximum of f on $[0, 3]$ or explain why f has no such maximum.



22 p **L14.** *(You will need a basic calculator for this problem.)*

Consider the function

$$f(t) = \frac{a}{t^2 - 3t + 25}$$

where a is an unspecified **positive** constant. Suppose the absolute minimum of f on $[0, 6]$ is 3.

(a) Find the value of a. ***Hint:*** First find the absolute minimum of f on $[0, 6]$ in terms of a.

(b) Calculate the absolute maximum of f on $[0, 6]$.

16 p **L15.** Consider the function below, where A is an unspecified, **positive** constant.

$$f(x) = \frac{A}{x - 8\sqrt{x} + 60}$$

For parts (c) and (d) only, assume the absolute minimum of f on $[0, 21]$ is 8.

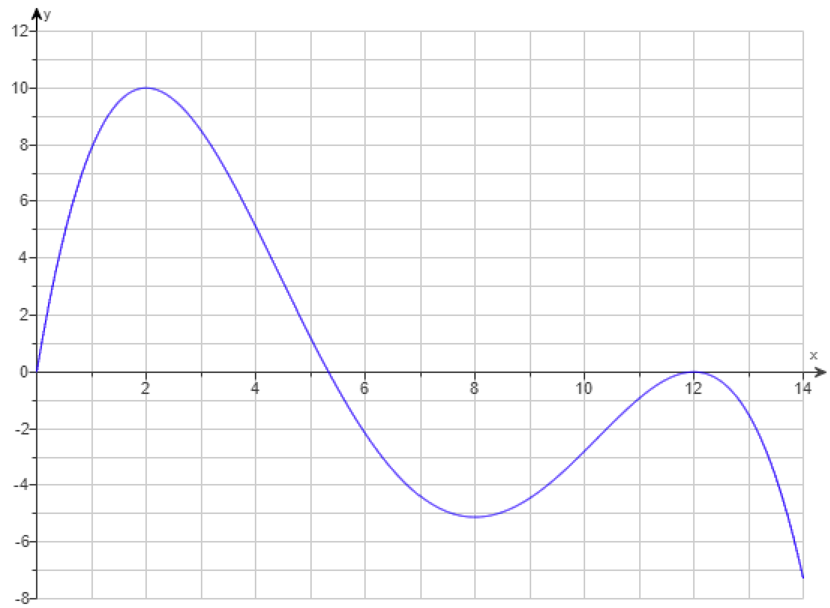
(a) List all x-values that must be tested to find the absolute extrema of f on $[0, 21]$.

(b) At which x-value does the absolute minimum of f occur on $[0, 21]$?

(c) Find the value of A.

(d) Find the absolute maximum of f on $[0, 21]$ and all x-values at which it occurs.

8 p **L16.** Use the graph of $y = f(x)$ on $[0, 14]$ below to answer the questions.



(a) List the critical points of f in $(0, 14)$.

(b) How many local extrema does f have on $(0, 14)$?

(c) Find the absolute maximum of f and the x-value at which it occurs.

(d) Find the absolute minimum of f and the x-value at which it occurs.

20 p **L17.** Find the absolute extreme values of $f(x) = x^3 - 6x^2 + 9x + 20$ on $[-3, 2]$ and the x-value(s) at which they occur.

20 p **L18.** Find the absolute extreme values of $f(x) = x\,(x-8)^{5/3}$ on the interval $[0, 9]$ and the x-values at which they occur.

15 p **L19.** Let $f(x) = Ax^B \ln(x)$, where A and B are unspecified constants. Suppose that $(e^5, 10)$ is a point of local extremum for $f(x)$.

(a) Calculate the values of A and B.

(b) Determine whether $(e^5, 10)$ is a point of local minimum or a point of local maximum for $f(x)$. Explain your answer.

20 p **L20.** For each part, find the absolute extreme values of the given function on the given interval. If a particular extreme value does not exist, write "DNE" as your answer, and explain why that extreme value does not exist.

(a) $f(x) = \frac{e}{x} + \ln(x)$ on $[1, e^3]$

(b) $g(x) = 12x - x^3$ on $[0, \infty)$

§4.3, 4.4: What Derivatives Tell Us and Graphing Functions

10 p **M1.** Suppose $f(x)$ satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

Information from $f(x)$:

- the points $(1, 2)$, $(3, 3)$, and $(5, 2)$ lie on the graph of $y = f(x)$
- $\lim_{x\to+\infty} f(x) = \lim_{x\to-\infty} f(x) = 1$
- $\lim_{x\to-2^+} f(x) = \lim_{x\to2^-} f(x) = +\infty$
- $\lim_{x\to-2^-} f(x) = \lim_{x\to2^+} f(x) = -\infty$

Information from $f'(x)$:

- $f'(1) = f'(3) = 0$
- $f'(x) < 0$ on the intervals $(-\infty, -2)$,$(-2, 1)$, and $(3, \infty)$
- $f'(x) > 0$ on the intervals $(1, 2)$ and $(2, 3)$

Information from $f''(x)$:

- $f''(5) = 0$
- $f''(x) < 0$ on the intervals $(-\infty, -2)$ and $(2, 5)$
- $f''(x) > 0$ on the interval $(-2, 1)$, $(1, 2)$, and $(5, \infty)$

M2. Consider the function $f(x) = (x - 5)(x + 10)^2 = x^3 + 15x^2 - 500$.

1 p (a) Calculate all x- and y-intercepts of f.

6 p (b) Find where f is increasing and find where f is decreasing. Then calculate the x- and y-coordinates of all local extrema, classifying each as either a local minimum or a local maximum.

6 p (c) Find where f is concave up and find where f is concave down. Then calculate the x- and y-coordinates of all inflection points.

4 p (d) Sketch the graph of $y = f(x)$ on the provided grid. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

10 p **M3.** Suppose $f(x)$ satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

domain of f:	$[-8, 8]$
specific points on graph:	$f(-2) = -3$ and $f'(-6) = 0$
asymptotes of f:	$x = -2$ and $y = -3$
f is decreasing on:	$[-8, -2)$, $(-2, 2)$
f is increasing on:	$(2, 8]$
f is concave down on:	$(-1, 1)$
f is concave up on:	$[-8, -1)$, $(1, 8]$

M4. Consider the function f and its derivatives below.

$$f(x) = \frac{x^2}{x^2-1} \quad , \quad f'(x) = \frac{-2x}{(x^2-1)^2} \quad , \quad f''(x) = \frac{6x^2+2}{(x^2-1)^3}$$

6 p (a) Find all horizontal asymptotes of f.

6 p (b) Find all vertical asymptotes of f. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of f.

6 p (c) Find where f is decreasing and find where f is increasing. Then calculate all points of local extrema, classifying each as either a local minimum, a local maximum, or neither.

6 p (d) Find where f is concave down and find where f is concave up. Then calculate all points of inflection.

M5. Consider the function f and its derivatives below.

$$f(x) = \frac{2x^3+3x^2-1}{x^3} \quad , \quad f'(x) = \frac{3-3x^2}{x^4} \quad , \quad f''(x) = \frac{6x^2-12}{x^5}$$

For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

4 p (a) Find all horizontal asymptotes of f.

3 p (b) Find all vertical asymptotes of f. Then at each vertical asymptote you find, calculate the corresponding one-sided limits of f.

7 p (c) Find where f is decreasing and find where f is increasing. Then calculate the x-coordinates of all points of local extrema.

7 p (d) Find where f is concave down and find where f is concave up. Then calculate the x-coordinates of all points of inflection.

18 p **M6.** Consider the function f and its derivatives below.

$$f(x) = 2x + \frac{8}{x^2} \quad , \quad f'(x) = \frac{2\left(x^3-8\right)}{x^3} \quad , \quad f''(x) = \frac{48}{x^4}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

5 p **M7.** Find the x-coordinate of each inflection point, if any, of $f(x) = x^3 - 12x^2 + 5x - 10$.

25 p **M8.** Consider the function f and its derivatives below.

$$f(x) = \frac{3x^3 - 2x + 48}{x} \quad , \quad f'(x) = \frac{6(x^3 - 8)}{x^2} \quad , \quad f''(x) = \frac{6(x^3 + 16)}{x^3}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

12 p **M9.** For each part, sketch the graph of a function that satisfies the given properties.

(a) $f(x)$ is decreasing for all x; $f''(x) < 0$ for $x < 13$; $f''(x) > 0$ for $x > 13$.

(b) $f(x)$ has a local minimum at $x = a$ where $f'(a) = 0$.

(c) $f(x)$ has a local maximum at $x = b$ where $f'(b)$ is undefined.

14 p **M10.** The first two derivatives of the function f are given below.

$$f'(x) = \frac{x}{(x-6)^2(x+48)} \quad , \quad f''(x) = \frac{-2(x+12)^2}{(x-6)^3(x+48)^2}$$

(Do not attempt to find a formula for $f(x)$.)

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

8 p **M11.** Consider the following function

$$g(x) = \frac{3}{2}x^4 + 8x^3 - 36x^2$$

(a) Where does g have a local minimum on $(-7, 3)$? local maximum?

(b) Where does g have a global minimum on $[-7, 3]$? global maximum?

25 p **M12.** Suppose f is continuous for all x and its first derivative is given by $f'(x) = (x-4)^2(x+2)$.

(a) Where is f decreasing?

(b) A student writes "since $f'(4) = 0$, there is a local extremum (either min or max) at $x = 4$". Is the student correct? Explain.

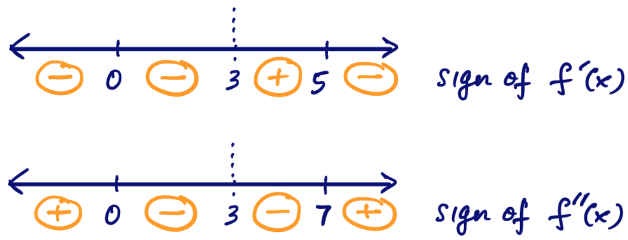
(c) Where is f concave up?

(d) Find the x-coordinate of each inflection point of f.

25 p **M13.** Suppose $f(x)$ satisfies all of the following properties.

- $f(x)$ is continuous and differentiable on $(-\infty, 3) \cup (3, \infty)$
- $x = 3$ is a vertical asymptote of $f(x)$
- $\lim\limits_{x\to\infty} f(x) = 1$
- the only x-values for which $f'(x) = 0$ are $x = 0$ and $x = 5$
- the only x-values for which $f''(x) = 0$ are $x = 0$ and $x = 7$

A sign chart for the first and second derivatives of f are given below.



Use this information to answer the following questions about $f(x)$. ***Note:*** Do not attempt to find an algebraic formula for $f(x)$.

(a) Where is f increasing?

(b) Where is f concave down?

(c) At which x-value(s) does f have a local minimum?

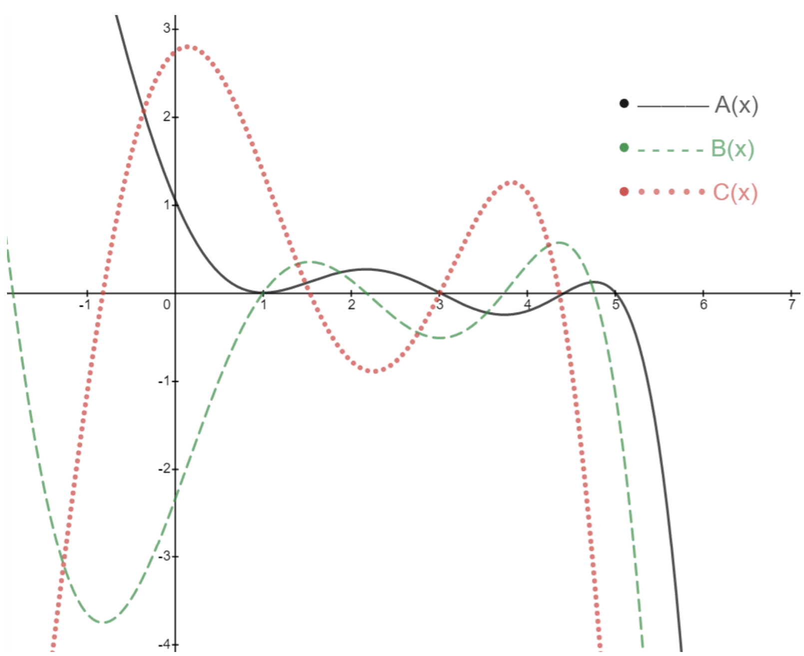
(d) At which x-value(s) does f have a local maximum?

(e) Calculate $\lim\limits_{x\to 3^+} f(x)$ or determine there is not enough information to do so.

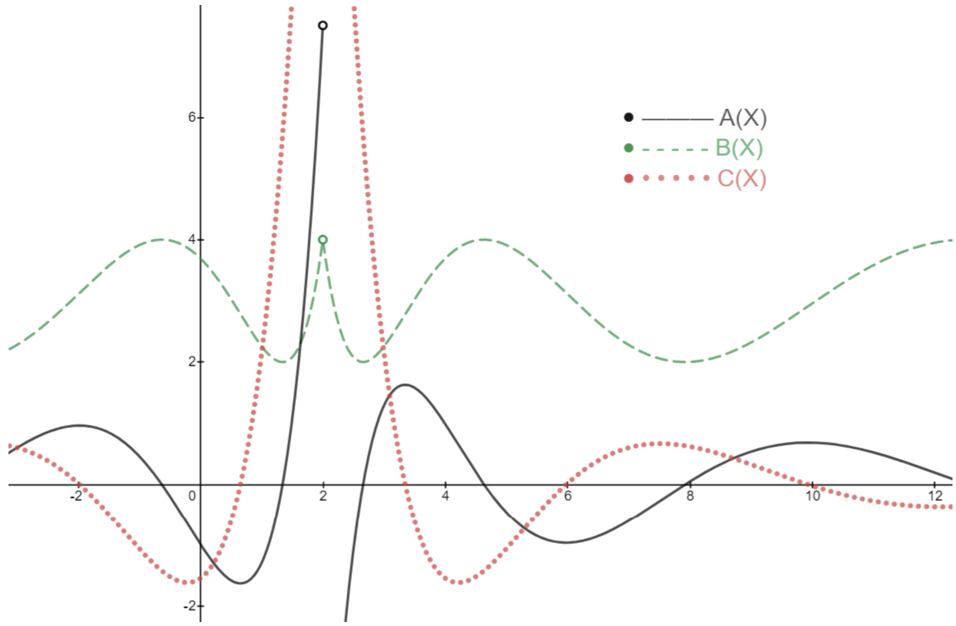
(f) Calculate $\lim\limits_{x\to -\infty} f(x)$ or determine there is not enough information to do so.

(g) Sketch a possible graph of $y = f(x)$. Clearly mark and label all of the following: local minima, local maxima, inflection points, vertical asymptotes, horizontal asymptotes. *Your graph does not have to be to scale, but the shape must be correct.*

6 p **M14.** The figure below shows the graphs of f, f', and f''. Identify which graph is that of f''.



8 p **M15.** The figure below shows the graphs of f, f', and f''. Identify which graph is which.



24 p **M16.** Suppose $f(x)$ satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

Information from $f(x)$:

- $\lim_{x\to-\infty} f(x) = 1$
- $\lim_{x\to\infty} f(x) = 6$
- $x = -3$ is a vertical asymptote for f

Information from $f'(x)$:

- $f'(x) > 0$ on $(2, \infty)$
- $f'(x) < 0$ on $(-\infty, -3)$ and $(-3, 2)$
- $f'(2) = 0$

Information from $f''(x)$:

- $f''(x) > 0$ on $(-3, 5)$
- $f''(x) < 0$ on $(-\infty, -3)$ and $(5, \infty)$
- $f''(5) = 0$

28 p **M17.** The first and second derivative of f are given below. You may assume that $f(x)$ has a vertical asymptote at $x = 25$ only, but do not attempt to calculate $f(x)$ explicitly.

$$f'(x) = \frac{(x+2)^{1/5}}{(x-25)^2} \quad , \quad f''(x) = \frac{-9(x+5)}{5(x-25)^3(x+2)^{4/5}}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

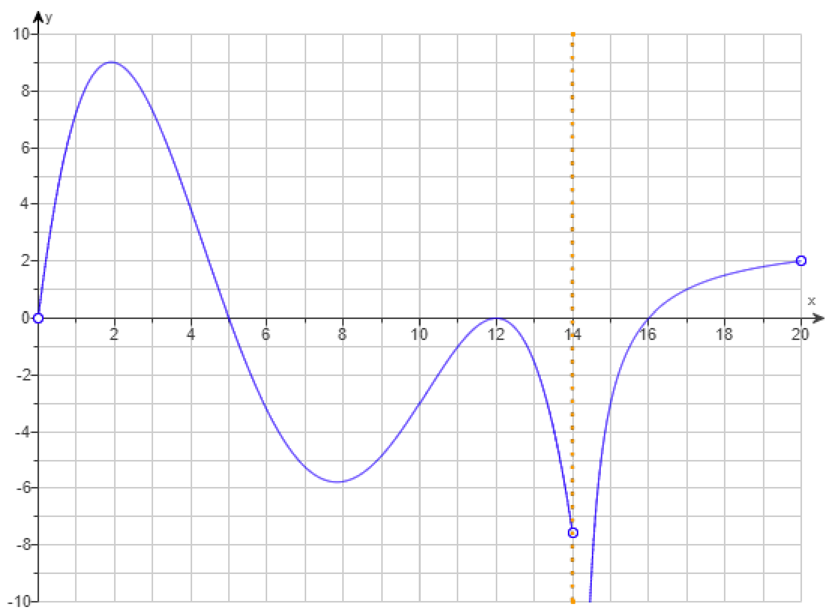
14 p **M18.** Consider the function $f(x)$ whose second derivative is given.

$$f''(x) = \frac{(x-2)^2(x-5)^3}{(x-9)^5}$$

You may assume the domain of $f(x)$ is $(-\infty, 9) \cup (9, \infty)$.

Find where $f(x)$ is concave down, where $f(x)$ is concave up, and where $f(x)$ has an inflection point. Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

18 p **M19.** Use the graph of $y = f'(x)$ below to answer the questions. You may assume that $f'(x)$ has a vertical asymptote at $x = 14$ and that the domain of f is $(0, 14) \cup (14, 20)$.

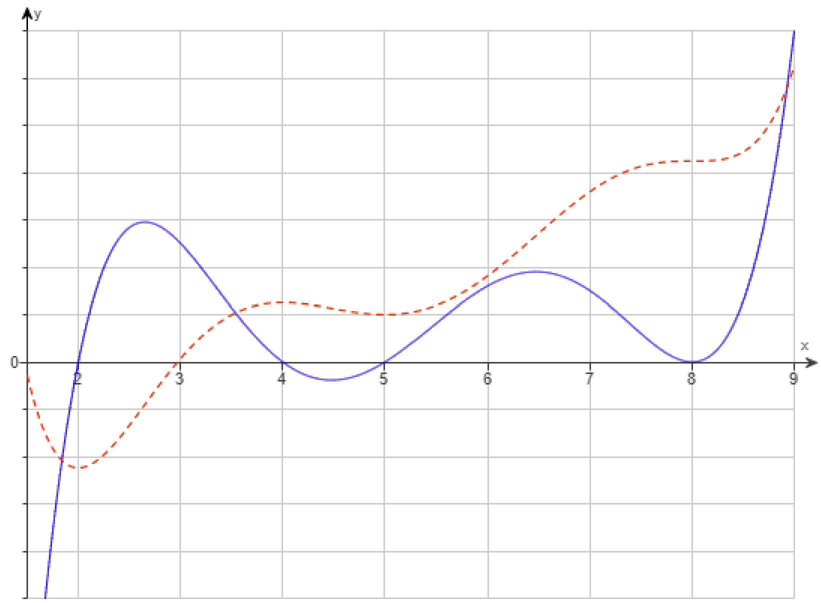


Note: You are given a graph of the first derivative of f, not a graph of f.

(a) Find the critical points of f.

(b) Find where f is decreasing, where f is increasing, where f has a local minimum, and where f has a local maximum. Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

12 p **M20.** The figure below shows the graphs of two functions. One function is $f(x)$ and the other is $f'(x)$, but you are not told which is which.



(a) Which graph is that of $y = f(x)$?

(b) Explain your answer to part (a) based on the behavior of the graphs at $x = 4$ only.

(c) Explain your answer to part (a) based on the behavior of the graphs near $x = 3.5$ only.

20 p **M21.** Consider the function f and its derivatives below.

$$f(x) = \frac{x-3}{x^2-6x-16} \quad , \quad f'(x) = \frac{-(x-3)^2-25}{(x^2-6x-16)^2} \quad , \quad f''(x) = \frac{2(x-3)\left((x-3)^2+75\right)}{(x^2-6x-16)^3}$$

Find where f is concave down and where f is concave up; write your answers using interval notation. Also find the x-coordinate of each inflection point of f.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

20 p **M22.** Suppose f is differentiable on $(-\infty, 1) \cup (1, \infty)$ and satisfies all of the following properties. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

(i) $\lim_{x\to-\infty} f(x) = -3$; $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to 1^-} f(x) = -\infty$; $\lim_{x\to 1^+} f(x) = \infty$;

(ii) $f'(x) > 0$ on $(-\infty, -2)$ and $(5, \infty)$; $f'(x) < 0$ on $(-2, 1)$ and $(1, 5)$; $f'(-2) = f'(5) = 0$

(iii) $f''(x) > 0$ on $(-\infty, -7)$ and $(1, \infty)$; $f''(x) < 0$ on $(-7, 1)$; $f''(-7) = 0$

20 p **M23.** Let $f(x) = -e^{-x}\left(x^2 - 5x - 23\right)$. Find all critical points of f. Then find where f is decreasing and where f is increasing; write your answers using interval notation. Also find where relative extrema of f occur.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

15 p **M24.** Let $f(x) = 4x^5 - 20x^4 + 7x + 32$. Find where f is concave down and where f is concave up; write your answer using interval notation. Also find where inflection points of f occur.

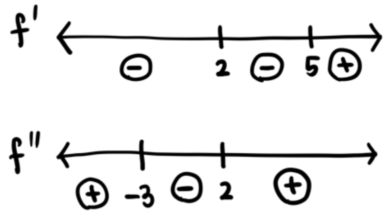
10 p **M25.** Suppose $f(x)$ satisfies all of the following properties. Sign charts for f' and f'' are also given below. Sketch a possible graph of $y = f(x)$ on the axes provided. *Label all asymptotes, local extrema, and inflection points. Your graph need not to be to scale, but it must have the correct shape.*

(i) f is continuous and differentiable on $(-\infty, 2) \cup (2, \infty)$

(ii) $\lim_{x\to-\infty} f(x) = \infty$; $\lim_{x\to\infty} f(x) = \infty$; $\lim_{x\to 2^-} f(x) = -\infty$; $\lim_{x\to 2^+} f(x) = \infty$

(iii) the only x-value for which $f'(x) = 0$ is $x = 5$

(iv) the only x-value for which $f''(x) = 0$ is $x = -3$



15 p **M26.** Let $f(x) = \dfrac{x^2 + 21}{x - 2}$. Find where f is decreasing and where f is increasing; write your answer using interval notation. Also find where the local extrema of f occur.

Write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.

15 p **M27.** Let $f(x) = Ax^B \ln(x)$, where A and B are unspecified constants. Suppose that $(e^5, 10)$ is a point of local extremum for $f(x)$.

(a) Calculate the values of A and B.

(b) Determine whether $(e^5, 10)$ is a point of local minimum or a point of local maximum for $f(x)$. Explain your answer.

20 p **M28.** For each part, find the absolute extreme values of the given function on the given interval. If a particular extreme value does not exist, write "DNE" as your answer, and explain why that extreme value does not exist.

(a) $f(x) = \dfrac{e}{x} + \ln(x)$ on $[1, e^3]$

(b) $g(x) = 12x - x^3$ on $[0, \infty)$

18 p **M29.** Consider the function f and its derivatives below.

$$f(x) = \frac{x^2}{x-7} \qquad f'(x) = \frac{x(x-14)}{(x-7)^2} \qquad f''(x) = \frac{98}{(x-7)^3}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

M30. Let $f(x) = x^2 e^x$.

7 p (a) Calculate the vertical and horizontal asymptotes of f.

10 p (b) Calculate the critical points of f. Then use the Second Derivative Test to classify each critical point of f as a local minimum or a local maximum. Show your work and label your answers clearly. ***Hint:*** The second derivative of f is $f''(x) = (x^2 + 4x + 2)e^x$.

§4.5: Optimization Problems

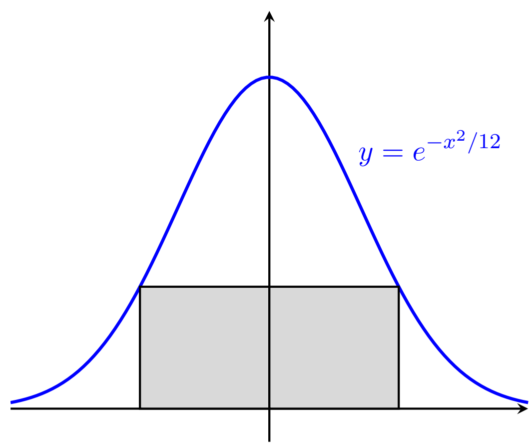
16 p **N1.** A wire of length 51 cm is cut into two pieces. One piece is bent into a square. The other piece is bent into a rectangle whose length is two times its width. How should the wire be cut and the pieces assembled so that the total area enclosed by both pieces is a minimum?

You must use calculus-based methods in your work. You must also justify that your answer really does give the minimum.

16 p **N2.** You are constructing a rectangular box with a total surface area (six sides) of 450 in^2. The length of the box is three times its width. Find the dimensions of the box, measured in inches, with the largest possible volume.

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.

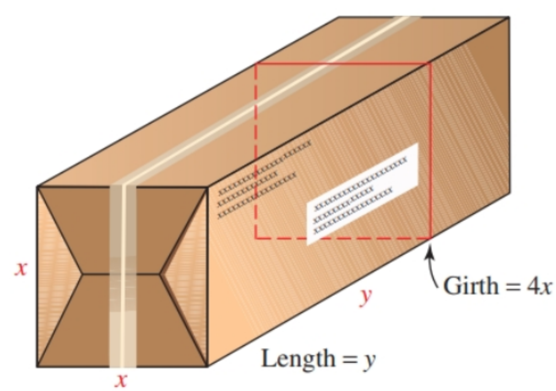
14 p **N3.** Find the maximum possible area of a rectangle inscribed in the region between the graph of $f(x) = e^{-x^2/12}$ and the x-axis. *You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.*



11 p **N4.** The cost of producing x units is $C(x) = 2x^2 + 5x + 8$. Find the level of production (value of x) that minimizes the average cost. ***Hint:*** Average cost is $AC(x) = \frac{C(x)}{x}$.

11 p **N5.** According to postal regulations, the sum of the girth and length of a parcel may not exceed 90 inches. What are the dimensions (in inches) of the parcel with the largest possible volume that can be sent, if the parcel is a rectangular box with two square sides?

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.



5 p **N6.** If x units of a certain product are produced, the total cost is $C(x) = 5x^2 + 104x + 80$. Find the level of production which minimizes the average cost per unit.

10 p **N7.** A rectangular container with a closed top and a square base is to be constructed. The top and all four sides of the container are to be made of material that costs \$2/ft^2, and the bottom is to be made of material that costs \$3/ft^2. Find the container with the largest volume that can be constructed for a total cost of \$60.

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.

8 p **N8.** Let x be the level of production for a certain commodity. The marginal cost is modeled by the function

$$\frac{dC}{dx} = 3x^2 + 2x$$

and the market price is modeled by the function

$$p(x) = 144 - 2x$$

Suppose that the cost of producing the 1st unit of the commodity is 70.

(a) What is the cost of producing the first 3 units of the commodity?

(b) What is the level of production that maximizes the total profit?

18 p **N9.** Suppose the local post office has a policy that all packages must be shaped like a rectangular box with a sum of length, width, and height not exceeding 144 inches. You plan to construct such a package whose length is 2 times its width. Find the dimensions of the package with the largest volume. For this problem, let L, W, and H be the length, width, and height of the package, respectively.

(a) What is the objective function for this problem in terms of L, W, and H?

(b) There are two constraints for this problem. In terms of L, W, and H, give the constraint equation which corresponds to...

(i) ...the policy set by the post office.

(ii) ...your specific plan to construct such a package.

(c) Find the objective function in terms of W only.

(d) What is the interval of interest for the objective function?

(e) Find the values of L, W, and H that give the largest volume.

(f) Suppose the post office adds the additional requirement that the width W of the package must be no smaller than 36 inches and no larger than 40 inches. With this additional policy, what is the width of the package with the largest volume?

25 p **N10.** Farmer Brown wants to create a rectangular pen that must enclose exactly 1800 ft^2. The fencing along the north and south sides of the fence costs \$10/ft and the fencing along the east and west sides costs \$5/ft. (The cost is different because some parts of the fence have to be taller than other parts.) Let x denote the length of the north side and let y denote the length of the east side.

(a) What are the dimensions and total cost of the cheapest pen?

(b) Justify that your answer really does give the cheapest pen.

25 p **N11.** In a certain video game, the player may adjust the values of their character's *Intelligence* (denoted by x) and *Dexterity* (denoted by y). These power values must be non-negative but can be any real number (they need not be whole numbers). The player cannot arbitrarily adjust their power, but rather these values must satisfy the equation $x^2 + y^2 = 100$. The total damage done (denoted by D) by the spell *Thunderbolt* is given by $D = x + 3y$.

(a) How should the player adjust their power so that *Thunderbolt* does the most possible damage?

(b) What is the minimum possible damage that *Thunderbolt* will do, regardless of how the player adjusts their character's power? How should a player adjust these power values to achieve the minimum possible damage?

34 p **N12.** A rectangular box with a square base and no top is being constructed to hold a volume of 150 cm^3. The material for the base of the container costs \$6/cm^2 and the material for the sides of the container costs \$2/cm^2. Find the dimensions of the cheapest possible container.

You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.

18 p **N13.** An airline policy states that all baggage must be shaped like a rectangular box with the sum of the length, width, and height not exceeding 122 inches. You plan to purchase a bag from a company that makes customized bagged whose height must be 3 times its width. Find the dimensions of the baggage with the largest volume. (Let L, W, and H be the length, width, and height of the baggage, respectively.)

(a) Before considering any constraints particular to this problem, find the objective function in terms of L, W, and H.

(b) There are two constraints for this problem. One constraint is from the airline and the other is from the baggage company. Find these constraints.

(c) Write the objective function in terms of W only.

(d) Find the interval of interest for the objective function in part (c).

(e) Find the dimensions of the baggage with the largest volume.

20 p **N14.** A storage shed with a volume of 1500 ft^3 is to be built in the shape of a rectangular box with a square base. The material for the base costs \$6/ft^2, the material for the roof costs \$9/ft^2, and the material for the sides costs \$2.50/ft^2. Find the dimensions of the cheapest shed. As you work, fill in the answer boxes below. Let x represent the length of the base of the shed.

objective function in terms of x:	
interval of interest:	
dimensions of cheapest shed (in ft):	______ × ______ × ______ length of base, width of base, height of shed

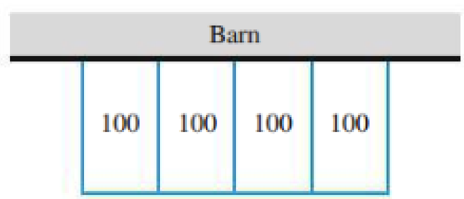
20 p **N15.** A rectangle (with base B and height H) is constructed with its base on the diameter of a semicircle with radius 5 and with its two other vertices on the semicircle. Find the dimensions of the rectangle with the maximum possible area.

As you work, fill in the answer boxes below. You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.

constraint equation in terms of B and H:	
objective function in terms of H only:	
interval of interest:	
dimensions of rectangle:	______ × ______ B (base) H (height)

18 p **N16.** A rancher plans to make four identical and adjacent rectangular pens against a barn, each with an area of 100 m^2 (see the figure below). What are the dimensions of each pen that minimize the amount of fence that must be used? ***Note:*** No fencing is needed on the side of the pen that borders the barn (the north side of the pen).

As you work, fill in the answer boxes below. You must use calculus-based methods in your work. You must also justify that your answer really does give the maximum.



constraint equation(s):	
objective function in one variable only:	
interval of interest:	
dimensions of one pen:	______ × ______ horizontal dimension vertical dimension

§4.6: Linear Approximation and Differentials

8 p **O1.** Use a linear approximation to estimate $\sqrt{33}$.

8 p **O2.** At a certain factory, the daily output is

$$Q(L) = 1500L^{2/3}$$

where L denotes the size of the labor force measured in worker-hours. Currently $1,000$ worker-hours of labor are used each day. Use a linear approximation to estimate the effect on the daily output if the labor force is cut to 975 worker-hours.

10 p **O3.** The concentration of a certain drug in the bloodstream t hours after the drug is injected is modeled by the following formula.

$$C(t) = \frac{100t}{t^2+1}$$

(The concentration is measured in micrograms per milliliter.) Use a linear approximation to estimate the change in the concentration over the time period from 2 to 2.1 hours after injection. Also indicate whether the concentration increases or decreases.

5 p **O4.** Use a linear approximation to estimate the value of $\sqrt{35.9}$. Do not simplify your answer.

5 p **O5.** The cost of producing x units is $C(x) = 3x^2 + 4x + 1000$. Use marginal analysis to estimate the cost of producing the 41st unit.

O6. ***Note:*** The parts of this problem are not related!

5 p (a) Use linear approximation to estimate the value of $\sqrt{79}$.

5 p (b) A manufacturer's total cost to produce x units is $C(x) = 25\ln(x^2 + 16)$. Use marginal analysis to estimate the cost of the 4th unit.

10 p **O7.** Use linear approximation or differentials to estimate the value of $\dfrac{1}{\sqrt[3]{8.48}}$.

10 p **O8.** Suppose the cost of manufacturing x units is given by $C(x) = x^3 + 5x^2 + 12x + 50$.

(a) What is the exact cost of producing the 3rd unit?

(b) Using marginal analysis, estimate the cost of producing the 3rd unit.

10 p **O9.** Use linear approximation to estimate the value of $(0.98)^3 - 5(0.98)^2 + 4(0.98) + 10$.

8 p **O10.** If x units are produced, the total cost is $C(x) = x^2 + 15x + 24$ and the selling price per unit is

$$p(x) = \frac{156}{x^2 - 4x + 16}$$

(a) What is the exact cost of producing the 3rd unit?

(b) Using marginal analysis, estimate the revenue from the 3rd unit sold.

18 p **O11.** Given that x units of a commodity are sold, the selling price per unit is $p(x) = \dfrac{5000}{x^2+64}$.

(a) Calculate the revenue function.

(b) Calculate the exact revenue derived from the 7th unit.

(c) Using marginal analysis, estimate the revenue derived from the 7th unit.

16 p **O12.** The total number of gallons in a water tank at t hours is given by $N(t) = 40t^{2/5}$. Use a linear approximation to estimate the number of gallons added to the water between $t = 32$ and $t = 35$.

6 p **O13.** Suppose f is differentiable on $(-\infty, \infty)$, $f(5) = 3$, and $f'(5) = -7$. Use linear approximation to estimate $f(5.1)$.

24 p **O14.** Use linear approximation to estimate $\sqrt[3]{29} - \sqrt[3]{27}$. *Your final answer must be exact and may not contain any radicals.*

10 p **O15.** Use the identity $4^2 + \sqrt{4} = 18$ and linear approximation to estimate $(3.81)^2 + \sqrt{3.81}$.

15 p **O16.** The total cost (in dollars) of producing x items is modeled by the function $C(x) = x^2 + 4x + 3$, and the price per item (in dollars) is $p(x) = \dfrac{98x + 49}{x + 3}$.

(a) Calculate the exact cost of producing the 5th item.

(b) Using marginal analysis, estimate the revenue derived from producing the 5th item.

§4.7: L'Hôpital's Rule

14 p

P1. For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0}(1-\sin(4x))^{6/x}$

(b) $\lim_{x\to 1}\left(\dfrac{xe^{4x}+4e^4-5e^4x}{(x-1)^2}\right)$

12 p

P2. For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 1}\left(\dfrac{x^{1/4}-1}{e^{2x}-e^2}\right)$

(b) $\lim_{x\to 1}\left((x-1)\tan\left(\dfrac{\pi x}{2}\right)\right)$

14 p

P3. For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to 0}\left(\dfrac{1-\cos(9x)}{x^2}\right)$

(b) $\lim_{x\to 0}(1-3x)^{5/x}$

P4. The parts of this problem *are* related!

3 p (a) Show that $\lim_{x\to\infty}\left(\dfrac{x}{x-3}\right)=1$.

8 p (b) Calculate the following limit or show it does not exist.

$$\lim_{x\to\infty}\left(\frac{x}{x-3}\right)^x$$

Hint: First use part (a) to identify the appropriate indeterminate form.

20 p

P5. For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to\pi}\left(\dfrac{1+\cos(x)}{(x-\pi)^2}\right)$

(b) $\lim_{x\to\infty}\left(1-\dfrac{12}{x}\right)^{5x}$

(c) $\lim_{x\to 1}\left(\dfrac{x^3-2x^2-5x+6}{x^3+x^2+x-3}\right)$

(d) $\lim_{x\to 4^+}\left(\dfrac{2x-x^2}{x-4}\right)$

10 p

P6. Calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 0^+}\left(\sqrt{12x+9}-\sqrt{2x+4}\right)^{1/x}$$

4 p

P7. Suppose you want to compute a limit that is in the form of a quotient, i.e., a limit of the form:

$$\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right)$$

Suppose you have already determined that L'Hospital's Rule is applicable. Explain the next step in your calculation, i.e., how do you apply L'Hospital's Rule? *Your answer may contain either English, mathematical symbols, or both.*

5 p

P8. Each of the following limits is written in the form of a quotient. Which limits can be calculated using L'Hospital's Rule directly, i.e., by applying L'Hospital's Rule as the immediately next step without any other algebra or modification? Select all that apply.

(a) $\lim_{x\to\pi}\left(\frac{\sin(7x)}{x}\right)$

(b) $\lim_{x\to 2}\left(\frac{x^3+3x-14}{x^2-5x+6}\right)$

(c) $\lim_{x\to\infty}\left(\frac{x^{-1}+5}{x^{-2}+8}\right)$

(d) $\lim_{x\to 9^-}\left(\frac{x^{3/2}+x-36}{x-\sqrt{x}-6}\right)$

(e) $\lim_{x\to\infty}\left(\frac{e^x+10}{e^x-3}\right)$

(f) $\lim_{x\to-\infty}\left(\frac{e^x+10}{e^x-3}\right)$

5 p **P9.** Which of the following limits are equal to $+\infty$? Select all that apply.

(a) $\lim_{x\to 5^-}\left(\frac{x^2+25}{5-x}\right)$

(b) $\lim_{x\to 5^+}\left(\frac{x^2+25}{5-x}\right)$

(c) $\lim_{x\to -3^-}\left(\frac{x^3}{|x+3|}\right)$

(d) $\lim_{x\to 0^-}\left(\frac{x^4-2x-5}{\sin(x)}\right)$

(e) $\lim_{x\to 1^+}\left(\frac{x^6-x^2}{x-1}\right)$

10 p **P10.** Calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to\infty}\left(5x^3+2x^2+8\right)^{1/\ln(x)}$$

5 p **P11.** Suppose you have determined

$$\lim_{x\to a} f(x)=0 \quad , \quad \lim_{x\to a} g(x)=\infty$$

and you want to calculate the following limit:

$$L=\lim_{x\to a}\left(f(x)g(x)\right)$$

You recall that to calculate L, you have to use L'Hospital's Rule. What is the next step you must take before you are able to apply L'Hospital's Rule directly to the limit L? *Your answer may contain either English, mathematical symbols, or both.*

4 p **P12.** Which of the following are indeterminate forms? Recall that in this course, we have learned that limits with indeterminate forms may often be computed using L'Hospital's Rule.

(a) $\frac{0}{0}$

(b) $0\cdot\infty$

(c) $\frac{\infty}{-\infty}$

(d) $\frac{0}{\infty}$

(e) 2^∞

(f) $3\cdot(-\infty)$

(g) $\infty\cdot(-\infty)$

(h) ∞^0

(i) ∞^∞

16 p **P13.** Calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 1}\left(\frac{\tan(\pi x)}{\sqrt{2+x^3}-\sqrt{2+x}}\right)$$

20 p **P14.** Consider the following limit.

$$L=\lim_{x\to -3}(4+x)^{7/(6+2x)}$$

(a) What indeterminate form does this limit have?

(b) Explain why l'Hospital's rule cannot be used on this limit in its current form.

(c) Calculate the value of L.

18 p **P15.** Consider the limit $\lim_{x\to 2^-}((x-2)\ln(2-x))$.

(a) Does this limit have an indeterminate form? If so, which indeterminate form?

(b) Explain why l'Hospital's rule cannot be used on this limit in its current form.

(c) Write the limit in an equivalent form to which l'Hospital's rule may be applied.

Note: Do not attempt to calculate the limit. You are not required to calculate the limit.

20 p **P16.** Suppose $f'(x)$ is continuous with $f(3) = 2$ and $f'(3) = -8$. Calculate the following limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 1}\left(\frac{2x^4 - f(3x^{1/4})}{x^2-4x+3}\right)$$

15 p **P17.** Suppose $f''(x)$ is continuous. You are also given the following values:

$$f\left(\frac{1}{8}\right) = 20 \quad , \quad f'\left(\frac{1}{8}\right) = -22$$

Calculate the following limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

$$\lim_{x\to 8}\left(\frac{20 - f\left(\frac{1}{x}\right)}{x^2+x-72}\right)$$

20 p **P18.** For each part, calculate the limit or show that it does not exist. *If the limit is "+∞" or "−∞", write that as your answer, instead of "does not exist".*

(a) $\lim_{x\to\pi}\left(\frac{\cos(6x)-1}{(x-\pi)^2}\right)$

(b) $\lim_{x\to 0}\left(e^{2x}+3x\right)^{1/x}$

P19. Let $f(x) = x^2e^x$.

7 p (a) Calculate the vertical and horizontal asymptotes of f.

10 p (b) Calculate the critical points of f. Then use the Second Derivative Test to classify each critical point of f as a local minimum or a local maximum. Show your work and label your answers clearly. ***Hint:*** The second derivative of f is $f''(x) = (x^2+4x+2)e^x$.

12 p **P20.** Let $f(x) = \frac{x\sin(Ax)}{\sin^2(2x)}$, where A is a constant. Suppose $\lim_{x\to 0} f(x) = -6$. Calculate A.

§4.9: Antiderivatives

14 p **Q1.** Given that x units of a commodity are sold, the marginal cost is

$$\frac{dC}{dx} = 9x^2 + 4x + 15x^{1/4} + 10$$

Suppose the total cost of producing the 1st unit is 100. Calculate the total cost of producing the first 16 units.

22 p **Q2.** Let $V(t)$ denote the volume of water, measured in gallons, in a tank at time t. The tank is initially filled with 5 gallons of water. At $t = 0$, water flows in at a rate in gal/min given by $V'(t) = 0.5(196 - t^2)$ for $0 \le t \le 10$. Find the total amount of water in the tank after 4 minutes.

14 p **Q3.** A particle travels along the x-axis in such a way that its velocity (measured in ft/sec) at any time t (measures in sec) is

$$v(t) = 4t^3 - 2t + 2$$

The particle is at $x = 3$ when $t = 2$.

(a) Find the position of the particle at any time t.

(b) Find the position of the particle at time $t = 4$.

(c) Find the acceleration of the particle when $t = 4$.

1.5 Chapter 5: Integration

§5.1–5.3, 5.5: Introduction to the Integral, Fundamental Theorem of Calculus, Substitution Rule

12 p **R1.** Suppose f is a continuous function such that all of the following hold:

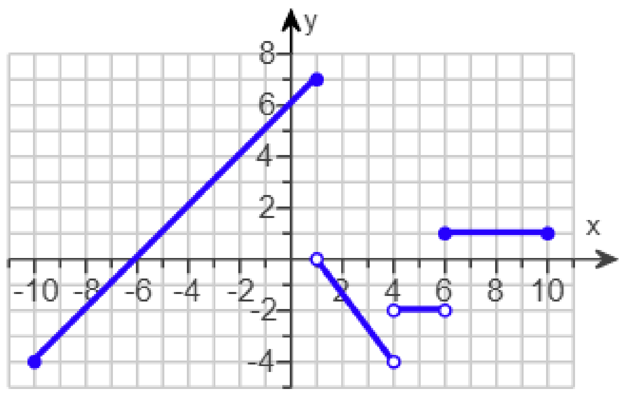
$$\int_{-1}^{6} f(x)\,dx = -15 \quad , \quad \int_{6}^{9} f(x)\,dx = 14 \quad , \quad \int_{0}^{9} f(x)\,dx = 19$$

Calculate the quantities below or determine there is not enough information.

(a) $\int_{-1}^{9} f(x)\,dx$ (c) $\int_{-1}^{6} |f(x)|\,dx$ (e) $\int_{-1}^{0} f(x)\,dx$

(b) $\int_{0}^{6} f(x)\,dx$ (d) $\left|\int_{-1}^{6} f(x)\,dx\right|$ (f) $\int_{6}^{9} (3f(x)+4)\,dx$

12 p **R2.** Use the graph of $y = f(x)$ to calculate the integrals below.



(a) $\int_{0}^{1} f(x)\,dx$ (b) $\int_{1}^{6} f(x)\,dx$ (c) $\int_{-10}^{10} f(x)\,dx$

20 p **R3.** Let $f(x) = 5 + \int_{-3}^{x} t^2 e^t\,dt$. Find an equation of the tangent line to f at $x = -3$.

20 p **R4.** Suppose f is continuous on $[0, 8]$ and has the following integrals:

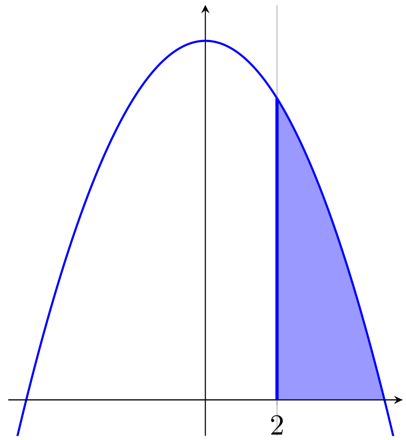
$$\int_{0}^{3} f(x)\,dx = 2 \qquad \int_{3}^{5} f(x)\,dx = 7 \qquad \int_{0}^{8} f(x)\,dx = 15$$

For each part, calculate the integral or determine there is not enough information to do so.

(a) $\int_{0}^{5} f(x)\,dx$ (b) $\int_{5}^{3} f(x)\,dx$ (c) $\int_{5}^{8} f(x)\,dx$ (d) $\int_{3}^{8} (2f(x)-6)\,dx$

20 p **R5.** Calculate $\int_{0}^{\sqrt{10}} \left(x + \sqrt{10 - x^2}\right) dx$ using geometry and properties of integrals only. Do not attempt to use the fundamental theorem of calculus.

20 p **R6.** The curve $y = 25 - x^2$ is shown in the figure below. Calculate the area of the shaded region.



20 p **R7.** Find the unique positive value of a such that $\int_0^a \frac{x}{x^2+1}\,dx = 3$.

2 Practice Worksheets

The exercises in these worksheets are marked as belonging to one of four difficulty categories.

C: Core

An exercise categorized as "Core" (or "C") is considered fundamental part of the course. Students must be able to solve C-level exercises to succeed in calculus. If a student uses calculus in other courses, these are the exercises they will almost certainly encounter and be expected to solve. These are generally the easiest exercises in the course, but they are not all necessarily easy.

B: Beyond Core

An exercise categorized as "Beyond Core" (or "B") is typically an exercise from the Core level but with an added complexity in algebra or precalculus skills. Most of the non-Core exercises done in lecture are B-level exercises, and students should generally have a lot of exposure (via lecture, recitation, or homework) to such exercises before the quizzes and exams.

A: Advanced

An exercise categorized as "Advanced" (or "A") is one of the hardest exercises students will encounter in this course. We expect most students not to be able to fully solve these exercises, but that does not mean we expect students not to attempt these exercises. These exercises are nevertheless still a part of this course. These are the hardest examples possibly covered in lecture, but they are not always emphasized and they are seen in the homework only sparingly. Students who can solve these exercises are those students who go beyond our minimum expectations and study beyond what was seen in lecture.

The A-level exercises were designed primarily to distinguish between what we expect from B-level and A-level students. An A-level student understands the concepts at a fundamental level and ***can apply these concepts correctly to exercises they have not seen before***.

R: Removed from Syllabus

These exercises cover topics or learning goals that have been removed the syllabus since they were introduced. They remain in these worksheets as an extra challenge to students and in case these topics are ever re-introduced into the course.

2.1 Chapter 1: Review of Algebra and Precalculus

§1.1, 1.2, 1.3, 1.4, 7.2, Appendix B

Difficulty guide for this worksheet:	
C*ore or* ***B****eyond Core*:	all
A*dvanced*:	none
R*emoved from syllabus*:	none

Computation

W1. For each of the following problems, zero or more of the choices are exact answers. Identify all of the exact answers, and ***explain why the other choices are wrong***. If the exact value of the correct answer does not appear as one of the choices, find the exact value of the correct answer.

(a) Find all real numbers x such that $x^2 = 2$.

A. 1.41 **B.** $\sqrt{2}$ **C.** ± 1.41 **D.** 1.41 and -1.41 **E.** $\pm\sqrt{2}$

(b) Find all real numbers t such that $t^3 + 4 = 0$.

A. -1.59 **B.** ± 1.59 **C.** $\pm\sqrt[3]{-4}$ **D.** $-2^{2/3}$ **E.** no real solution

(c) Find the circumference of a circle whose radius is 1.

A. 6.28 **B.** ± 6.283185 **C.** $\dfrac{44}{7}$

(d) Find all real solutions to the equation $2^x = 3$.

A. 1.585 **B.** ± 1.585 **C.** 3^{-2} **D.** $\log_2(3)$ **E.** $\log_3(2)$ **F.** $\dfrac{\ln(3)}{\ln(2)}$ **G.** $\dfrac{1}{2}\log_2(9)$

Simplifying Algebraic Expressions

W2. Zero or more of the following statements are true for all real numbers a, x, and y. Determine which statements are true and determine which statements are false. For each false statement, find values of a, x, and y that make the statement false.

(a) $a(x+y) = ax + ay$

(b) $a(x+y)^2 = (ax+ay)^2$

(c) $a(x+y)^2 = ax^2 + ay^2$

(d) $a\sqrt{x+y} = \sqrt{a^2x + a^2y}$

(e) $\sin(x+y) = \sin(x) + \sin(y)$

(f) $\cos(ax) = a\cos(x)$

(g) $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$

(h) $\dfrac{a}{x+y} = \dfrac{a}{x} + \dfrac{a}{y}$

W3. Simplify each of the following expressions according to the instructions.

(a) Positive exponents and integer coefficients only (assume $x, y > 0$): $\left(\dfrac{x^8y^{-4}}{16y^{4/3}}\right)^{-1/4}$

(b) Positive exponents only (assume $a, b > 0$): $\dfrac{(9ab)^{3/2}}{(27a^3b^{-4})^{2/3}} \cdot \left(\dfrac{3a^{-2}}{4b^{1/3}}\right)^{-1}$

(c) Common factors canceled (assume $h \neq 5$): $\dfrac{2h - 10}{\sqrt{5} - \sqrt{h}}$

(d) Expand and fully simplify: $\left(\sqrt{9s^2+4} + 2\right)\left(\sqrt{9s^2+4} - 2\right)$

(e) Factor completely: $5y^2(y-3)^5 + 10y(y-3)^4$

(f) Factor completely: $3x^3 + x^2 - 12x - 4$

(g) Factor completely: $3x^{-1/2} + 4x^{1/2} + x^{3/2}$

(h) Common factors canceled, positive exponents only ($x \neq y$ and $x, y \neq 0$): $\dfrac{y^{-1} - x^{-1}}{x^{-2} - y^{-2}}$

(i) Common factors canceled ($u \neq 1$ and $u \neq -2$): $\dfrac{\dfrac{4}{u-1} - \dfrac{4}{u+2}}{\dfrac{3}{u^2+u-2} + \dfrac{3}{u+2}}$

W4. For each given function $f(x)$, fully simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$. Assume $h \neq 0$.

(a) $f(x) = 2x^2 - 2x$ (b) $f(x) = 9 - 5x$ (c) $f(x) = -4$ (d) $f(x) = \dfrac{1}{x}$

Solving Algebraic Equations and Inequalities

W5. Solve each equation or inequality. (Parts (b) – (d) are related!)

(a) $p^2 = p + 1$

(b) $2u^2 - 3u + 1 = 0$

(c) $2x^{5/2} - 3x^{3/2} + x^{1/2} = 0$

(d) $2\sin(\theta)^2 - 3\sin(\theta) + 1 = 0$

(e) $2x = x^2$

(f) $\dfrac{1-x}{1+x} + \dfrac{1+x}{1-x} = 6$

(g) $3\cos(x) + 2\sin(x)^2 = 3$

(h) $|2x + 1| = 1$

(i) $|3x - 5| = 4x$

(j) $\dfrac{x+5}{x-2} = \dfrac{5}{x+2} + \dfrac{28}{x^2-4}$

(k) $t^2 - 4t - 5 > 0$

(l) $\dfrac{x-4}{2x+1} < 0$

(m) $\dfrac{x-4}{2x+1} < 5$

Equations of Lines

W6. Find an equation of each described line.

(a) line through the point $(4, -6)$ with slope 3

(b) line through the points $(1, 2)$ and $(-3, 4)$

(c) line through the point $(5, 5)$ and perpendicular to the line described by $2x - 4y = 3$

(d) line through the point $(-1, -2)$ and parallel to the line described by $3x + 8y = 1$

(e) horizontal line through the point $(3, -1)$

(f) vertical line through the point $(2, -4)$

Functions, Domains, and Compositions

W7. If $f(x)$ and $g(x)$ are functions, then $f(g(x))$ is also a function, called the composition of f and g. We also write $f \circ g$ to mean $f(g(x))$. Similarly, $g \circ f$ means $g(f(x))$.

(a) Let $f(x) = \sin(3x) + 7$ and $g(x) = e^{2x} + 1$. Write expressions for both $f(g(x))$ and $g(f(x))$.

(b) Let $h(x) = \log_{10}(\sin(\sqrt{x}) + 1)$. Find four functions f_1, f_2, f_3, and f_4 such that $h(x) = f_4(f_3(f_2(f_1(x))))$. You may not use the function $f(x) = x$ for any of your choices.

W8. For each of the following function pairs, find a simplified formula for $f \circ g$ and $g \circ f$. Then find the domain of f, g, $f \circ g$, and $g \circ f$.

(a) $f(x) = \sin(x)$ and $g(x) = 2x + 3$

(b) $f(x) = \dfrac{2+x}{1-2x}$ and $g(x) = \dfrac{x-2}{2x+1}$

Exponential and Logarithmic Functions

W9. Find the exact value of each expression. Your final answer cannot contain "log" or "ln".

(a) $\log_2(48) - \log_2(6)$

(b) $\log_2(48) - \log_4(144)$

(c) $\ln(\log_{10}(10^e))$

(d) $3^{\log_3(4e) - \log_3(e)}$

W10. Sketch the graph of each of the following functions.

(a) $f(x) = e^{-x}$ (b) $f(x) = \log_5(x)$ (c) $f(x) = -2^x$ (d) $f(x) = \log_{1/3}(x)$

W11. Find all solutions to the following equations.

(a) $3^{x^2-x} = 9$ (b) $e^{2x+3} = 1$ (c) $\log_3(x) + \log_3(2x+1) = 1$

W12. Suppose $\log_{b^3}(5) = \frac{1}{6}$. Find the exact value of $\sqrt{b-16}$.

Trigonometric Functions

W13. Write the exact values of the sine, cosine, and tangent of each of the following angles: $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, $2\pi/3$, π, $-\pi/6$, and $-3\pi/4$. *(You should do this without any reference or calculator.)*

W14. Graph each of the following curves.

(a) $y = \sin(\theta)$

(b) $y = 3\cos(\pi\theta)$

Modeling with Equations and Functions

W15. A bank pays 6% annual interest compounded continuously. How long will it take for \$835 to triple?

W16. The number of bacteria in a certian petri dish obeys a law of exponential growth. Suppose there are initially 1000 bacteria and the number of bacteria doubles every 20 minutes. When will the number of bacteria reach 5000?

W17. A rectangular box is constructed according to the following rules.

- the length of the box is twice its width
- the height of the box is 5 feet more than three times the length

(a) If x is the width of the box in feet, write an expression for $V(x)$, the volume of the box in cubic feet as a function of its width.

(b) Suppose the rules also require that the sum of the box's width and height to be no more than 26 feet. Under this condition, what is the domain of the function $V(x)$?

W18. The total cost (in \$) of producing q units of some product is $C(q) = 30q^2 + 400q + 500$.

(a) Compute the cost of making 20 units.

(b) Compute the cost of making the 20th unit.

(c) What is the initial setup cost?

W19. The speed of blood that is a distance r from the central axis in an artery of radius R is $v(r) = C(R^2 - r^2)$, where C is some constant.

(a) What is the speed of the blood on the central axis?

(b) What is the speed halfway between the central axis and the artery wall?

Miscellaneous

W20. Simplify the expression $\dfrac{|2-x|}{x-2}$ if $x > 2$.

W21. Find all solutions to the equation $2^{x^2-2x} = 8$.

W22. Simplify the expression $2^{\log_2(3)-\log_2(5)}$.

W23. Find an equation of the line through the point $(-1, 4)$ with slope 2.

W24. Find the domain of $f(x) = \dfrac{\ln(x)}{x-2}$. Write your answer in interval notation.

W25. Solve the inequality $\dfrac{3x+6}{x(x-4)} \leq 0$. Write your answer in interval notation.

W26. An account in a certain bank pays 5% annual interest, compounded continuously. An initial deposit of \$200 is made into the account. How many years does it take for the \$200 to double? ***You must write an exact answer in terms of logarithms.***

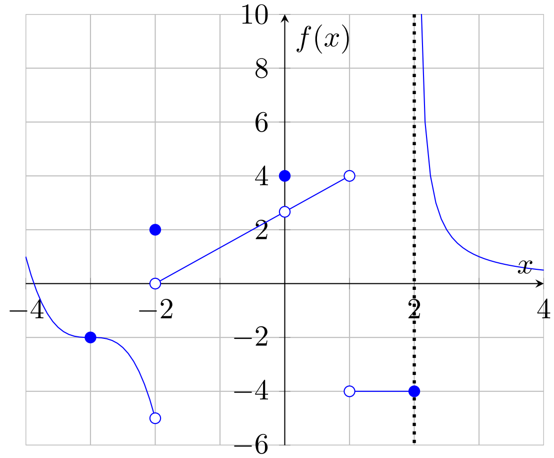
W27. A radioactive frog hops out of a pond full of nuclear waste. If its level of radioactivity declines to 1/3 of its original value in 30 days, when will its level of radioactivity reach 1/100 of its original value? *Hint:* Use the exponential growth formula $P(t) = P_0e^{rt}$.

2.2 Chapter 2: Limits

§2.1, 2.2: Introduction to Limits

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	28, 30
***A**dvanced*:	29
***R**emoved from syllabus*:	none

W28. Evaluate the following using the given graph.



(a) $\lim\limits_{x\to -3^-} f(x)$ (e) $\lim\limits_{x\to -2^-} f(x)$ (i) $\lim\limits_{x\to 0^-} f(x)$ (m) $\lim\limits_{x\to 1^-} f(x)$ (q) $\lim\limits_{x\to 2^-} f(x)$

(b) $\lim\limits_{x\to -3^+} f(x)$ (f) $\lim\limits_{x\to -2^+} f(x)$ (j) $\lim\limits_{x\to 0^+} f(x)$ (n) $\lim\limits_{x\to 1^+} f(x)$ (r) $\lim\limits_{x\to 2^+} f(x)$

(c) $\lim\limits_{x\to -3} f(x)$ (g) $\lim\limits_{x\to -2} f(x)$ (k) $\lim\limits_{x\to 0} f(x)$ (o) $\lim\limits_{x\to 1} f(x)$ (s) $\lim\limits_{x\to 2} f(x)$

(d) $f(-3)$ (h) $f(-2)$ (l) $f(0)$ (p) $f(1)$ (t) $f(2)$

W29. Suppose $\lim\limits_{x\to 0}(f(x)+g(x))$ exists. Is it true that $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to 0} g(x)$ also exist? Explain your answer.

W30. Suppose $\lim\limits_{x\to 2}\left(\dfrac{f(x)-3}{x-2}\right) = 5$ and $\lim\limits_{x\to 2} f(x)$ exists (and is equal to $f(2)$). What is the value of $f(2)$? Explain your answer.

§2.3: Techniques for Computing Limits

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	31 (all parts except j, k, l, or u)
***A**dvanced*:	31j, 31k, 31l, 31u
***R**emoved from syllabus*:	none

W31. For each of the following, evaluate the limit or explain why it does not exist. Show all work.

(a) $\lim\limits_{x\to 2}\left(\dfrac{x^2+3x-1}{x+\sin(\pi x)}\right)$

(b) $\lim\limits_{x\to 1}(x^4-9x)^{1/3}$

(c) $\lim\limits_{x\to -3}\left(\dfrac{x^2-9}{x^3+x^2-6x}\right)$

(d) $\lim\limits_{x\to 1}\left(\dfrac{\sqrt{23-7x}-4}{x-1}\right)$

(e) $\lim\limits_{h\to 0}\left(\dfrac{(x+h)^{-2}-x^{-2}}{h}\right)$

(f) $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{x^2+x}\right)$

(g) $\lim\limits_{x\to 1}\left(\dfrac{\frac{1}{x}-1}{\sqrt{x}-1}\right)$

(h) $\lim\limits_{x\to 0}|x|$

(i) $\lim\limits_{x\to 8}\left(\dfrac{|x-8|}{x-8}\right)$

(j) $\lim\limits_{x\to 8^-}\left(\dfrac{|x^2-64|}{x-8}\right)$

(k) $\lim\limits_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$

(l) $\lim\limits_{x\to 0^-}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$

(m) $\lim\limits_{x\to 0}\left(\dfrac{\sin(\pi x)}{x}\right)$

(n) $\lim\limits_{x\to 0}\left(\dfrac{\sec(x)-1}{x\sec(x)}\right)$

(o) $\lim\limits_{x\to 0}\left(\dfrac{1-\cos(x)}{\sin(x)}\right)$

(p) $\lim\limits_{x\to 2}\left(\dfrac{\sin(6-3x)}{5x-10}\right)$

(q) $\lim\limits_{x\to \pi}\left(\dfrac{\tan(x-\pi)}{x-\pi}\right)$

(r) $\lim\limits_{x\to 0}\left(\dfrac{\sin(2x)^2\cos(3x)}{\tan(5x)\sin(7x)}\right)$

(s) $\lim\limits_{x\to -1} g(x)$ where

$$g(x)=\begin{cases} 4x-5 & , \quad x<-1 \\ x^3+x & , \quad x\geq -1 \end{cases}$$

(t) $\lim\limits_{x\to 2} f(x)$ where

$$f(x)=\begin{cases} \dfrac{x^2-2x}{x-2} & , \quad x<2 \\ \sqrt{x+2} & , \quad x>2 \end{cases}$$

(u) $\lim\limits_{x\to a}\dfrac{\cos\left(\frac{\pi a}{2x}\right)}{x-a}$

§2.4: Infinite Limits

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	32 (all parts except c), 33 (all parts except f)
***A**dvanced*:	32c, 33f
***R**emoved from syllabus*:	none

W32. For each part, calculate the limit or show that it does not exist. Show all work.

(a) $\displaystyle\lim_{x\to 0^+}\left(\frac{x^2-x+4}{2x+\sin(x)}\right)$ (b) $\displaystyle\lim_{x\to 3^-}\left(\frac{2x^2+8}{x^2-9}\right)$ (c) $\displaystyle\lim_{x\to 4^+}\left(\frac{|16-x^2|}{x-4}\right)$

W33. For each function, find the vertical asymptotes and, at each vertical asymptote of f, find both corresponding one-sided limits.

(a) $f(x)=\dfrac{(x-1)(2x+5)}{(x+1)(3x-6)}$

(b) $f(x)=\dfrac{x^2-18x+81}{x^2-81}$

(c) $f(x)=\dfrac{(x-4)\sin(x)}{x^3-8x^2+16x}$

(d) $f(x)=\ln(x)$

(e) $f(x)=\dfrac{2e^x+3}{1-e^x}$

(f) $f(x)=e^{-1/x}$

§2.5: Limits at Infinity

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	34, 35 (all parts except e)
***A**dvanced*:	35e
***R**emoved from syllabus*:	none

W34. For each part, calculate the limit or show that it does not exist. Show all work.

(a) $\lim_{x\to\infty}\left(\frac{3x-5}{x+1}\right)$

(b) $\lim_{x\to-\infty}\left(\frac{3x}{\sqrt{4x^2+9}}\right)$

(c) $\lim_{x\to\infty}\left(\frac{(x-3)(2x+4)(x-5)}{(3x+1)(4x-7)(x+2)}\right)$

(d) $\lim_{x\to-\infty}\left(\frac{(x-3)(2x+4)(x-5)}{(3x+1)(4x-7)(x+2)}\right)$

(e) $\lim_{x\to\infty}\cos\left(\frac{1}{x}\right)$

(f) $\lim_{x\to\infty} e^{-x^3}$

W35. For each function, find all horizontal asymptotes.

(a) $f(x)=\frac{(x-1)(2x+5)}{(x+1)(3x-6)}$

(b) $f(x)=\ln(x)$

(c) $f(x)=\frac{2e^x+3}{1-e^x}$

(d) $f(x)=e^{-1/x}$

(e) $f(x)=\frac{2x}{x-\sqrt{x^2+10}}$

§2.6: Continuity

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	36, 37, 38, 39, 40
***A**dvanced*:	41
***R**emoved from syllabus*:	42, 43

W36. Detemine all points where the following function is continuous.
Make sure you give a justification for any x-value at which you claim f is continuous.

$$f(x) = \begin{cases} 3x^2 - x + 1 & , \quad x < -2 \\ 15 + \sin(2\pi x) & , \quad -2 \le x < 3 \\ 2x - 4 & , \quad 3 \le x \end{cases}$$

W37. Let $f(x) = \dfrac{x^3 - 9x}{x + 3}$.

(a) What is the domain of f?

(b) Find all points where f is discontinuous.

(c) For each x-value you found in part (b), determine what value should be assigned to f, if any, to guarantee that f will be continuous there.

(For example, if you claim f is discontinuous at $x = a$, then you should determine the value that should be assigned to $f(a)$, if any, to guarantee that f will be continuous at $x = a$.)

W38. Let $f(x) = \dfrac{\sqrt{2x^2 + 1} - 1}{x^2(x - 3)}$.

(a) What is the domain of f?

(b) Find all points where f is discontinuous.

(c) For each x-value you found in part (b), determine what value should be assigned to f, if any, to guarantee that f will be continuous there.

(For example, if you claim f is discontinuous at $x = a$, then you should determine the value that should be assigned to $f(a)$, if any, to guarantee that f will be continuous at $x = a$.)

W39. Find the values of the constants a and b that make f continuous for all real numbers.

$$f(x) = \begin{cases} ax^2 - x & , \quad x < 4 \\ 6 & , \quad x = 4 \\ x^3 + bx & , \quad x > 4 \end{cases}$$

W40. For what values of a and b is the following function continuous for all x?

$$g(x) = \begin{cases} ax + 2b & , \quad x \le 0 \\ x^2 + 3a - b & , \quad 0 < x \le 2 \\ 3x - 5 & , \quad x > 2 \end{cases}$$

W41. Find the values of the constants a and b that make f continuous at $x = 0$. *You may assume* $a > 0$.

$$f(x) = \begin{cases} \dfrac{1-\cos(ax)}{x^2} & , \quad x < 0 \\ 2a + b & , \quad x = 0 \\ \dfrac{x^2 - bx}{\sin(x)} & , \quad x > 0 \end{cases}$$

W42. Prove that the equation $\sqrt{x} + x^3 = 1$ has a solution in the interval $[0, 1]$.

W43. Prove that the equation $x^4 + 3x^2 + 2 = 4x^3 + 8x$ has a solution.

2.3 Chapter 3: Derivatives

§3.1, 3.2: Introduction to the Derivative

Difficulty guide for this worksheet:	
Core or Beyond Core:	44, 45 (all parts except f), 46
Advanced:	45f, 48
Removed from syllabus:	47

W44. Suppose the line described by $y = 5x - 9$ is tangent to the graph of $y = f(x)$ at $x = 4$.

(a) Calculate $f(4)$. If there is not enough information to do so, explain why.

(b) Calculate $f(3)$. If there is not enough information to do so, explain why.

(c) Calculate $f'(4)$. If there is not enough information to do so, explain why.

(d) Calculate $f'(3)$. If there is not enough information to do so, explain why.

W45. Use the limit definition of the derivative to calculate the derivative of f at $x = 5$. Then find an equation for the line tangent to the graph of $y = f(x)$ at $x = 5$.

(a) $f(x) = 2x - 1$

(b) $f(x) = (2x - 1)^2$

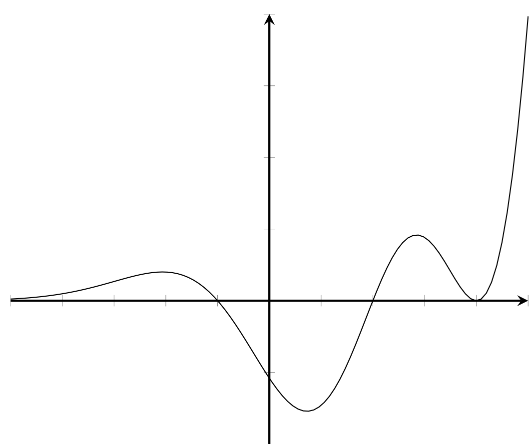
(c) $f(x) = \sqrt{2x - 1}$

(d) $f(x) = \dfrac{1}{2x - 1}$

(e) $f(x) = \dfrac{1}{\sqrt{2x - 1}}$

(f) $f(x) = \dfrac{1}{\sqrt{2x} - 1}$

W46. The graph of $y = f(x)$ is given below. Sketch a graph of $y = f'(x)$. *Only the general shape is important. Do not worry about scales.*



W47. Consider the following function.

$$f(x) = \begin{cases} -x^2 & , \quad x < 0 \\ x^2 + 2x & , \quad 0 \le x < 1 \\ 6x - x^2 + c & , \quad x \ge 1 \end{cases}$$

(a) Is f differentiable at $x = 0$?

(b) Is there a value of c that makes f differentiable at $x = 1$? If so, calculate it. If not, explain why.

W48. Use the limit definition of derivative to find the derivative of $f(x) = x^{2/3}$.

§3.3, 3.4, 3.5, 3.9: Rules for Computing Derivatives

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	49, 50, 51, 52
***A**dvanced*:	53
***R**emoved from syllabus*:	none

W49. Calculate $f'(x)$ for each function below. *After computing the derivative, do not simplify your answer.*

(a) $f(x) = \sqrt{2x} + 3x^2 + e^4$

(b) $f(x) = \dfrac{4}{x} + \ln(4)$

(c) $f(x) = \dfrac{8x^4 - 5x^{1/3} + 1}{x^2}$

(d) $f(x) = \dfrac{x^2 + 3}{x - 1}$

(e) $f(x) = x^3 e^x$

(f) $f(x) = \sqrt{x}\cos(x) - e^x \sin(x)$

(g) $f(x) = \dfrac{\tan(x) + 9x^2}{\ln(x) - 4x}$

(h) $f(x) = \dfrac{x\sin(x)}{1 - e^x \cos(x)}$

W50. Use the quotient rule to prove a derivative rule for $f(x) = \cot(x)$.

W51. Find the x-coordinate of each point on the graph of the given function where the tangent line is horizontal.

(a) $f(x) = \dfrac{1}{x^2} - \dfrac{1}{x^3}$

(b) $f(x) = (x^2 - 8)e^x$

(c) $f(x) = \dfrac{1}{\sqrt{x}}(x + 9)$

(d) $f(x) = (1 - \sin(x))\sin(x)$

W52. Find equations for two tangent lines to the graph of $f(x) = \dfrac{3x+5}{x+1}$ that are perpendicular to the line $2x - y = 1$.

W53. Find all points P on the graph of $y = 4x^2$ with the property that the tangent line at P passes through the point $(2, 0)$.

§3.7: The Chain Rule

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	54, 55 (all parts except d), 56
***A**dvanced*:	55d, 57
***R**emoved from syllabus*:	none

W54. Calculate $f'(x)$ for each function below. *After computing the derivative, do not simplify your answer.*

(a) $f(x) = \sqrt{\sin(x)}$

(b) $f(x) = \sin(\sqrt{x})$

(c) $f(x) = \sqrt{\sin(\sqrt{x})}$

(d) $f(x) = (x^3 - 3x + 2)^2$

(e) $f(x) = \dfrac{1}{(3x+1)^2}$

(f) $f(x) = (2x + \sec(x))^2$

(g) $f(x) = e^{-2x}\sin(x)$

(h) $f(x) = \dfrac{\ln(2x+1)}{(2x+1)^2}$

(i) $f(x) = (\tan(x)+1)^4\cos(2x)$

(j) $f(x) = \left(\dfrac{6}{9-2x}\right)^8$

(k) $f(x) = \left(\sin\left((4x-5)^2\right)\right)^4$

(*Many authors will write this function as* $f(x) = \sin^4(4x-5)^2$, *despite the ambiguous and inconsistent notation.*)

(l) $f(x) = \sqrt[3]{\sin(x)\cos(x)}$

(m) $f(x) = \sqrt{\dfrac{x^2-1}{x^3+x}}$

(n) $f(x) = \ln(\ln(x))$

(o) $f(x) = \sin(\sin(\sin(x)))$

(p) $f(x) = \left(x + \left(x + \sin(x)^2\right)^3\right)^4$

(q) $f(x) = |x|$

(*Hint: use the identity* $|x| = \sqrt{x^2}$.)

W55. Find the x-coordinate of each point at which the graph of $y = f(x)$ has a horizontal tangent line.

(a) $f(x) = (2x^2 - 7)^3$

(b) $f(x) = x^2 e^{1-3x}$

(c) $f(x) = \ln(3x^4 + 6x^2 - 4x^3 - 12x + 6)$

(d) $f(x) = \dfrac{(e^{3x} + e^{-3x})^2}{e^{3x}}$

W56. It is estimated that t years from now, the population (in thousands of people) of a certain suburban community is modeled by the formula

$$p(t) = 20 - \frac{6}{t+1}$$

A separate environmental study indicates that the average daily level of carbon monoxide in the air (measured in ppm) will be

$$L(p) = 0.5\sqrt{p^2 + p + 58}$$

when the population is p thousand. Find the rate at which the level of carbon monoxide will be changing with respect to time two years from now. (*Make sure to indicate units in your answer.*)

W57. Suppose g and h are differentiable functions. Selected values of g, h, and their derivatives are given below.

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
2	1	7	2	3
4	-3	-9	1	5
16	5	-1	1	-6

Define the function f by the formula

$$f(x) = g\left(\sqrt{x}\right) h\left(x^2\right)$$

(a) Calculate $f(4)$ or explain why there is not enough information to do so.

(b) Calculate $f'(4)$ or explain why there is not enough information to do so.

§3.8: Implicit Differentiation

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	58, 60, 61
***A**dvanced*:	59, 62
***R**emoved from syllabus*:	none

W58. For each of the following parts, calculate $\dfrac{dy}{dx}$.
If y is given as an explicit function of x, then the derivative must also be an explicit function of x.

(a) $x^2 + y^3 = 12$

(b) $y + \dfrac{1}{xy} = x^2$

(c) $y = \sqrt[18]{\left(x^{10}+1\right)^3\left(x^7-3\right)^8}$

(d) $y = \dfrac{e^{3x^2}}{\left(x^3+1\right)^2(4x-7)^{-2}}$

(e) $y = x^{\ln(\sqrt{x})}$

(f) $\sin(x+y) = x + \cos(y)$

(g) $\ln\left(\dfrac{x-y}{xy}\right) = \dfrac{1}{y}$

(h) $6x^2 + 3xy + 2y^2 + 17y = 6$

W59. Suppose $x^2 + y^2 = R^2$, where R is a constant. Find y'' and fully simplify your answer as much as possible.

W60. Find an equation of the line tangent to the graph of

$$xe^y = 2xy + y^3$$

at the point $\left(\dfrac{1}{e-2}, 1\right)$.

W61. Find an equation of the line tangent to the graph of

$$\sin(x-y) = xy$$

at the point $(0, \pi)$.

W62. Suppose x and y satisfy the following equation.

$$x^2 + xy + 3y^2 = 99$$

(a) Find all points on the graph where the tangent line is horizontal.

(b) Find all points on the graph where the tangent line is vertical.

§3.11: Related Rates

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	63, 64, 65, 67, 68, 69, 70
***A**dvanced*:	66, 68
***R**emoved from syllabus*:	none

W63. A rock is dropped into a lake and an expanding circular ripple results. When the radius of the ripple is 8 inches, the radius is increasing at a rate of 3 inches per second. At what rate is the area enclosed by the ripple changing at this time?

W64. An environmental study of a certain community indicates that there will be

$$Q(p) = 2p^2 + 6p + 1$$

units of a harmful pollutant in the air when the population is p thousand. The population is currently $30,000$ and is increasing at a rate of of $2,000$ per year. At what rate is the level of the air pollution increasing currently?

W65. Every day, a flight to Los Angeles flies directly over a man's home at a constant altitude of 4 miles. If we assume that the plane is flying at a constant speed of 400 miles per hour, at what rate is the angle of elevation of the man's line of sight changing with respect to time when the horizontal distance between the approaching plane and the man's location is exactly 3 miles?

W66. A person 6 feet tall stands 10 feet from point P, which is directly beneath a lantern hanging 30 feet above the ground. The lantern starts to fall, thus causing the person's shadow to lengthen. Given that the lantern falls $16t^2$ feet after t seconds, how fast will the shadow be lengthening exactly 1 second after the lantern has started to fall?

W67. The volume of a spherical balloon is increasing at constant rate of 3 in^3/s. At what rate is the radius of the balloon changing when the radius is 2 in.?

W68. At noon, a ship sails due north from a point P at 8 knots (nautical miles per hour). Another ship, sailing at 12 knots, leaves the same point 1 hour later on a course due east. How fast is the distance between the ships increasing at 2:00 PM?

W69. Recall that a baseball diamond is a square of side length 90 ft. The corners of the diamond are labeled, in anti-clockwise order, home plate, first base, second base, and third base. Player A runs from home plate to first base at a speed of 20 ft/s. How fast is the player's distance from second base changing when the player is halfway to first base?

W70. A particle moves along the elliptical path given by $x^2 + 9y^2 = 13$ in such a way that when it is at the point $(-2, 1)$, its x-coordinate is decreasing at the rate of 7 units per second. How fast is the y-coordinate changing at that instant?

2.4 Chapter 4: Applications of the Derivative

§4.1: Maxima and Minima

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	71 (all parts except f), 72
***A**dvanced*:	none
***R**emoved from syllabus*:	71f

W71. For each part, find the absolute maximum and the absolute minimum of the function f on the given interval. *You may use a scientific calculator for parts (k) and (l) only.*

(a) $f(x) = x^4 - 8x^2$ on $[-3, 3]$

(b) $f(x) = x^3 + 3x^2 - 24x - 72$ on $[-4, 4]$

(c) $f(x) = \sqrt{x}(x-5)^{1/3}$ on $[0, 6]$

(d) $f(x) = e^{-x}\sin(x)$ on $[0, 2\pi]$

(e) $f(x) = x\left(\ln(x) - 5\right)^2$ on $[e^{-4}, e^4]$

(f) $f(x) = \begin{cases} 9-4x & , \quad x < 1 \\ -x^2+6x & , \quad x \geq 1 \end{cases}$ on $[0, 4]$

(g) $f(x) = 2x^3 - 9x^2 + 12x$ on $[0, 3]$

(h) $f(x) = \dfrac{1-x}{x^2+3x}$ on $[1, 4]$

(i) $f(x) = x - 2\sin(x)$ on $[0, 2\pi]$

(j) $f(x) = (x - x^2)^{1/3}$ on $[-1, 2]$

(k) $f(x) = x^3 - 24\ln(x)$ on $[\frac{1}{2}, 3]$

(l) $f(x) = 3e^x - e^{2x}$ on $[-\frac{1}{2}, 1]$

W72. A particle moves along the x axis with position

$$x(t) = t^4 - 2t^3 - 12t^2 + 60t - 10$$

Find the particle's minimum velocity for $0 \leq t \leq 3$.

§4.3, 4.4: What Derivatives Tell Us and Graphing Functions

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	73 (all parts except g, j, k, and l), 74, 75
***A**dvanced*:	73g, 73j, 73k, 73l
***R**emoved from syllabus*:	none

This worksheet assumes knowledge of §4.7 (Lôspital's Rule).

W73. For each function, do all of the following.

- Calculate and fully simplify $f'(x)$ and $f''(x)$.
- Find all vertical asymptotes and all horizontal asymptotes.
- Find all first-order critical numbers.
- Find where the function is increasing and where the function is decreasing.
- Classify each critical value as a relative maximum, relative minimum, or neither.
- Find all second-order critical numbers.
- Find where the graph of $y = f(x)$ is concave up and where it is concave down.
- Identify any inflection points.
- Sketch the graph of $y = f(x)$.

(a) $f(x) = \frac{1}{3}x^3 - 9x + 2$

(b) $f(x) = (x+1)^2(x-5)$

(c) $f(x) = \dfrac{x}{x^2+1}$

(d) $f(x) = x - \sin(2x)$ (on $[0, \pi]$ only)

(e) $f(x) = 1 + 2x + 18x^{-1}$

(f) $f(x) = 1 - \dfrac{x}{4-x}$

(g) $f(x) = \sqrt[3]{x^3 - 48x}$

(h) $f(x) = \ln(4 - x^2)$

(i) $f(x) = 10x^3 - x^5$

(j) $f(x) = \dfrac{1}{x^3+8}$

(k) $f(x) = \dfrac{x^3}{x-1}$

(l) $f(x) = \dfrac{1}{x^3 - 3x}$

W74. Sketch the graph of a function f that satisfies all of the following conditions.

- $f'(x) > 0$ when $x < 2$ and when $2 < x < 5$
- $f'(x) < 0$ when $x > 5$
- $f'(2) = 0$
- $f''(x) < 0$ when $x < 2$ and when $4 < x < 7$
- $f''(x) > 0$ when $2 < x < 4$ and when $x > 7$

W75. Sketch the graph of a function f that satisfies all of the following conditions.

- the lines $y = 1$ and $x = 3$ are asymptotes
- f is increasing for $x < 3$ and $3 < x < 5$, and f is decreasing elsewhere
- the graph of $y = f(x)$ is concave up for $x < 3$ and for $x > 7$
- the graph of $y = f(x)$ is concave down for $3 < x < 7$
- $f(0) = f(5) = 4$ and $f(7) = 2$

§4.5: Optimization Problems

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	76, 77, 78, 79, 80, 81, 84, 85, 86, 88, 89
***A**dvanced*:	82, 83, 87
***R**emoved from syllabus*:	none

W76. The sum of two numbers is 80. Find the largest possible product.

W77. The sum of two numbers is 10. Find the smallest possible value for the sum of their squares.

W78. Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 4, assuming that one side of the rectangle lies on the diameter of the semicircle.

W79. Find the dimensions of the rectangle of largest area whose lower vertices lie on the x-axis and whose upper vertices lie on the graph of $y = e^{-x^2}$.

W80. A farmer is constructing a rectangular fence on a straight river. The side of the rectangle bordering the river does not need any fencing. If the farmer has 1000 feet of fencing, what is the largest possible area he may enclose?

W81. A farmer with 1600 feet of fencing wants to enclose a rectangular area and then divide it into four equal-area pens with fencing parallel to one side of the rectangle. What is the largest possible area that a single pen can enclose?

W82. A truck is 250 miles east of a sports car and is traveling west at a constant speed of 60 miles per hour. Meanwhile, the sports car is going north at 80 miles per hour. When will the truck and car be closest to each other? What is the minimum distance between them?

W83. Suppose we want to construct a rectangular aquarium that must hold a volume of 4000 in^3. The length of the base will be twice the width of the base. The top and bottom bases of the tank cost $\$1.50/\text{in}^2$. Each of the sides of the tank costs $\$3/\text{in}^2$. Find the dimensions (length, width, height) of the cheapest tank.

W84. The total cost of producing x widgets is

$$C(x) = x^3 + 9x^2 + 18x + 200$$

and the selling price per unit is

$$p(x) = 45 - 2x^2$$

What is the optimal price? (That is, what price maximizes total profit?)

W85. Suppose the total cost of producing x units is

$$C(x) = 2x^4 - 10x^3 - 18x^2 + x + 5$$

Find the smallest and largest values of marginal cost for $0 \le x \le 5$.

W86. Suppose the total cost of manufacturing x widgets is

$$C(x) = 3x^2 + 5x + 75$$

What level of production minimizes the average cost per unit?

W87. A tour agency is booking a tour and has 100 people signed up. The price of a ticket is $2000 per person. The agency has booked a plane seating 150 people at a cost of $125,000. Additional costs to the agency are incidental fees of $500 per person. For each $10 that the price is lowered, a new person will sign up. How much should the price be lowered for all participants to maximize the profit to the tour agency?

W88. The total cost of producing x widgets is

$$C(x) = x^3 - 6x^2 + 15x$$

and the selling price per unit is fixed at $p(x) = 6$. Show that if you want to set a level of production to maximize total profit, the best you can do is break even.

W89. The reaction of the body to a dose of medicine can sometimes be represented by an equation of the form

$$R = M^2\left(\frac{C}{2} - \frac{M}{3}\right)$$

where C is a positive constant and M is the amount of medicine absorbed in the blood. If the reaction is a change in blood pressure, R is measure in millimeters of mercury. If the reaction is a change in temperature, R is measured in degrees, and so on.

The quantity $\frac{dR}{dM}$ is called the *sensitivity* of the body to the medicine. Find the amount of medicine to which the body is most sensitive.

§4.6: Linear Approximation and Differentials

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	90, 91, 92
***A**dvanced*:	none
***R**emoved from syllabus*:	93

W90. Use a linear approximation to estimate the value of each of the following.
You must express your answer as a single exact rational number.

(a) $e^{0.1}$

(b) $\ln(1.04)$

(c) $\dfrac{1}{\sqrt[3]{25}}$

(d) $\left(\sec\left(\frac{\pi}{4} - 0.02\right)\right)^2$

(e) $\sqrt{96}$

(f) $(5.01)^3 - 2(5.01) + 3$

W91. A manufacturer's total cost (in dollars) when the level of production is q units is

$$C(q) = q^5 - 2q^3 + 3q^2 - 2$$

The current level of production is 3 units, and the manufacturer is planning to increase this to 3.01 units. Estimate how the total cost will change as a result.

W92. A manufacturer's total cost (in dollars) when the level of production is q units is

$$C(q) = 3q^2 + q + 500$$

(a) What is the exact cost of manufacturing the 41st unit?

(b) Use marginal anaysis to estimate the cost of manufacturing the 41st unit.

W93. You measure the radius of a sphere to be 6 inches, and then you use your measurement to calculate the volume of the sphere with the formula $V = \frac{4\pi}{3}r^3$. If your measurement of the radius is accurate to within 1%, approximately how accurate (to the nearest percent) is your calculation of the volume?

§4.7: L'Hôpital's Rule

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	94 (all parts except e, g, k, m, and p)
***A**dvanced*:	94e, 94g, 94k, 94m, 94p
***R**emoved from syllabus*:	none

W94. For each part, calculate the limit or show that it does not exist. Show all work.
If you use L'Hospital's Rule, you must justify its use.

(a) $\lim_{x\to 0}\left(\frac{e^{2x}-1-2x-2x^2}{x^3}\right)$

(b) $\lim_{x\to 1}\left(\frac{x^3-1}{x^4-x}\right)$

(c) $\lim_{x\to 2}\left(\frac{x^3-8}{x^4-x}\right)$

(d) $\lim_{x\to \infty}\left(\frac{x-1}{x+2}\right)$

(e) $\lim_{x\to \infty}\left(\frac{x-1}{x+2}\right)^x$

(f) $\lim_{x\to \pi/2}\left(\frac{\sec(x)}{\tan(x)}\right)$

(g) $\lim_{x\to 0^+}\left(\sin(2x)\ln(x)\right)$

(h) $\lim_{x\to 0^+}\left(x^{-4}\ln(x)\right)$

(i) $\lim_{x\to 4}\left(\frac{1}{\sqrt{x}-2}-\frac{4}{x-4}\right)$

(j) $\lim_{x\to 3}\left(\frac{\sqrt{x+1}-2}{x^3-7x-6}\right)$

(k) $\lim_{x\to \infty}\left(\sqrt{x^2+x}-x\right)$

(l) $\lim_{x\to 0}\left(\frac{1}{\sin(x)}-\frac{1}{x}\right)$

(m) $\lim_{x\to 0}\left(\cos(x)\right)^{3/x^2}$

(n) $\lim_{x\to \infty}\left(\frac{x}{\sqrt{3x^2+4}}\right)$

(o) $\lim_{x\to \infty}\left(\frac{e^x-e^{-x}}{e^x+e^{-x}}\right)$

(p) $\lim_{x\to 0}\left(1-\sin(2x)\right)^{1/\tan(3x)}$

(q) $\lim_{x\to 0}\left(\frac{x\sin(x)}{1-\cos(x)}\right)$

(r) $\lim_{x\to \pi/2}\left(\left(x-\frac{\pi}{2}\right)\tan(x)\right)$

§4.9: Antiderivatives

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	95, 96, 97
***A**dvanced*:	none
***R**emoved from syllabus*:	none

W95. Find each of the following antiderivatives.

(a) $\displaystyle\int \frac{\cos(\theta)}{4}\,d\theta$

(b) $\displaystyle\int (4-9x+x^2)\,dx$

(c) $\displaystyle\int (12e^x+\sin(x))\,dx$

(d) $\displaystyle\int (6y-y^3)^2\,dy$

(e) $\displaystyle\int (86t^7-\sqrt[3]{t})\,dt$

(f) $\displaystyle\int \frac{3t^3-6\sqrt{t}-\frac{9}{t}}{t}\,dt$

(g) $\displaystyle\int \left(1-\frac{1}{u}\right)\left(2+\frac{3}{\sqrt{u}}\right)du$

W96. The marginal revenue of a certain commodity is

$$MR(x) = -9x^2+24x+48$$

Find the price that maximizes total revenue. (Assume that $R(0)=0$.)

W97. A particle moves along the x-axis in such a way that its acceleration at time $t>0$ is

$$a(t) = 1-\frac{1}{t^2}$$

The particle's velocity at time $t=2$ is $v(2)=5.5$. What is the net distance the particle travels between the times $t=3$ and $t=6$?

2.5 Chapter 5: Integration

§5.1, 5.2: Introduction to the Integral

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	99, 100
***A**dvanced*:	none
***R**emoved from syllabus*:	98

W98. For each part, first sketch the region under the graph of $y = f(x)$ on the given interval. Then approximate the area of each region by using a Riemann sum with right endpoints and the indicated number of rectangles.

(a) $f(x) = \dfrac{1}{x+4}$ on $[0, 2]$ for $n = 4$

(b) $f(x) = \sqrt{3 + x^2}$ on $[1, 4]$ for $n = 6$

W99. Use geometry to calculate each integral.

(a) $\displaystyle\int_{-1}^{9} (27 - 3x)\, dx$

(b) $\displaystyle\int_{-2}^{4} (3x + 15)\, dx$

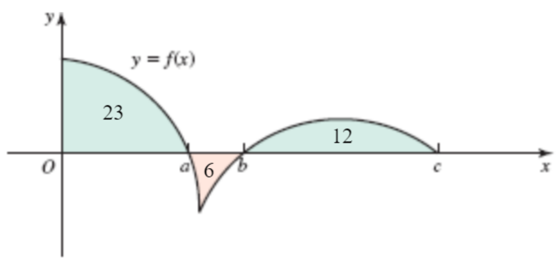
(c) $\displaystyle\int_{0}^{12} (2x - 10)\, dx$

(d) $\displaystyle\int_{-3}^{5} (|x| - 1)\, dx$

(e) $\displaystyle\int_{-4}^{0} \sqrt{16 - x^2}\, dx$

(f) $\displaystyle\int_{2}^{10} \sqrt{64 - (x - 10)^2}\, dx$

W100. Use the graph below to calculate the following integrals. Write your answer in terms of a, b, and c, if necessary. If there is not information to calculate the integral, explain why.



(a) $\displaystyle\int_{0}^{a} f(x)\, dx$

(b) $\displaystyle\int_{0}^{b} f(x)\, dx$

(c) $\displaystyle\int_{a}^{c} f(x)\, dx$

(d) $\displaystyle\int_{0}^{c} |f(x)|\, dx$

(e) $\displaystyle\int_{0}^{c} (2\,|f(x)| + 3f(x))\, dx$

(f) $\displaystyle\int_{a}^{0} f(x)\, dx$

(g) $\displaystyle\int_{c}^{a} |f(x)|\, dx$

(h) $\displaystyle\int_{0}^{c} (2f(x) + 3)\, dx$

(i) $\displaystyle\int_{0}^{a} f(x)^2\, dx$

§5.3: Fundamental Theorem of Calculus

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	101 (all parts except j), 102 (all parts except b and c), 103, 104
***A**dvanced*:	101j
***R**emoved from syllabus*:	102b, 102c

W101. Evaluate each of the following integrals.

(a) $\displaystyle\int_{-3}^{5} (-8)\,dx$

(b) $\displaystyle\int_{4}^{36} \sqrt{2x}\,dx$

(c) $\displaystyle\int_{-\ln(3)}^{\ln(8)} 5e^x\,dx$

(d) $\displaystyle\int_{0}^{9} \sqrt{x}(x^2 - x + 1)\,dx$

(e) $\displaystyle\int_{9}^{10} \frac{a}{x}\,dx$

(f) $\displaystyle\int_{-4}^{4} \sqrt{16 - x^2}\,dx$

(g) $\displaystyle\int_{-2}^{5} (2x - |x|)\,dx$

(h) $\displaystyle\int_{-\pi}^{\pi/2} \sin(x)\,dx$

(i) $\displaystyle\left|\int_{-\pi}^{\pi/2} \sin(x)\,dx\right|$

(j) $\displaystyle\int_{-\pi}^{\pi/2} |\sin(x)|\,dx$

W102. Find the derivative of each function.

(a) $\displaystyle F(x) = \int_{-3}^{x} \frac{t^4 - t^2 + 1}{\sqrt{t^6 + 1}}\,dt$

(b) $\displaystyle F(u) = \int_{u}^{0} \frac{\ln(|y| + 4)}{e^y}\,dy$

(c) $\displaystyle F(t) = \int_{1}^{t^2} \frac{\sin(x)}{x}\,dx$

(d) $\displaystyle F(x) = \int_{-\pi}^{x} \sqrt[3]{w}(w^2 - 2w + 5)\,dw$

W103. Let $f(x)$ be the function below.

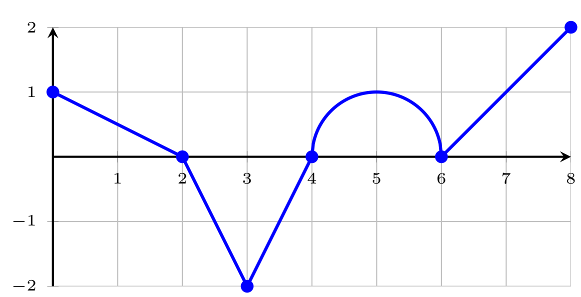
$$f(x) = \begin{cases} 4x - x^2 & , \quad x \le 2 \\ \dfrac{8}{x} & , \quad x > 2 \end{cases}$$

(a) Show that f is continuous on the interval $[-1, 4]$.

(b) Draw a sketch of a region whose area is given by the integral $\displaystyle\int_{-1}^{4} f(x)\,dx$.

(c) Evaluate the integral $\displaystyle\int_{-1}^{4} f(x)\,dx$.

W104. The graph of the function f is given below. The graph consists of line segments and a semicircle.



Define $\displaystyle g(x) = \int_{0}^{x} f(t)\,dt$.

(a) Where is g increasing?

(b) At what x-values does g have a local extremum in $(0, 8)$? Classify each as either a local maximum or a local minimum.

(c) Where is the graph of g concave down?

(d) At what x-values does g have an inflection point?

(e) Evaluate $g(8)$.

(f) Is the statement "$g(4) > g(2)$" true or false? Explain your answer?

§5.5: Substitution Rule

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	105 (all parts except d), 106 (all parts except f), 107 (all parts except b)
***A**dvanced*:	105d, 106f, 107b
***R**emoved from syllabus*:	none

W105. Find the following antiderivatives.

(a) $\displaystyle\int (5x-7)^{14}\,dx$

(b) $\displaystyle\int \frac{x^3}{\sqrt{9-x^4}}\,dx$

(c) $\displaystyle\int \cos(4-x)\,dx$

(d) $\displaystyle\int x\sqrt{2x+1}\,dx$

(e) $\displaystyle\int \frac{1}{x\ln(x)\ln(\ln(x))}\,dx$

(f) $\displaystyle\int \frac{1}{\sqrt{w}(\sqrt{w}+7)}\,dw$

W106. Calculate the following integrals.

(a) $\displaystyle\int_0^1 \frac{5x^2}{3x^3+2}\,dx$

(b) $\displaystyle\int_{\pi/4}^{\pi/3} \tan(3\theta)\,d\theta$

(c) $\displaystyle\int_0^2 (e^{3x}-e^{-3x})^2\,dx$

(d) $\displaystyle\int_0^{\ln(2)} \frac{1}{1+e^{-t}}\,dt$

(e) $\displaystyle\int_1^{e^3} \frac{\ln(x)}{x}\,dx$

(f) $\displaystyle\int_{-1}^1 \frac{2x}{2x-9}\,dx$

W107. Find the area of the region under the given curve.

(a) $y = t\sqrt{t^2+9}$ on $[0,4]$

(b) $y = x(x-3)^{1/3}$ on $[3,11]$

(c) $y = \sin(2x)^2\cos(2x)$ on $[0,\frac{\pi}{4}]$

(d) $y = \dfrac{e^{\sqrt{x}}}{\sqrt{x}}$ on $[1,9]$

2.6 Unit Review

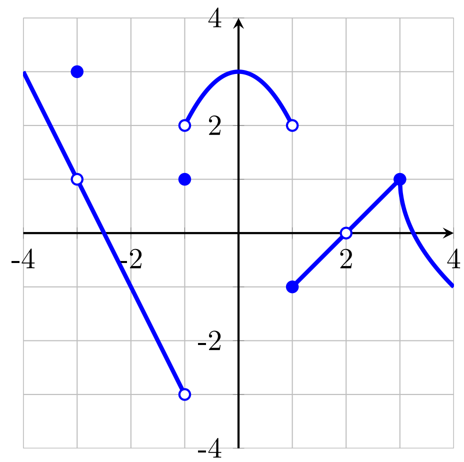
Unit #2 Review: Limits and Continuity (2.1 – 2.6, 3.5)

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	108, 109, 110, 111, 112, 113, 114
***A**dvanced*:	none
***R**emoved from syllabus*:	none

W108. For each part, calculate the limit or show that it does not exist.

(a) $\lim_{x\to 0}\left(\frac{\sin(5x)}{3x}\cos(4x)\right)$ (b) $\lim_{x\to -2}\left(\frac{x^2+3x+2}{x^2+x-2}\right)$ (c) $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{x^2+x}\right)$

W109. The graph of $f(x)$ is given below. Find all values of x in the interval $(-4,4)$ for which f is not continuous.



W110. Find the values of the constants a and b that make f continuous at $x=9$.

$$f(x)=\begin{cases}\sin(2\pi x)-2ax & , \quad x<9\\ b & , \quad x=9\\ \dfrac{x-9}{\sqrt{x}-3} & , \quad x>9\end{cases}$$

You must use proper calculus and notation to give a complete and clear justification for your answer.

W111. Find the equation of each horizontal asymptote of $f(x)=\frac{2e^x-5}{3e^x+2}$. *Write "NONE" as your answer if appropriate.*

W112. Find all vertical asymptotes of $f(x)=\frac{x^2+x-2}{x^2-4x+3}$. Justify your answer. At each vertical asymptote, also calculate the corresponding one-sided limits.

W113. For what values of a and b is the following function continuous for all x?

$$g(x)=\begin{cases}ax+2b & , \quad x\le 0\\ x^2+3a-b & , \quad 0<x\le 2\\ 3x-5 & , \quad x>2\end{cases}$$

W114. Let $f(x) = \dfrac{2e^x + 3}{1 - e^x}$.

(a) Find all horizontal asymptotes of f, if any.

(b) Find all vertical asymptotes of f. Then at each vertical asymptote, find both corresponding one-sided limits.

Unit #3 Review: Derivatives and Tangent Lines (3.1 – 3.9)

Difficulty guide for this worksheet:

***C**ore or **B**eyond Core*:	115, 116, 117, 118, 119, 120, 121, 123a, 124, 125, 126, 127
***A**dvanced*:	122, 123b
***R**emoved from syllabus*:	none

W115. For each part, calculate $f'(x)$. Do not simplify your answer after computing the derivative.

(a) $f(x) = \dfrac{\tan(x)}{\pi - \sec(x)}$

(b) $f(x) = \cos\left(e^{-3x}\right)$

(c) $f(x) = \sqrt{\ln(x^2+4) + x\sin(2x)}$

(d) $f(x) = \dfrac{e^{1/x}}{x^{2/3} + x^{1/3}}$

W116. Some values of g, h, g', and h' are given below. Use this table to answer parts (a) and (b).

x	$g(x)$	$g'(x)$	$h(x)$	$h'(x)$
0	1	7	2	3
2	−3	−9	1	5
4	5	−1	1	−6

(a) Let $f(x) = 3g(x)h(x)$. Calculate $f'(2)$.

(b) Let $F(x) = g\left(\sqrt{x}\right)$. Calculate $F'(4)$.

W117. Find an equation of the line normal to the graph of $f(x) = 2x^2 - \ln(x) + 3$ at $x = 1$. (Recall that the normal line is perpendicular to the tangent line.)

W118. Let $f(x) = 3\sqrt{x}$. Use the limit definition of the derivative to find $f'(x)$. Show all work.

W119. Find the x-coordinate of each point on the graph of $y = \dfrac{1}{\sqrt{x}}(x^3 + 15)$ where the tangent line is perpendicular to the line $x + 5y = 1$.

W120. Find the slope of the tangent line to the curve $x^3 - y^3 = y - 1$ at the point $(1, 1)$.

W121. Calculate the derivative of $f(x) = x^x$. Your final answer must contain only x.

W122. Suppose x and y satisfy the following equation.

$$x^2 + xy + 3y^2 = 99$$

(a) Find all points on the graph where the tangent line is horizontal.

(b) Find all points on the graph where the tangent line is vertical.

W123. For each part, find $f'(x)$ as a function of x only. Do not simplify your answer.

(a) $f(x) = (x^3 + x)^{10}$

(b) $f(x) = x^{\sin(2x)}$

W124. If $x^3 + xy + y^2 = 7$, find $\dfrac{dy}{dx}$ at $(1, 2)$.

W125. Let $f(x) = \dfrac{x+2}{x-3}$. Use the limit definition of derivative to find $f'(2)$.

W126. Find the derivative of each function.

(a) $f(x) = \tan(3x^2 + e)$

(b) $f(x) = e^{x/(x+1)}$

W127. Find the equation of the line normal to the curve $5x^2y + 2y^3 = 22$ at the point $(2, 1)$.

Unit #4 Review: Applications of the Derivative (3.11, 4.1, 4.3 – 4.7, 4.9)

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	all except 131b, 141, and 150
***A**dvanced*:	131b, 141, 150
***R**emoved from syllabus*:	none

W128. The total revenue from selling x units of a certain product is $R(x) = 40 - \dfrac{200}{x+5}$. Using marginal analysis, estimate the revenue from selling the 6th unit.

W129. Use a linear approximation to estimate the value of $(16.32)^{1/4}$.

W130. The surface area of a sphere is changing at a rate of 16π in^2/s when its radius is 3 in. At what rate is the volume of the sphere changing at that time?
You must include correct units as part of your answer.
Hint: If a sphere has radius R, then its surface area A and volume V are given by

$$A = 4\pi R^2 \quad , \quad V = \frac{4\pi}{3} R^3$$

W131. For each part, calculate the limit or show that it does not exist. If the limit is infinite, write "∞" or "$-\infty$" as your answer, as appropriate.

(a) $\displaystyle\lim_{x\to 3^-} \left(\frac{x^2+6}{3-x}\right)$ (b) $\displaystyle\lim_{x\to 0} (1-\sin(3x))^{1/x}$ (c) $\displaystyle\lim_{x\to -3} \left((x+3)\tan\left(\frac{\pi x}{2}\right)\right)$

W132. The position of a particle on the x-axis (measured in meters) at time t (measured in seconds) is modeled by the equation $f(t) = 100 + 8t^{3/4} - 5t$. Use a linear approximation to estimate the change in the particle's position between $t = 81$ and $t = 83$.

W133. A hot-air balloon flying at 10 ft/sec. and traveling in a straight line at a constant elevation of 400 ft passes directly over a spectator at an air show. How quickly is the angle of elevation (between the ground and the line from the spectator to balloon) changing 40 seconds later?

W134. If x units are produced, then the total cost is $C(x) = x^3 + 4x^2 + 60x + 200$ and the selling price per unit is $p(x) = 100 - 3x$. Find the level of production that maximizes the total profit.

W135. Find the absolute extreme values of $f(x) = 3x^4 - 4x^3 - 12x^2$ on $[-2, 1]$.

W136. Find the absolute extreme values of $f(x) = x^2(x+5)^3$ on $[-6, 0]$.

W137. Consider the function

$$f(x) = e^{-x^2/2}$$

Find where f is concave down and find where f is concave up. Then find all inflection points (x- and y- coordinates). *Write "NONE" for your answer if appropriate.*

W138. Consider the function

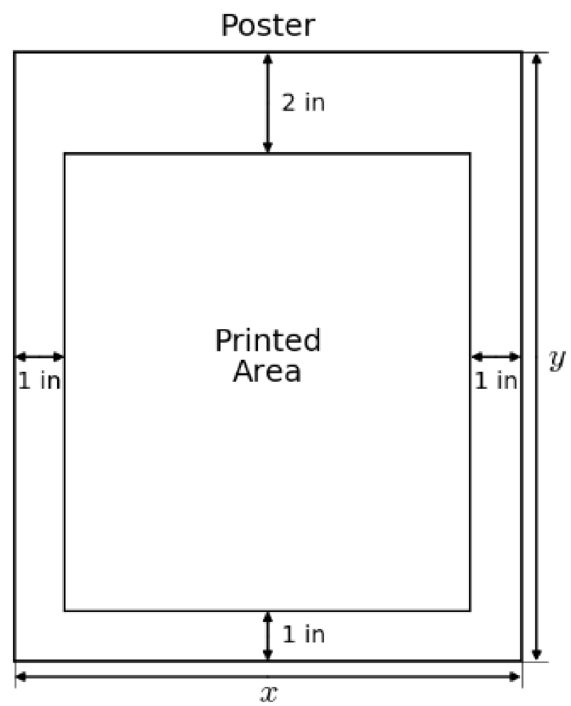
$$f(x) = \frac{1}{x^2 - 6x}$$

Find all vertical asymptotes of f. Then find where f is decreasing and find where f is increasing. Finally determine the x-coordinates of all local extrema of f (and classify them as either a local minimum or a local maximum). *Write "NONE" for your answer if appropriate.*

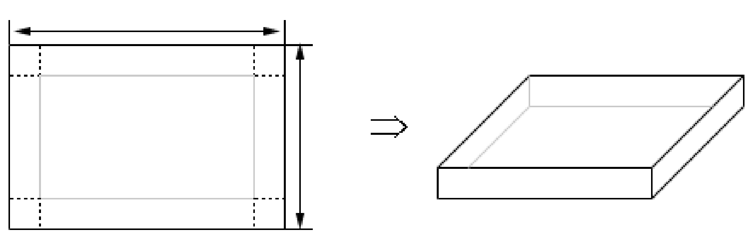
W139. Calculate each limit.

(a) $\lim_{x\to 0}\left(\dfrac{\sin(x)^2}{\sin(2x^2)}\right)$ (b) $\lim_{x\to 1}\left(\dfrac{\ln(x^2+2)-\ln(3)}{x-1}\right)$

W140. A poster is to have a total area of 150 in^2, which includes a central printed area, 1-inch margins at the bottom and sides, and a 2-inch margin at the top. What poster dimensions (in inches) will give the largest printed area? Use calculus to justify your answer.
You must demonstrate that your answers really are the optimal dimensions.



W141. A piece of cardboard that is 24 inches wide and 15 inches long is to be used to construct a box with an open top. To do this, congruent squares are cut from each corner of the cardboard, and the flaps are folded up and taped to form the sides of the box. What is the largest possible volume of such a box? Use calculus to justify your answer.
You must demonstrate that your answers really are the optimal dimensions.



W142. Find the largest possible area of a rectangle whose base lies on the x-axis and whose upper vertices lie on the parabola $y = 6 - x^2$.

W143. A car traveling north at 40 mi/hr and a truck traveling east at 30 mi/hr leave an intersection at the same time. At what rate will the distance between them be changing 4 hours later?

W144. Find the absolute minimum and maximum of $f(x) = (6x+1)e^{3x}$ on the interval $[-1000, 1000]$.

W145. The marginal revenue of a certain product is $R'(x) = -9x^2 + 17x + 30$, where x is the level of production. Assume $R(0) = 0$. Find the market price that maximizes revenue.

W146. Use linear approximation or differentials to estimate $(33.6)^{1/5}$.

W147. Consider the function f and its derivatives below.

$$f(x) = \frac{(x-1)^2}{(x+2)(x-4)} \quad , \quad f'(x) = \frac{-18(x-1)}{(x+2)^2(x-4)^2} \quad , \quad f''(x) = \frac{54((x-1)^2+3)}{(x+2)^3(x-4)^3}$$

Find the vertical and horizontal asymptotes of f. Then find where f is decreasing, where f is increasing, where f is concave down, and where f is concave up. Calculate the x-coordinates of all local minima, local maxima, and points of inflection.

W148. For each part, calculate the limit or show it does not exist.

(a) $\displaystyle\lim_{x\to 2}\left(\frac{\sqrt{x+2}-\sqrt{2x}}{x^2-2x}\right)$ (b) $\displaystyle\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}$ (c) $\displaystyle\lim_{x\to 0}\left(\frac{\sin(5x)-5x}{x^3}\right)$

W149. Find the absolute minimum and absolute maximum values of $f(x) = x^3 - 12x + 5$ on $[-5, 3]$.

W150. An open cylindrical can (without top) must have a volume of 16π cm^3. The cost of the bottom is \$2/cm^2 and the cost of the curved surface is \$1/cm^2. Find the radius and height of the least expensive can. Justify that your answer does, in fact, give the minimum cost.
Hint: The volume of a cylinder is $\pi r^2 h$. The surface area of the curved surface is $2\pi rh$, and the surface area of the top or bottom is πr^2.

W151. Use linear approximation or differentials to estimate $\sqrt[4]{78}$.

W152. The altitude of a triangle is increasing at a rate of 1 ft/min. while the area is increasing at a rate of 2 ft/min. At what rate is the base of the triangle changing when the altitude is 10 ft. and the area is 100 ft^2?

W153. Consider the function f and its derivatives below.

$$f(x) = \frac{24}{x^3+8} \quad , \quad f'(x) = \frac{-72x^2}{(x^3+8)^2} \quad , \quad f''(x) = \frac{288x(x^3-4)}{(x^3+8)^3}$$

Find the vertical and horizontal asymptotes of f. Then find where f is decreasing, where f is increasing, where f is concave down, and where f is concave up. Calculate the x-coordinates of all local minima, local maxima, and points of inflection.

Unit #5 Review: Integration (5.1 – 5.3, 5.5)

Difficulty guide for this worksheet:	
***C**ore or **B**eyond Core*:	154, 155, 156 (all parts except d), 158
***A**dvanced*:	156d
***R**emoved from syllabus*:	157

W154. Find each antiderivative or integral.

(a) $\displaystyle\int \frac{2x+\sqrt{x}-1}{x}\,dx$

(b) $\displaystyle\int (2x+3)^{12}\,dx$

(c) $\displaystyle\int_0^1 e^x(1+e^{-2x})\,dx$

(d) $\displaystyle\int_0^{\pi/2} (1+\sin(x))^5\cos(x)\,dx$

W155. The parts of this question are not related.

(a) Find $F'(x)$ if $F(x)=\displaystyle\int_{-1}^{x}\frac{t^5}{3+t^6}\,dt$.

(b) Find $\displaystyle\int_0^5 f(t)\,dt$ if $f(x)=\begin{cases} x & , \quad x<1 \\ \dfrac{1}{x} & , \quad x\geq 1 \end{cases}$.

W156. Find each antiderivative or integral.

(a) $\displaystyle\int t^2\cos(1-t^3)\,dt$

(b) $\displaystyle\int \sqrt{x-1}\,dx$

(c) $\displaystyle\int_2^3 \frac{\ln(x)}{x}\,dx$

(d) $\displaystyle\int_0^{\ln(3)} e^{2x}\sqrt{e^{2x}-1}\,dx$

W157. Estimate the area under the graph of $f(x)=x^2+5x$ from $x=0$ to $x=4$ using a Riemann sum with right endpoints and 4 rectangles. Simplify your answer.

W158. The marginal cost (in dollars) of a certain product is $C'(x)=6x^2+30x+200$. If it costs \$250 to produce 1 unit, how much does it cost to produce 10 units?

3 Quizzes

3.1 Spring 2018

X1. Find an equation of the line whose slope is -3 and which passes through the point $(1, 4)$.

X2. Simplify the expression $\dfrac{x^3 - 4x}{x^3 - x^2 - 6x}$ as much as possible.

X3. Write the expression $\dfrac{\sqrt{xy^3}}{(x^{2/3}y^{-5/2})^6}$ in the form $x^a y^b$.

X4. Let $f(x) = 3x^2$. Simplify the expression $\dfrac{f(x+h) - f(x)}{h}$ as much as possible.

X5. Evaluate the expression $\log_6(9) + \log_6(4)$.

X6. Let $f(x) = 5x + 1$. Evaluate $f^{-1}(1)$.

X7. Write the solution to the inequality $x^2 - 3x + 2 < 0$ using interval notation.

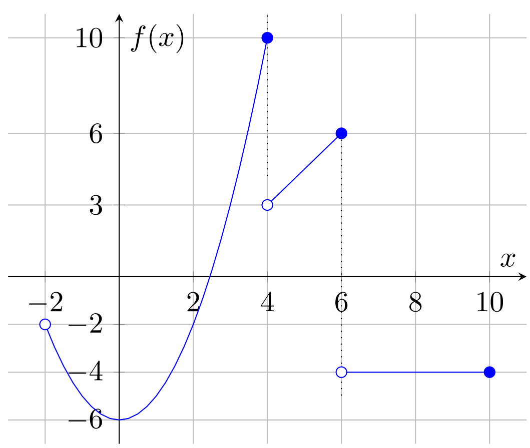
X8. Find all values of θ in the interval $[0, 2\pi)$ such that $2\sin(2\theta) = 1$.

X9. Find an equation of the line that passes through the point $(-\pi, 1)$ with slope $\sqrt{2}$.

X10. Find the center and radius of the circle described by the equation $x^2 - 6x + y^2 + 2y - 6 = 0$.

X11. Let $f(x) = \dfrac{1}{x}$. Simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ as much as possible.

X12. Evaluate the limits using the given graph.



(a) $\displaystyle\lim_{x\to -2^+} f(x) =$

(b) $\displaystyle\lim_{x\to 4^-} f(x) =$

(c) $\displaystyle\lim_{x\to 4^+} f(x) =$

(d) $\displaystyle\lim_{x\to 6} f(x) =$

X13. Evaluate each of the following limits or show why it does not exist.

(a) $\displaystyle\lim_{x\to 2} \left(\frac{2x^2 - 3x - 2}{x^2 + 2x - 8}\right)$

(b) $\displaystyle\lim_{x\to 4} \left(\frac{3 - \sqrt{x+5}}{x - 4}\right)$

X14. Consider the following function.

$$f(x) = \begin{cases} x^3 + 27 & , \quad x \leq -3 \\ \dfrac{x+3}{2-\sqrt{1-x}} & , \quad -3 < x < 1 \\ 4 & , \quad x = 1 \\ x^2 + 2x - 1 & , \quad 1 < x \end{cases}$$

(a) Find all points where f is discontinuous. *Be sure to give a full justification here.*

(b) For each x-value you found in part (a), determine what value should be assigned to f, if any, to guarantee that f will be continuous there. Justify your answer.

(For example, if you claim f is discontinuous at $x = a$, then you should determine the value that should be assigned to $f(a)$, if any, to guarantee that f will be continuous at $x = a$.)

X15. Find all real solutions to the following equation.

$$\log_2(x) + \log_2(x-3) = 2$$

X16. Let $f(x) = \dfrac{3-x}{1+x}$. Use the limit definition of derivative to calculate $f'(1)$.
If you simply quote a rule, you will receive zero credit. You must use the definition of derivative.

X17. Let $g(x) = x^2 \ln(x)$. Find an equation of the tangent line at $x = e$.

X18. At a certain factory, the total cost (in dollars) of manufacturing q tables during the daily production run is

$$C(q) = 0.2q^2 + 10q + 900$$

From experience, it has been determined that approximately

$$q(t) = t^2 + 99t$$

tables are manufactured during the first t hours of a production run.
Make sure to indicate the units of your answer in each question below.

(a) Calculate $C'(50)$ and explain its precise meaning.

(b) Compute the rate at which the total manufacturing cost is changing with respect to time one hour after production begins.

X19. Calculate $\frac{d}{dx}\left(4x^3 e^{\sin(2x)}\right)$. *After computing the derivative, do not simplify your answer.*

X20. At a certain factory, the total cost (in dollars) of manufacturing q tables during the daily production run is

$$C(q) = 0.2q^2 + 10q + 900$$

From experience, it has been determined that approximately

$$q(t) = t^2 + 99t$$

tables are manufactured during the first t hours of a production run.
Make sure to indicate the units of your answer in each question below.

(a) Calculate $C'(50)$ and explain its precise meaning.

(b) Compute the rate at which the total manufacturing cost is changing with respect to time one hour after production begins.

X21. Calculate $\frac{d}{dx}\left(4x^3e^{\sin(2x)}\right)$. *After computing the derivative, do not simplify your answer.*

X22. Use a linear approximation to estimate the value of $\dfrac{1}{\sqrt[4]{0.96}}$.
You must express your answer as a single exact rational number.

X23. Find the absolute maximum and absolute minimum values of $f(x) = \dfrac{10x}{x^2+1}$ on the interval $[0, 2]$.

X24. The function f and its derivatives are given below.

$$f(x) = \frac{x}{x^2+1} \quad , \quad f'(x) = \frac{1-x^2}{(x^2+1)^2} \quad , \quad f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}$$

(a) Fill in the table below with information about the graph of $y = f(x)$. Write "NONE" as your answer if appropriate.

domain of $f(x)$	
vertical asymptotes	
horizontal asymptotes	
intervals where f is decreasing	
intervals where f is increasing	
x- and y-coordinates of local minima	
x- and y-coordinates of local maxima	
intervals where graph is concave down	
intervals where graph is concave up	
points of inflection	

(b) Sketch the graph of $y = f(x)$ on the provided graph paper. Make sure to label the scales on the axes! For each relative extremum or inflection point, identify its coordinates and label the point "rel. min", "rel. max", or "infl. pt." as appropriate.

X25. Calculate the following limit or show it does not exist. Show all work.

$$\lim_{x\to 0}\left(\frac{x - \ln(1+x)}{1-\cos(2x)}\right)$$

X26. The product of two positive numbers is 25. Find the smallest value of their sum.

X27. An apartment complex has 200 units. When the monthly rent for each unit is \$1200, all units are occupied. Experience indicates that for each \$40-increase in rent, 10 units will become vacant. Each rented apartment costs the owners of the complex \$480 per month to maintain. What monthly rent should be charged to maximize the owner's profit?

X28. Calculate the following antiderivatives.

(a) $\displaystyle\int (\cos(w) + 2\sin(w) - 3e^{w})\, dw$

(b) $\displaystyle\int \frac{3t^3 - \sqrt[3]{t} + 2t}{t^2}\, dt$

X29. Let $f(x) = x^2 + 3x$ and let R be the region under the graph of $y = f(x)$ and above the interval $[0, 2]$ on the x-axis.

(a) Sketch the region R.

(b) Estimate the area of R using a Riemann sum with right endpoints and 4 rectangles.

Do not simplify your answer.

(c) Calculate the exact area of R.

Simplify your answer as much as possible.

3.2 Spring 2020

X30. Evaluate each of the following limits or show why it does not exist.

(a) $\lim\limits_{x\to 1}\left(\dfrac{\sqrt{7x+9}-4}{x-1}\right)$

(b) $\lim\limits_{x\to 5}\left(\dfrac{\frac{1}{5}-\frac{1}{x}}{\frac{x}{5}-\frac{5}{x}}\right)$

X31. Evaluate $\lim\limits_{x\to 3}\left(\dfrac{3-\sqrt{12-x}}{x-3}\right)$ or show that the limit does not exist.

X32. Find the values of a and b for which f is continuous for all x, or show that no such values of a and b exist. You must use proper calculus methods and clearly explain your work using limits.

$$f(x)=\begin{cases} ax^2-bx-6 & , \quad x<3 \\ b & , \quad x=3 \\ 10x-x^3 & , \quad x>3 \end{cases}$$

X33. Let $f(x)=x^{-1}-3x^{-2}$. Use the limit definition of the derivative to calculate $f'(1)$. *(If you simply use shortcut rules, you will receive no credit.)*

X34. Let $f(x)=\dfrac{50e^x}{x^2+1}$. Find an equation of the line tangent to the graph of $y=f(x)$ at $x=3$.

X35. For each part, find $f'(x)$. After computing the derivative, do not simplify your answer.

(a) $f(x)=\sqrt{\tan(x^3)}$

(b) $f(x)=x^{3/4}\ln\left(\sin(x)+x+e^3\right)$

X36. Let $f(x)=x^{12}e^{5-3x}$. Find the x-coordinate of each point at which the graph of $y=f(x)$ has a horizontal tangent line.

X37. Use a linear approximation to estimate the value of $(2.01)^5-5\cdot(2.01)^3+9$. Write your answer as an exact decimal or as a fraction of integers.

X38. The total cost of producing x units is $C(x)=10x^3+500x^2+1000x+24000$.

(a) Write a numerical expression equal to the exact cost of the 11th unit.

(b) Use marginal analysis to estimate the cost of the 11th unit. Write your answer as an exact decimal or as a fraction of integers.

X39. Find an equation of the line tangent to the graph of $xe^y=x^3+(y-1)^2-1$ at the point $(0,2)$.

X40. The image of a certain rectangle of area 30 cm^2 is changing in such a way that its length is decreasing at a rate of 2 cm/sec. and its area remains constant. At what rate is its width changing when its length is 6 cm?

X41. Let $f(x)=3x^2-5$. Use the limit definition of derivative to find $f'(x)$.

X42. For each part, calculate the limit or show that it does not exist. If the limit is infinite, your answer should be "$+\infty$" or "$-\infty$".

(a) $\lim\limits_{x\to e}\left(\dfrac{1-\ln(x)}{x^2\ln(x)-e^2}\right)$

(b) $\lim\limits_{x\to 2^+}\left(\dfrac{\cos(\pi x)}{x^2-6x+8}\right)$

X43. Determine where $f(x)$ is continuous. Write your answer using interval notation.

$$f(x) = \begin{cases} 4x^2 - 10 & , \quad x < -1 \\ 6\sin(\pi x/2) & , \quad -1 \le x \le 4 \\ x - 4^{x-3} & . \quad x > 4 \end{cases}$$

X44. Use a linear approximation to estimate $\sqrt{14}$. Write your answer as an exact decimal or as a fraction of integers.

3.3 Summer 2022

X45. Find all solutions to the given equation.

$$2x^{5/2} + x^{3/2} + x^{1/2} = 0$$

X46. Simplify the expression; assume $x \neq -10$.

$$\frac{x^3 + 10x^2}{\sqrt{15 - x} - 5}$$

X47. Simplify the expression; assume any common factors are non-zero.

$$\frac{\frac{x-1}{x+1} + \frac{6}{x}}{\frac{2}{x^2+x} + \frac{1}{x+1}}$$

X48. Calculate the following limit or show that it does not exist.

$$\lim_{x\to 9}\left(\frac{x^3 - 81x}{(x-4)^2 - 25}\right)$$

X49. Calculate the following limit or show that it does not exist.

$$\lim_{x\to 1}\left(\frac{\sqrt{x+3} - 2}{x - 1}\right)$$

X50. Calculate $\lim_{x\to 0} f(x)$ or show the limit does not exist, where $f(x)$ is the function given below. Your work must be coherent and clearly explain your answer.

$$f(x) = \begin{cases} 10e^x & x < 0 \\ 7 & x = 0 \\ \dfrac{\sin(10x)}{x} & x > 0 \end{cases}$$

X51. Calculate all of the vertical and horizontal asymptotes of $f(x) = \dfrac{x^2 - 100}{10x - x^2}$.

Then find the two one-sided at $x = a$, where $x = a$ is the leftmost vertical asymptote of f.

X52. If $f(x)$ is not defined at $x = a$, then which of the following must be true?

(a) $\lim_{x\to a} f(x)$ cannot exist

(b) $\lim_{x\to a^+} f(x)$ must be infinite (either $+\infty$ or $-\infty$)

(c) $\lim_{x\to a} f(x)$ could be 0

(d) none of the above

X53. If $\lim_{x\to a^-} f(x) = \lim_{x\to a^-} g(x) = 0$, then which of the following is true about $\lim_{x\to a^-} \dfrac{f(x)}{g(x)}$?

(a) The limit does not exist, and is not infinite.
(b) The limit is infinite (either $+\infty$ or $-\infty$).
(c) The limit must exist.
(d) There is not enough information to say anything about the limit's value.

X54. Calculate the limit below.

$$\lim_{x\to -\infty} \left(\frac{2 - 3e^{x} + 4e^{-x}}{5 + 7e^{x} - 15e^{-x}} \right)$$

X55. Consider the function $f(x)$ below, where a and b are unspecified constants.

$$f(x) = \begin{cases} ax^2 - 7x + b & x < 2 \\ 10 & x = 2 \\ ae^{x-2} + b\ln(x-1) & x > 2 \end{cases}$$

Find the values of a and b for which f is continuous for all x, or determine that no such values exist. Write "NONE" in the answer boxes if no such values exist.

In your work, you must use proper notation and limit-based methods to solve this problem. Solutions that have work that does not have proper notation or which is not based on limits will not receive full credit.

X56. Let $f(x) = \dfrac{x^3 - 7x^2 + 10x}{x^2 - 6x}$.

(a) Find the domain of f. Write your answer using interval notation.
(b) Find all values of x where f is discontinuous.
(c) For each value of x where f is discontinuous, classify the type of discontinuity as "removable", "jump", "infinite", or "essential". Clearly label your work and justify your answers.

X57. Let $f(x) = \dfrac{x^2 - 3}{x - 1}$. Use the limit definition of derivative to calculate $f'(2)$.

X58. Use the limit definition of derivative to find an equation of the tangent line to $f(x) = 2x^2 + x + 5$ at $x = -1$.

X59. Find the x-coordinate of each point on the graph of $y = x^3 e^{-5x}$ where the tangent line is horizontal.

X60. Calculate each derivative below. Do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\dfrac{x\sin(x)}{\pi^3 + \ln(x)} \right)$

(b) $\dfrac{d}{dx}\left(\left(\sqrt{5x-8} + x^2\right)^{1/3} \right)$

X61. Suppose x and y are implicitly related by the following equation.

$$5 + xy^2 = \frac{y}{2 - x^3}$$

Find $\dfrac{dy}{dx}$ for a general point on the curve.

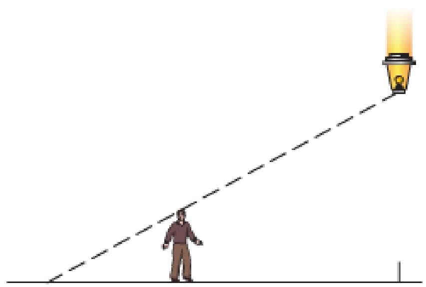
X62. Suppose x and y are implicitly related by the following equation.

$$6x^2 - 3xy + 2y^2 = 52$$

Find all points (both x- and y-coordinates) on the curve where the tangent line is horizontal.

X63. The total surface area of a cube is changing at a rate of 8 in^2/s when the length of one of the sides is 20 in. At what rate is the volume of the cube changing at that time? *You must include correct units as part of your answer.*

X64. A 2-meter tall person is initially standing 4 meters from point P directly beneath a lantern hanging 14 meters above the ground, as shown in the figure below. The person then begins to walk towards point P at 1.5 m/sec. Let x denote the distance between the person's feet and the point P. Let y denote the length of the person's shadow.



(a) Write an equation that relates x and y.

(b) Write an equation that expresses the English sentence *"The person begins to walk towards point P at 1.5 m/sec."*

(c) At what rate is the length of the person's shadow changing when the person is 3 meters from point P? *You must include correct units as part of your answer.*

X65. Find the absolute extrema of $f(x) = 10 + 8x^2 - x^4$ on the interval $[-1, 3]$.

X66. Let $f(x) = x^2(5x + 9)^{1/5}$. Observe that the domain of f is $(-\infty, \infty)$. Calculate the critical numbers of f. For each critical number you find, explain precisely why your answer is a critical number.

X67. Consider the function f and its derivatives below.

$$f(x) = x^2 - \frac{27}{x} \qquad f'(x) = 2x + \frac{27}{x^2} \qquad f''(x) = 2 - \frac{54}{x^3}$$

Fill in the table below with information about the graph of $y = f(x)$. For each part, write "NONE" as your answer if appropriate. (You may use the bottom or back of this page for scratch work.) ***You do not have to show work, and each part of the table will be graded with no partial credit.***

vertical asymptote(s) of f	
horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

X68. Calculate the limit or show that it does not exist. If the limit is infinite, write "$+\infty$" or "$-\infty$" as your answer, instead of "does not exist", as appropriate.

$$\lim_{x\to 0}\left(\frac{xe^{-2x}+\cos(x)-1-x}{x^2}\right)$$

X69. An airline policy states that all carry-on baggage must be box-shaped with a sum of length, width, and height not exceeding 60 in. Suppose the length of a particular carry-on is three times its width. Under the airline's policy, what are the dimensions of such a carry-on with the greatest volume?

You must use calculus-based methods to solve this problem, and you must demonstrate that your answer really does give the greatest volume.

X70. Use linear approximation to estimate the number $\dfrac{1}{(2.9)^2}$. Do not simplify your answer.

X71. The position of a particle at time t is given by $x(t) = 5 + 20t^{3/5} + t$. Use linear approximation to estimate the change in the particle's position between $t = 32$ and $t = 35$. Do not simplify your answer.

X72. If x units of a certain product are being produced, the marginal cost is

$$\frac{dC}{dx} = 5 + 12x + 20x^{3/2}$$

Suppose the total cost of producing 1 unit is 100 (measured in thousands of dollars). Calculate the total cost of producing 4 units.

X73. Calculate each of the following. You do not have to simplify your answers.

(a) $\displaystyle\int\left(\frac{3t^2-\sqrt{t}+4}{5t}\right)\,dt$

(b) $\displaystyle\int_{-1}^{3}\left(3x^2+2e^x\right)\,dx$

3.4 Fall 2022

X74. Find all solutions to the given equation.

$$\log_2(x-3)+2=\log_2(x+9)$$

X75. Fully simplify the given expression. Assume any common factors are non-zero.

$$\frac{100}{x^2-25}-\frac{2x}{x+5}$$

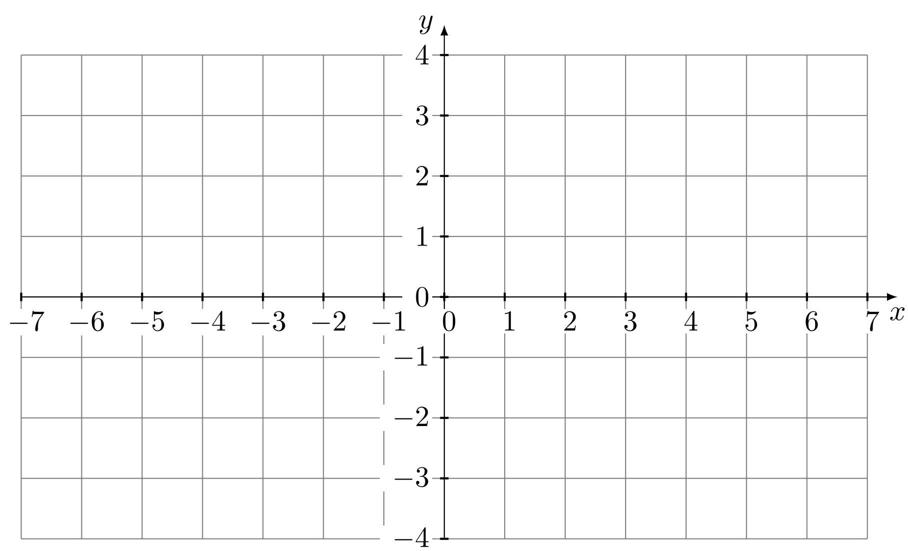
X76. For each part, calculate the limit or determine it does not exist. You must show all work, and your work will be graded on its correctness and coherence.

(a) $\lim\limits_{x\to 6}\left(\dfrac{x^2-36}{2x^2-11x-6}\right)$

(b) $\lim\limits_{x\to 2}\left(\dfrac{\frac{3x+1}{x-1}-7}{x-2}\right)$

X77. On the axes below, sketch the graph of a function $f(x)$ that satisfies the following properties:

- the domain of $f(x)$ is $[-7,4)\cup(4,7]$
- $\lim\limits_{x\to -5} f(x)\neq f(-5)$
- $\lim\limits_{x\to -3^-} f(x)=f(-3)$ but $\lim\limits_{x\to -3} f(x)$ does not exist
- $\lim\limits_{x\to 2} f(x)=f(2)=4$
- $\lim\limits_{x\to 4^+} f(x)=2$ but $\lim\limits_{x\to 4} f(x)$ does not exist



X78. Use rationalization to simplify the expression below. All common factors must be canceled.

$$\frac{3-\sqrt{2-x}}{x+7}$$

X79. Find all vertical asymptotes of $f(x)$. You must justify your answers precisely.

$$f(x) = \frac{\sin(2x)}{x^2 - 10x}$$

X80. Find all horizontal asymptotes of $g(x)$. You must justify your answers precisely.

$$g(x) = \frac{3e^{-2x} + 4e^{5x} - 10}{6e^{-9x} - 7e^{8x} + 1}$$

X81. Find the value of A that makes $f(x)$ continuous for all x, or determine that no such value exists. Write "DNE" if no such value of A exists. Your solution must be based on limits to receive full credit.

$$f(x) = \begin{cases} \dfrac{\sin(Ax)}{x} - 2 & x < 0 \\ 9 & x = 0 \\ 3x^3 - A\cos(x) + 10 & x > 0 \end{cases}$$

X82. Calculate $\lim\limits_{x \to -3} \left(\dfrac{\sqrt{2x+15} - 3}{x^2 + 8x + 15} \right)$ or determine that it does not exist. If the limit is infinite, write "$+\infty$" or "$-\infty$" as your answer, as appropriate, instead of "DNE".

X83. The limit below is equal to the derivative of some function $f(x)$ at some point $x = a$. Identify both the function f and the value of a. No work is required.

$$f'(a) = \lim_{h \to 0} \left(\frac{\frac{1}{(3+h)^2+1} - \frac{1}{10}}{h} \right)$$

X84. Let $f(x) = 2x^2 - 6x + 10$.

(a) Use the limit definition of derivative to calculate $f'(-1)$.

(b) Find the tangent line to $y = f(x)$ at $x = -1$.

X85. For each part, calculate the derivative. You do not have to show work and there is no partial credit.

(a) $\dfrac{d}{dx}\left(\cos(x) - \dfrac{5}{x^7}\right)$

(b) $\dfrac{d}{dx}\left(8\sin(x)\ln(x)\right)$

(c) $\dfrac{d}{dx}\left(\dfrac{2x^4}{10 - 3x}\right)$

X86. For each part, calculate the derivative. Do not simplify your answer.

(a) $\dfrac{d}{dx}\left(\sqrt[5]{4\sin(x) + e^{3x-7}}\right)$

(b) $\dfrac{d}{dx}\left(\dfrac{2x^4\tan(x)}{3x + 10}\right)$

X87. Find the x-coordinate of each point on the graph of $y = 3x^2 + \frac{60}{x}$ where the tangent line is horizontal.

X88. Find the x-coordinate of each point on the graph $y = \left(x^2 + x - 1\right) e^{3x}$ where the tangent line is horizontal.

X89. Find $\frac{dy}{dx}$ for a general point on the following curve.

$$x \sin(y) + 10 = \ln(y^2 + x)$$

X90. Find the slope of the line tangent to the given curve at the point $(1, \frac{1}{4})$.

$$x \tan(\pi y) = 16y^2 + 3\ln(x)$$

X91. A pebble is dropped into a lake and an expanding circular ripple results. When the radius of the ripple is 8 inches, the area enclosed by the ripple is changing at a rate of 48π in^2/sec. What is the rate at which the radius is changing at this time? *You must include correct units as part of your answer.*

X92. Let $f(x) = 3x^{4/3} - 300x^{1/3}$. Find all critical points of f. You must make clear why each of your answers is a critical point.

X93. Find the absolute extrema of $f(x) = \sqrt{2}\sin(x) + \cos^2(x)$ on the interval $[0, \pi]$. *Hint:* You will need the approximation $\sqrt{2} \approx 1.4$.

X94. Compute $\lim_{x \to 0} \left(\frac{e^{-5x} - 1}{\ln(1 + 13x)} \right)$. If the limit is infinite, write "$+\infty$" or "$-\infty$" instead of "DNE".

X95. Consider the function f and its derivatives below.

$$f(x) = \frac{x^4}{3 - x} \qquad f'(x) = \frac{x^3(12 - 3x)}{(3 - x)^2} \qquad f''(x) = \frac{6x^2((x - 4)^2 + 2)}{(3 - x)^3}$$

Fill in the table below with information about the graph of $y = f(x)$. Write your answers using interval notation if appropriate. For each part, write "NONE" as your answer if appropriate.

vertical asymptote(s) of f	
horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

X96. Calculate the limit below or determine it does not exist.

X97. A rectangle is constructed with its lower two vertices on the x-axis and its upper two vertices on the parabola $y = 75 - 3x^2$. Find the dimensions of the rectangle with the greatest area.

In your work, you must clearly define your variables, identify any constraint equations, and identify your objective function (in terms of one variable). You must also verify that your answer really does give a maximum.

X98. Use linear approximation to estimate the value below. Do not simplify your answer.

X99. The total number of rabbits in a certain region t weeks after observations have begun is modeled by the equation $N(t) = 200 + 36t^{2/3}$. Use a linear approximation to estimate the increase in the rabbit population between $t = 64$ and $t = 67$.

X100. When x units of a product are produced, the derivative of the total cost C (measured in \$) is:

$$\frac{dC}{dx} = 3x^2 + 40x + 100$$

Suppose the total cost of producing 1 unit is \$150. Find the total cost of producing the first 2 units.

X101. Let $f(x) = 12 - 3x$. Calculate each of the following integrals using geometry. If you use the Fundamental Theorem of Calculus, you will receive no credit.

(a) $\displaystyle\int_0^5 f(x)\,dx$

(b) $\displaystyle\int_0^5 |f(x)|\,dx$

X102. Find the area of the region bounded by the graph of $y = (x^4 + 1)^2$, the x-axis, and the lines $x = 0$ and $x = 1$.

X103. Calculate each of the following integrals using any valid method taught in this course. You may need to use basic geometry, the Fundamental Theorem of Calculus, substitution rule, or some combination.

(a) $\displaystyle\int_{-5}^0 \sqrt{25 - x^2}\,dx$

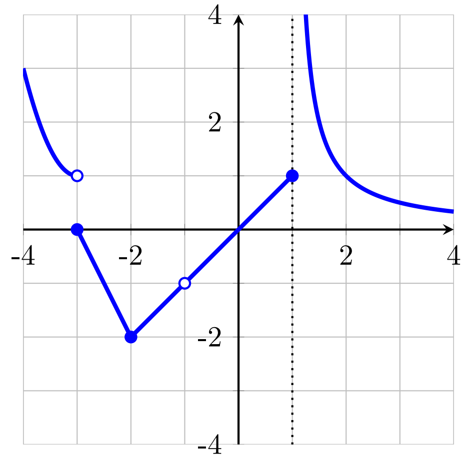
(b) $\displaystyle\int_0^1 6x^2(x^3 + 26)^{1/2}\,dx$

(c) $\displaystyle\int_{-\ln(5)}^{\ln(6)} (2e^x + 3)\,dx$

4 Final Exams

4.1 Spring 2020

3 p **Z1.** Find all values of a in $(-4, 4)$ such that $\lim_{x \to a} f(x)$ does not exist, where the graph of $y = f(x)$ is given below.



3 p **Z2.** Which statement is true about the graph of $f(x) = |x| + 91$ at the point $(0, 91)$?

(a) The graph has a tangent line at $y = 91$.
(b) The graph has infinitely many tangent lines.
(c) The graph has no tangent line.
(d) The graph has two tangent lines: $y = x + 91$ and $y = -x + 91$.
(e) None of the above statements is true.

3 p **Z3.** Suppose the cost (in dollars) of manufacturing q units is given by

$$C(q) = 6q^2 + 34q + 112$$

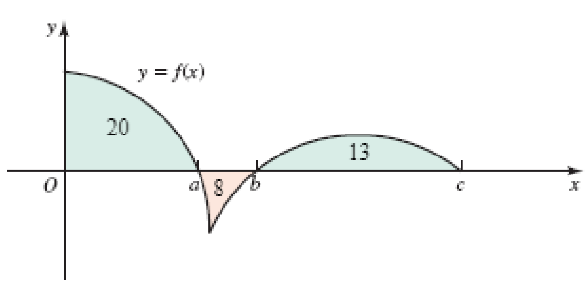
Use marginal analysis to estimate the cost of producing the 5th unit.

3 p **Z4.** Consider the function $f(x)$, where k is an unspecified constant. Find the value of k for which f continuous for all x, or show that no such value of k exists.

$$f(x) = \begin{cases} 38 + kx & x < 3 \\ kx^2 + x - k & x \geq 3 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

3 p **Z5.** The figure below shows the area of regions bounded by the graph of $y = f(x)$ and the x-axis, where $a = 4$, $b = 6$, and $c = 15$. Evaluate $\int_a^c (11f(x) - 6)\, dx$.



13 p

Z6. Consider the function f and its first two derivatives below.

$$f(x) = \frac{99e^x}{(x-25)^{47}} + 98 \quad , \quad f'(x) = \frac{99e^x(x-72)}{(x-25)^{48}} \quad , \quad f''(x) = \frac{99e^x\left((x-72)^2+47\right)}{(x-25)^{49}}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

equation(s) of vertical asymptote(s) of f	
equation(s) of horizontal asymptote(s) of f	
where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

6 p

Z7. A student is asked to calculate the following limit using l'Hospital's Rule and to show all their work.

$$L = \lim_{x\to 0}\left(\frac{\sin(2x)+17x^2+2x}{4x^2+\tan(x)}\right)$$

The student decides to cheat, so they find the solution online (shown below) and they submit the work as their own!

$$\begin{aligned}
L &= \lim_{x\to 0}\left(\frac{\sin(2x)+17x^2+2x}{4x^2+\tan(x)}\right) && (1)\\
&= \lim_{x\to 0}\left(\frac{2\cos(2x)+34x+2}{8x+\sec(x)^2}\right) && (2)\\
&= \lim_{x\to 0}\left(\frac{-4\sin(2x)+34}{8+2\sec(x)^2\tan(x)}\right) && (3)\\
&= \frac{-4\sin(0)+34}{8+2\sec(0)^2\tan(0)} && (4)\\
&= \frac{0+34}{8+0} && (5)\\
&= \frac{17}{4} && (6)
\end{aligned}$$

Unfortunately, this solution contains an error, and so the student lost all credit for the problem. The student was also later determined to be responsible for cheating, and so they earned a grade of 0 on the entire exam!

Your task is to find and correct the error(s). Answer the following questions.

(a) There may be several errors in this solution. Which line is the first incorrect line?

(b) Explain the error in the first incorrect line in your own words.

(c) Calculate the correct value of L (the original limit).

6 p **Z8.** Consider the integral below.

$$\int_{-2}^{1} \sqrt{9-(x-1)^2}\,dx$$

(a) Explain in your own words how you would calculate this integral without using Riemann sums or the fundamental theorem of calculus. ***Hint:*** Try graphing the integrand!

(b) Find the exact value of the integral.

6 p **Z9.** Consider the curve described by the following equation.

$$e^{12x+2y} = 6y - 3xy + 1$$

(a) Find $\dfrac{dy}{dx}$ at a general point on this curve.

(b) Calculate the slope of the line tangent to the curve at $(2, -12)$.

(c) There is a point on the curve close to the origin with coordinates $(0.07, b)$, and the line tangent to the curve at the origin is $y = 3x$. Use linear approximation to estimate the value of b.

6 p **Z10.** Suppose the derivative of f is $f'(x) = 3x^2 - 6x - 9$ and that $f(1) = 10$.

(a) Find an equation of the line tangent to the graph of $y = f(x)$ at $x = 1$.

(b) Find the critical points of f.

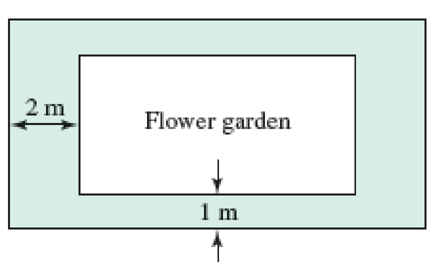
(c) Where does f have a local minimum value? local maximum value?

(d) Calculate $f(0)$.

(e) Calculate the absolute maximum value of f on the interval $[0, 6]$. At what x-value does it occur?

12 p **Z11.** A local park has hired you to construct a rectangular flower garden surrounded by a grass border that is 1 m wide on two sides and 2 m wide on the other two sides. (See the figure blow.) The area of the garden only (the small rectangle) must be 126 m^2.

Your primary task is to find the dimensions of the garden that give the smallest possible combined area of the garden and the grass border. For this problem, let W be the horizontal width of the garden and let H be the vertical height of the garden.



(a) What is the objective function for this problem in terms of W and H?

(b) What is the constraint equation for this problem in terms of W and H?

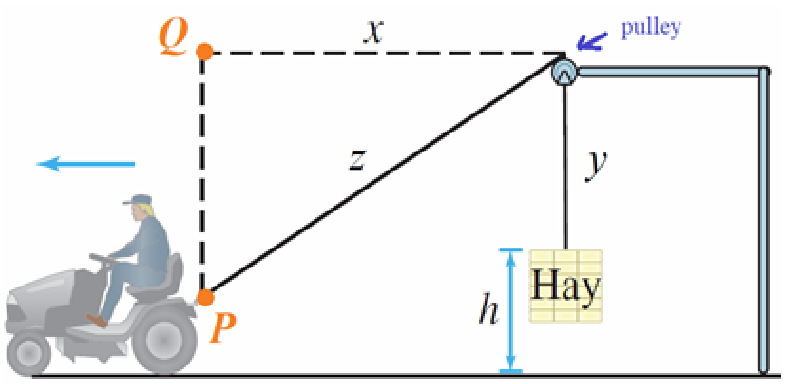
(c) Find the objective function in terms of W only.

(d) What is the interval of interest for the objective function?

(e) Find the values of W and H that minimize the total combined area.

(f) What horizontal width W of the garden will *maximize* the total area?

12 p **Z12.** A farmer's tractor pulls a rope of length 12 m attached to a bale of hay through a pulley is 8 m above the ground. The vertical distance between the tractor and the pulley (the distance from P to Q) is 7 m. The tractor is moving to the left at rate of 2 m/sec, which causes the bale of hay to rise off the ground.



(a) The rate of change (with respect to time) of which variable is equal to the speed of the tractor?

(b) Use the Pythagorean theorem to find an equation that holds for all time and involves only the variables x and z.

(c) Use the fact that the length of the rope is constant to find an equation that holds for all time and involves only the variables z and y.

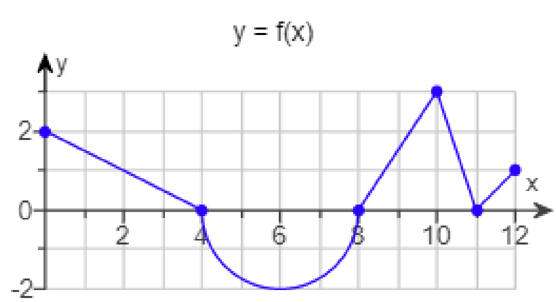
(d) Use the fact that the height of the pulley is constant to find an equation that holds for all time and involves only the variables h and y.

(e) Combine the equations from parts (b), (c), and (d) to find an equation that holds for all time and involves only the variables x and h.

(f) The rate of change (with respect to time) of which variable is equal to the rate at which the bale of hay is rising?

(g) Find the rate at which the bale of hay is rising off the ground when the horizontal distance between the tractor and the bale of hay is 8 m.

12 p **Z13.** Define the function g by $g(x) = \int_0^x f(t)\,dt$, where the graph of $y = f(x)$ is given below. The graph consists of four line segments and one semicricle. ***Note:*** f and g are different functions!



(a) Calculate $f'(9)$.

(b) Calculate $f'(6)$.

(c) Calculate $g'(6)$.

(d) Calculate $g(11) - g(8)$.

(e) Is the statement "$g(4) > g(0)$" true or false?

(f) Find the critical numbers of g in the interval $(0, 12)$.

12 p **Z14.** For this problem, you will explore the substitution rule for two different integrals.

(a) Consider the first (definite) integral:

$$J_1 = \int_{e^{-3}}^{e^2} \frac{2\ln(x) - 3}{5x}\, dx$$

Use the substitution $u = 2\ln(x) - 3$ to compute this integral. After you do the the substitution and translate the integral from being in terms of x to be being in terms of u, you have an integral of the following form:

$$J_1 = \int_a^b g(u)\, du$$

where $a < b$ and there is no number to the left of the integral sign.

(i) After the substitution, what is the integrand $g(u)$?

(ii) After the substitution, what is the lower limit of integration? upper limit of integration?

(b) Now use the fundamental theorem of calculus to calculate J_1, giving the following:

$$J_1 = \int_a^b g(u)\, du = G(b) - G(a)$$

(i) What is the relationship between g and G?

(ii) Calculate J_1.

(c) Now consider the second (indefinite) integral:

$$J_2 = \int \frac{\ln(x)}{3x^2}\, dx$$

Use the substitution $u = \ln(x)$. After you do the the substitution and translate the integral from being in terms of x to be being in terms of u, you have an integral of the following form:

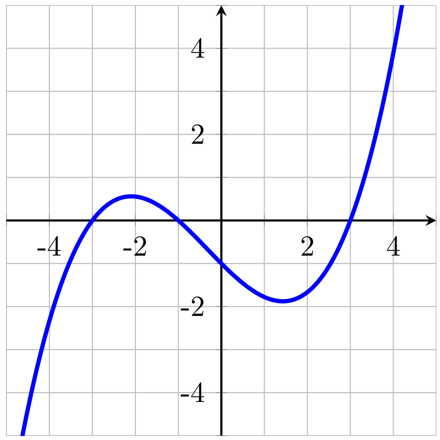
$$J_2 = \int f(u)\, du$$

where there is no number to the left of the integral sign.

(i) After the substitution, what is the integrand $f(u)$?

4.2 Fall 2021

Z15. For each part, use the graph of $y = g(x)$.



2 p (a) How many solutions does the equation $g'(x) = 0$ have?

2 p (b) Order the following quantities from least to greatest: $g'(-2.5)$, $g'(-2)$, $g'(0)$, and $g'(4)$. In your answer, write these quantities symbolically; do not give a numerical estimate.

2 p (c) What is the sign of $g''(-3)$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

2 p (d) Let $h(x) = g(x)^2$. What is the sign of $h'(-4)$(negative, positive, or zero)? If there is not enough information to determine the value, explain why.

6 p **Z16.** Let $f(x)$ be the following function, where k is an unspecified constant. Find the value of k that makes f continuous at $x = 2$ or determine that no such value of k exists.

$$f(x) = \begin{cases} 27x - kx^2 & x < 2 \\ -6 & x = 2 \\ 3x^3 + k & x > 2 \end{cases}$$

In your work, you must use limit-based methods to solve this problem. Solutions that have work that is not based on limits will not receive full credit.

Z17. Consider the curve described by the following equation: $2x^2 - 2xy + 3y^2 = 60$.

4 p (a) Find $\dfrac{dy}{dx}$ for a general point on the curve.

2 p (b) Find the x-coordinate of each point on the curve where the tangent line is horizontal.

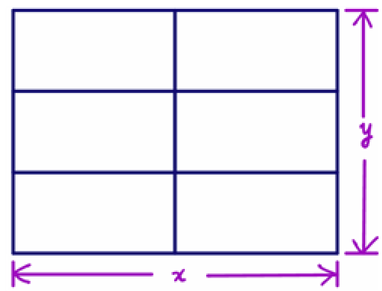
Z18. The parts of this problem are not related.

4 p (a) Calculate the integral $\displaystyle\int_2^4 \frac{18t - 3t^2}{t}\,dt$.

4 p (b) Calculate the area of the region below the curve $y = 23\sin(x)\cos^2(x)$ and above the interval $\left[0, \frac{\pi}{2}\right]$ on the x-axis. (Note that $y \geq 0$ on this interval.)

6 p **Z19.** Farmer Green is building an enclosure that must have a total area of 48 m^2. The pen will also be subdivided into 6 pens of equal area, as shown on the right. Find the dimensions of the enclosure that will require the least amount of fencing. *As you work, fill in the answer boxes below. You must*

use calculus-based methods in your work. You must also justify that your answer really does give the least fencing.



constraint equation in terms of x and y:	
objective function in terms of x only:	
interval of interest:	
dimensions of desired enclosure (in meters):	______ $\times$ ______ total length (x) total width (y)

12 p **Z20.** For each part, calculate the limit or show that it does not exist. *If the limit is "$+\infty$" or "$-\infty$", write that as your answer, instead of "does not exist".*

(a) $\displaystyle\lim_{x\to 1}\left(\frac{x^4 - x}{\ln(77x - 76)}\right)$

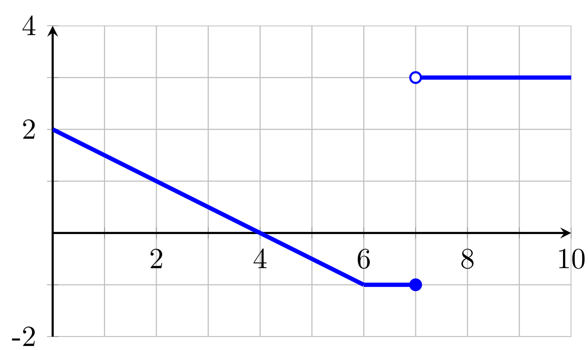
(b) $\displaystyle\lim_{x\to -\infty}\left(\frac{\sqrt{36x^2+63}}{31x}\right)$

(c) $\displaystyle\lim_{x\to 2^+} f(x)$, with $f(x) = \begin{cases} 1+4x & x \le 2 \\ \dfrac{x^2-4}{x-2} & x > 2 \end{cases}$

(d) $\displaystyle\lim_{x\to 5^-}\left(\frac{\cos(\pi x)}{x^2 - 25}\right)$

4 p **Z21.** For any time $t > 0$, the acceleration of a particle is given by $a(t) = 1 + \dfrac{3}{\sqrt{t}}$, and the particle has velocity $v = -20$ when $t = 1$. Find the velocity of the particle when $t = 16$.

8 p **Z22.** Let $F(x) = \displaystyle\int_0^x f(t)\,dt$, where the graph of $y = f(t)$ is given below. For each part, use this information to calculate the indicated item.



(a) $F(10)$

(b) $F'(6)$

(c) $\displaystyle\int_0^6 |f(t)|\,dt$

(d) $\displaystyle\int_0^4 \big(f'(t) + 5\big)\,dt$

4 p **Z23.** Use linear approximation to estimate $\tan\left(\frac{\pi}{4}+0.12\right)-\tan\left(\frac{\pi}{4}\right)$.

Z24. Let $f(x) = x^3(3x-4)$.

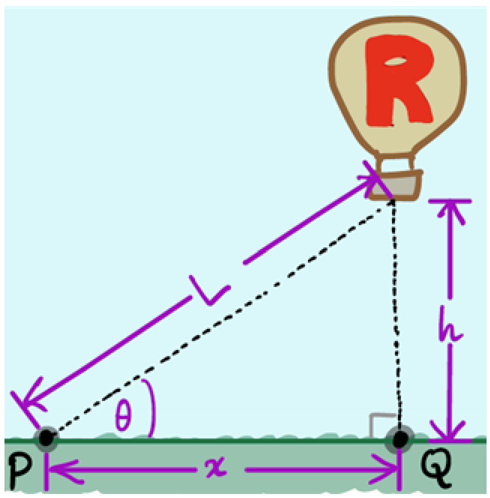
4 p (a) Find where relative extrema of f occur (if any). Classify each as a local minimum or a local maximum.

2 p (b) Find the absolute extrema of f on $[-1, 2]$ and the x-values at which they occur.

6 p **Z25.** For each part, find all vertical asymptotes of the given function.

(a) $f(x) = \dfrac{x^2-8x+15}{x^2-9}$ (b) $g(x) = \dfrac{e^{x+3}-1}{x^2-9}$

Z26. A hot-air balloon is floating directly above the point Q on the ground and is descending at a constant rate of 10 ft/sec. A camera is on the ground at point P, which is 500 feet from point Q. See the figure below.



2 p (a) What is the sign of $\frac{dh}{dt}$ (negative, positive, or zero)? If there is not enough information to determine the value, explain why.

2 p (b) How is $\cos(\theta)$ changing over time? Circle your answer below.

(i) increasing over time
(ii) decreasing over time
(iii) constant over time
(iv) sometimes increasing and sometimes decreasing
(v) not enough information to determine

4 p (c) What is the rate of change of the distance between the camera and the balloon when the balloon is 600 feet above the ground? *You must give correct units as part of your answer.*

6 p **Z27.** Consider the function $g(x)$, whose first two derivatives are given below. ***Note:*** Do not attempt to calculate $g(x)$. Also assume that $g(x)$ has the same domain as $g'(x)$.

$$g'(x) = \frac{8x^{17}}{x-32} \qquad g''(x) = \frac{128x^{16}(x-34)}{(x-32)^2}$$

Fill in the table below with information about the graph of $y = f(x)$. *For each part, write "NONE" as your answer if appropriate. Where applicable, give a comma-separated list of intervals that are as inclusive as possible.*

You do not have to show work, and each table item will be graded with no partial credit.

where f is decreasing	
where f is increasing	
x-coordinate(s) of local minima of f	
x-coordinate(s) of local maxima of f	
where f is concave down	
where f is concave up	
x-coordinate(s) of inflection point(s) of f	

Z28. The parts of this problem are not related.

3 p (a) Suppose that when x units are produced, the total cost is $C(x) = 2x^2 + 10x + 18$ and the selling price per unit is $p(x) = 46 - x$. Find the level of production that maximizes total proft.

3 p (b) Suppose the total cost of producing q units is $C(q) = q^3 + 20q^2 + 200q + 2000$. Use marginal analysis to estimate the cost of the 3rd unit.