Using the lower bound set by the universal modal to investigate the status of partial objects and count nouns

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Abstract. Prior research has demonstrated that when given objects (i.e., forks) broken into pieces, children deviate from adults by counting each discrete object-piece as on par with a whole. A recent proposal ties this behavior to the vagueness and context-sensitivity inherent to count noun semantics. The present study leverages the universal modal *have to* in order to investigate how a linguistic context, one which sets lower bounds on numerals in its scope, regulates nominal application. Our results show that for children, who prefer the 'exact' reading of numerals, the partial object not only serves to meet the lower bound, but also exceeds a numerical upper bound. Adults, on the other hand, do not consider the partial object as meeting the lower bound induced by the modal. Because we cannot determine the explanation for this finding with our current design, we plan to adapt it to use the universal modal *allowed to.*

Keywords. partial objects; modals; numerals; gradability; context-sensitivity; count noun semantics

1. Introduction. Count nouns such as *ball* and *fork* are among the first words children produce. Yet, children show a surprising, non-adultlike willingness to apply such words not just to whole balls and forks, but also their discrete parts. In a seminal study, Shipley and Shepperson (1990) gave children sets of whole objects and object parts ("partial objects") with specific instructions of what to count. When shown a set as in Figure 1, with four whole forks and two broken fork-pieces, and asked to "count the forks", children tended to count 6, as if treating the partial objects on par with the wholes.

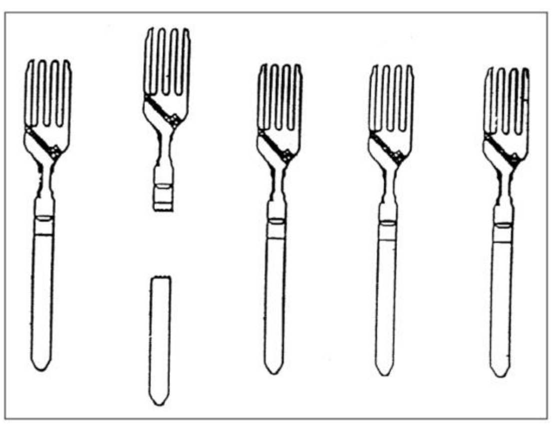

Figure 1: Example counting prompt from Shipley & Shepperson (1990)

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Adults either counted 4, ignoring the partial objects entirely, or 5, combining the two fork-parts into an imagined whole and counting it as 1. The authors took these findings to indicate that children have a conceptual bias to treat discrete physical objects as “default units” for counting. Since then, many have replicated and extended Shipley and Shepperson’s original findings, and proposed different accounts of children’s part-counting behavior. The details of these theories vary substantially: some argue that the problem lies in children's still-developing conceptual knowledge (Wagner & Carey, 2003), others suggest it stems from the interaction between cognitive defaults and developing noun semantics (Brooks et al., 2011), and some point to children’s more limited access to lexical alternatives to refer to parts of objects or degraded objects (Srinivasan et al., 2013). Despite these differences, most of these theories converge on the idea that children's behavior fundamentally differs from how adults handle partial objects.

In contrast, a more recent account by Syrett and Aravind (2022) proposed that children’s performance may be consistent with an adult noun semantics at its core. Count nouns, for both adults and children, have meanings that depend on context to determine what counts as a unit (Krifka, 1989; Rothstein, 2010), and in certain contexts, either a whole or a partial object could fall under the extension of the count noun. This idea gains support from anecdotal examples of adults using count nouns to label partial objects similarly as they would whole objects—for example, when presenting findings from an archeological dig, a shard of pottery could be described as a plate *tout court*. What differentiates adults from children, they argue, is adults’ more sophisticated ability to integrate context-specific information to restrict the noun’s application on a case-by-case basis.

To test their *context-sensitivity* hypothesis, they presented the participants with a task in which they had to determine whether a partial object counted as an instance of a count noun like *ball*, given the presence or absence of a speaker goal related to the object. Children – but not adults – were influenced by the degree of contextual support in deciding whether a partial object was a suitable referent for a count noun. When children were told, e.g., that someone intended to play tennis with a ball, they were less likely to accept a partial object as a referent for “a ball.”

These results demonstrate that there are limits to children’s application of count nouns to partial objects, modulated by contextual information. But whereas Syrett & Aravind’s task probed object reference (and therefore, category membership as indicated by the count noun and what falls under its extension), much of the prior work was focused on counting and quantification of sets of objects. Our goal in this paper is to test the context-sensitivity account in a task that calls upon participants to quantify objects, without overtly counting them. To achieve this, we leverage the bounding conditions induced by modals and their effect on the interpretation of numerical expressions to determine the status of partial objects relative to wholes. If children malleably treat partial objects as either parts of wholes or wholes depending on the context, then in a quantification task where the goal is to meet and exceed a lower bound, children should allow partial objects to serve this purpose. Adults, however, should continue to recognize partial objects as such, and not allow them to meet the lower bound. Previewing our results, we find that the adult-child difference in how partial objects are treated re-emerges in this task.

2. Bounding conditions on numerals in the scope of modals. Depending on the environment in which they appear, numerals can receive different interpretations — either an ‘exact’, ‘at least’ or ‘at most’ reading. In a discourse context such as (1), the numeral *three* is naturally understood to mean ‘*exactly* three’ – i.e, as providing both upper and lower limits on the number of mistakes made. If B made exactly three mistakes, answering A's question with any other numeral would be misleading, or even untruthful.

(1) A: How many mistakes did you make?
B: I made three mistakes.

In a somewhat different context, as in (2), the numeral is understood as providing a lower bound, i.e. as meaning '*at least* three'. B's response can be understood as felicitous and truthful even if they, in fact, have four children.

(2) A: People with three children get a tax break. Do you have three children?
B: Yes, I have three children.

The surrounding *linguistic* context can highlight different readings of numerals. In the scope of a universal modal such as *have to*, numerals are most naturally understood as having a lower bounded, 'at least', reading. Thus, (3) is understood as implying that anyone with three or more children will receive the tax break. Numerals in the scope of an existential modal like *allowed to*, in contrast, typically receive an upper bounded reading. Thus, (4) implies that anyone who makes three or fewer mistakes will pass the test.

(3) You have to have three children to receive the tax break.
(4) You are allowed to make three mistakes and still pass the test.

Prior developmental work suggests that children can access these different interpretations of numerals, despite sometimes showing less flexibility than adults in their interpretation of numerals. In unembedded contexts, children have been shown to prefer an 'exact' interpretation of numerals (e.g., Huang & Snedeker, 2009; Huang, Spelke, & Snedeker, 2013; Papafragou & Musolino, 2003). When the numeral is in the scope of modals, however, children more readily access the upper and lower bounded readings.

Musolino (2004) used a Truth-Value Judgment Task to test four- and five-year-olds' understanding of the interaction of numerals and modals by pairing stories with modal statements containing a conditional, such as (5).

(5) If the horse made it over two obstacles he would win.

In his experiment, children correctly assigned lower-bounded interpretations of numerals under universal modals (after controlling for the children's independent expectations about games), and upper-bounded interpretations for numerals under existential modals. Syrett and Kennedy (2022) expanded on this work by testing a condition missing from Musolino's experiment: a context in which the upper bound, induced by the universal modal *allowed to*, was exceeded. While the stories they created were similar to those in Musolino's experiment, they were paired with simplified modal statements that did *not* contain conditionals, such as (6).

(5) You are allowed to use two lemons.

They then manipulated the quantity of objects seen by participants to vary between <2, exactly 2, and >2. In their experiment, children successfully rejected the action when the upper bound was exceeded, but were also significantly less likely to accept actions that did not meet the upper limit.

These studies provide a good starting point for our experiment, which aims to assess children's and adults' treatment of partial objects and the role of language in mediating how such objects are quantified. In our experiment, we focus on the universal modal *have to*, which induces a lower bound on numerals in its scope, to investigate how a linguistic context could affect children's or adults' numerical judgments. We ask, can partial objects be flexibly included or excluded from counts depending on the bounds induced by modal statements? Specifically, do partial objects satisfy the numerical lower bound in a modal statement such as (3)? If they can be both flexibly treated as category members denoted by the count noun in the right contexts and also counted as one unit, partial objects should count towards meeting the limit. Our question is whether this is the case for both children and adults, in a task focused on quantification.

3. Experiment.

3.1. METHODS. Sample size, procedures, and analyses were pre-registered at https://osf.io/phyds.

3.1.1. PARTICIPANTS. 40 English-acquiring children (4;6-5;6, M=4;11) and 21 English-speaking adults (N=40 preregistered, in prog.) participated in the experiment. An additional 10 children were tested but excluded from the full sample for failing comprehension checks (4), not completing the experiment (3), inattentiveness (1), less than 50% home exposure to English (1), or experimenter error (1). All children were recruited from a database of families interested in participating in research with the MIT Language Acquisition Lab, and Zoom-tested by a live experimenter. 4 additional adults were tested as well, but excluded from the full sample for failing comprehension checks (2) or not completing the experiment (2). All adults were undergraduate students from Rutgers University who received extra credit in a Linguistics or Cognitive Science course for their participation, and took an asynchronous variant of the child experiment, in which video clips of each trial were inserted into a self-paced Qualtrics survey.

3.1.2. MATERIALS. Participants were introduced to a game in which characters had to satisfy a rule to receive a reward. Across trials, object kind and the distribution of whole and partial objects in the sets shown to participants varied, while the numeral in the instruction sentences was always 'three'. Participants gave a star when they found the numerical conditions to be met, and a calculator (or 'counting machine') otherwise. See Fig 2.

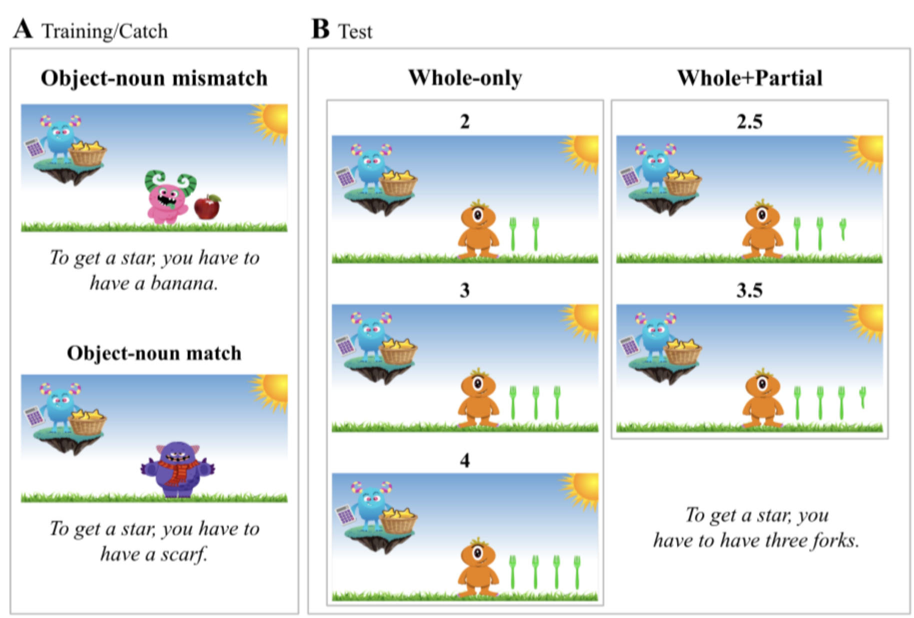

Figure 2: (**A**) During training, participants were shown a trial in which the pictured object did not match the noun in the modal statement, and one in which it did. Catch items were designed similarly. (**B**) Test trials split by collection type (Whole-only and Whole+Partial), all for which participants heard the same pre-recorded modal statement.

All participants saw two types of trials based on the nature of the object collections on display. The 'Whole-only' collection type included trials with 2, 3, or 4 identical whole objects. The 'Whole+Partial' collection type consisted of object sets with either 2 or 3 identical whole objects, plus a single partial object. The partial object stimuli were created by removing portions from the images of the whole objects, and so were identical in all other respects. Altogether, there were object sets with the following five cardinalities: 2, 2.5, 3, 3.5, and 4. Each object set was paired with a modal statement such as (7), in which we used count nouns denoting everyday objects (e.g. balls, cups, forks).

(7) To get a star, you have to have three forks.

3.1.2. PROCEDURES. All participants saw trials in both the Whole-only and Whole+Partial collection types in a pseudorandomized order. The experimental session began with an introduction to Zoryn, the protagonist, and her friends, a group of aliens visiting Earth. Participants were invited to play a counting game with them, in which they had to listen to a rule provided by Zoryn and then determine whether the set of objects in a friend's possession was rule-compliant. If they had followed the rule, they would be rewarded with a star; if not, they would receive a calculator to count better the next time.

To ensure that participants understood the task, they saw two training trials prior to the experimental phase. The instruction sentence for these trials involved the universal modal, but without a numeral, e.g. "To get a star, you have to have a banana." In one trial, the friend had an object that matched the noun in the rule, and in the other, one that did not.

The test phase consisted of 18 total trials: 2 per cardinality for the Whole-only trials (6 total), 4 per cardinality for the Whole+Partial collection trials (8 total), and 4 catch items. These catch items followed a similar structure as the training trials, and served as both task-comprehension and attention checks. For each trial, we coded whether participants judged a set as compliant with the modal statement, with a choice of star indicating 'Yes' (1) and a choice of calculator indicating 'No' (0).

3.2. PREDICTIONS. We begin with outlining the expectations in each of the Whole-only trials. For a participant who interprets the modal statement "You have to have three…" as specifying a lower bound for the numeral, sets of 2 objects would not satisfy the bound condition. Thus, a No-response is expected on 2-trials. Sets of 3, on the other hand, meet the bound conditions and should be accepted. Sets of 4 should also be accepted, as they not only meet but *exceed* the lower bound, but participants who prefer 'exact' interpretations of numerals could reject them as exceeding an upper bound. This might be the case for children, who have previously been shown to most readily access 'exact' readings of numerals.

For the Whole+Partial trials, expectations vary based on two factors: how participants quantify partial objects based on context, and their preferred reading of the numeral. If partial objects can count as 1 unit as the context demands, then sets with 2 whole objects and 1 partial object could be treated as on par with a set containing 3 wholes, leading to a 'Yes' response in the 2.5-trials. On the other hand, if the partial objects do *not* count as having a cardinality of 1, 2.5-trials should yield 'No' responses, in contrast to 3-trials. As for the numeral, if participants access a lower bounded reading, 3.5-trials should be accepted irrespective of their treatment of the partial object. If they access only an ‘exact’ reading, 3.5-trials may yield ‘No’ responses if the partial object counts as an instance of the noun, pushing the set beyond the limit. See Fig. 3.

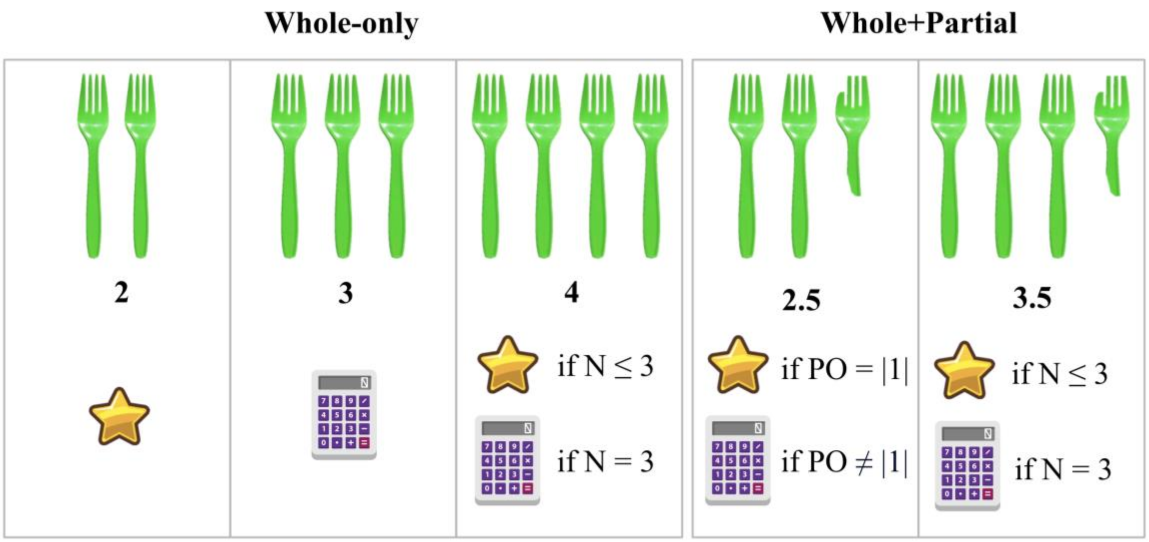

Figure 2: Predictions for both collection types, with a star corresponding to a ‘Yes-response’ and a calculator corresponding to a ‘No-response’

3.2. PREDICTIONS. Our primary question was how the rates of accepting a set as rule-compliant vary based on set composition, and whether this differs across adult and child populations. To test these questions, we fit separate logistic mixed effects models for the Whole-only and Whole+Partial collection types. For each, we predicted the probability of responding ‘Yes’ as a function of set type and age group, with random intercepts for participants. For the Whole-only model, collection type had three levels (2, 3, 4). For the Whole+Partials model, collection type had two levels (2.5, 3.5). Age group had two levels (adults, children) in both models. All factors were treatment-coded. To test for main effects and interactions, we used log-likelihood chi-squared tests to compare models with and without the relevant effect.

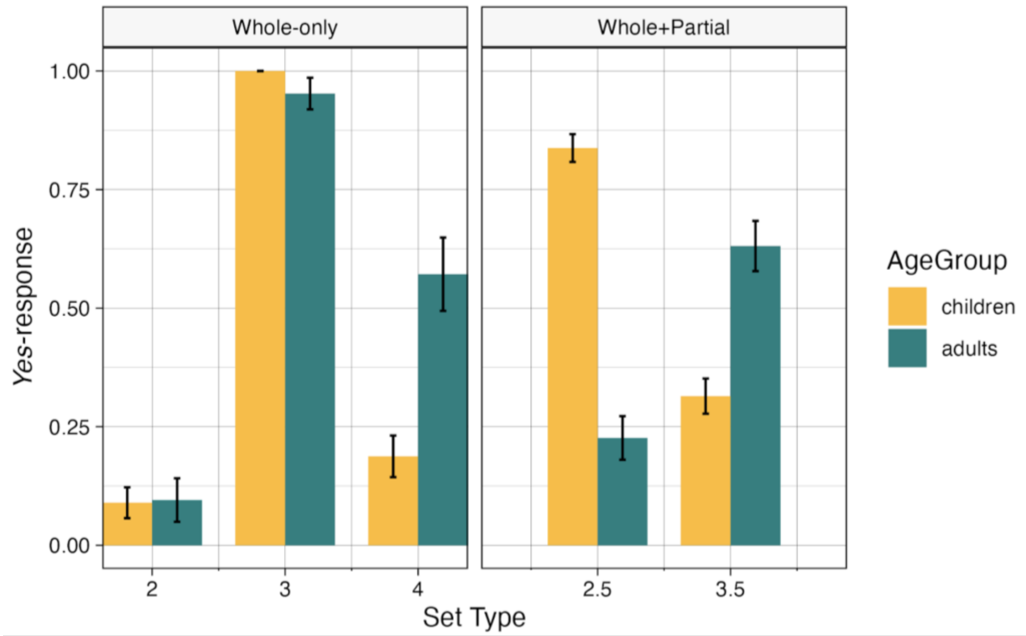

Figure 4: Mean 'Yes'-responses (+/- 1 SEM) for each population, split by collection type

For Whole-only trials (Fig.4, left), model comparisons revealed that both age group ($\chi2(1) = 4.14$, $p = 0.04$), set type ($\chi2(2) = 48.2$, $p < .001$) and their interaction ($\chi2(2) = 9.34$, $p = 0.001$) significantly improved model fit. The effect of age group reflects the fact that adults were more likely to respond "Yes" than children. Participants in both groups were also more likely to respond "Yes" when there were 3 objects in the set – corresponding to an *exact* reading of the numeral in the rule. Post-hoc tests exploring the interaction effect reveal that while both populations were alike in their judgments of sets of 2 and 3, they diverge crucially in their responses to sets of 4. Children were significantly less likely than adults to respond Yes when there were 4 whole objects (β=-2.53, SE= 0.732 z=-3.458 p <.001), suggesting that they accessed the *at least* reading less often.

In the Whole+Partial collection type (Fig. 4, right), children and adults again differed in their judgments. Model comparisons revealed a significant interaction of age group and set type ($\chi2(1) = 73.71$, $p < 0.001$). This interaction was driven by the fact that children's and adults' Yes-responses patterned in opposite directions for 2.5 and 3.5. Children were significantly more likely than adults to accept sets of 2.5 as rule-compliant (β=3.91 SE=0.62, z=6.314 <.001), but were significantly less likely than adults to do so for sets of 3.5 (β=-1.83 SE=0.55 z=-3.347 p=0.001). In other words, children saw a set containing 2 wholes and 1 partial object as satisfying the bounds conditions set by "have to have *three* N", whereas a set containing 3 wholes and 1 partial object exceeded it.

4. Discussion. In our study, we used a task probing numerical judgments to better understand children's and adults' treatment of partial objects. For example, does participants' willingness to treat a partial fork as falling under the extension of a count noun like *fork*, and counting it as one fork-unit, depend on contextual factors as previously proposed (e.g., Syrett & Aravind 2022)? If so, can information contained in the sentence itself modulate whether a partial object counts as 1? To address these questions, we capitalized on the interaction of numerals and universal modals like *have to*, which lead to a lower-bounded, or 'at least', interpretation of numerals in their scope. Specifically, we asked, when given a rule such as "You have to have three forks", paired with a set of whole and partial forks, are participants willing to count a partial fork as 1 if doing so would serve to meet the lower bound conditions set by the modal?

Our results answer this question in interestingly different ways for our two populations. Adults were able to access the lower bounded, ‘at least’ interpretation of the numeral, as evidenced by their acceptance of sets containing 4 whole objects. However, they rarely accepted sets containing two wholes and one partial object. This result indicates for adults, the partial object was treated on par with a whole, so as to satisfy the lower bound.

Children diverged from adults in having a strong preference for the ‘exact’ reading of numerals, demonstrated by their low acceptance of sets of 4. This is consistent with prior work on numerals, where such preferences have also been reported – though not in this specific linguistic environment. Crucially, they accepted sets involving two wholes and one partial object almost as often as they did sets of three wholes. This highlights a second, critical divergence from adults: for children, a partial object *does* count as one instance of the relevant count noun, thereby meeting the lower bound of three. This finding is in line with earlier work showing that in tasks involving counting and quantification, children treat partial objects on par with wholes.

Like adults, children also failed to show flexibility in their treatment of partial objects. This is demonstrated by their treatment of sets containing three wholes and one partial object. Children responded “No” in this case, suggesting that for them, the partial object not only served to satisfy the numerical lower bound, but also served to push the cardinality of a set beyond the upper bound. Thus, while both age groups differed at the surface level, they were internally consistent in their interpretations: neither group flexibly included or excluded the partial object to meet the bounds induced by the modal.

This result is surprising in light of prior work, which found that children are generally sensitive to context. What makes this task different? One possibility is that the linguistic environment—in this case, the bounds set by the modal—may not provide sufficient or the right type of contextual information about the inclusion or exclusion of partial objects from the counts. Another possibility is that, for both children and adults, noun application and individuation may diverge; that is, the question of whether an object can serve as a referent for a noun and how to count or measure that entity may not always align perfectly. This leads us to our next open question.

Although adults did not treat the partial object as meeting the lower bound, we cannot say that they excluded it from their counts entirely. There is, in fact, suggestive evidence that this is not the case. Had adults ignored the partial object altogether, we would expect their treatment of the 3.5 set to be comparable to their treatment of the 3 set: the 3.5 set contains 3 whole objects and an irrelevant object that doesn’t count. But numerically, adults’ Yes-responses to the 3.5 set were comparable to their treatment of the 4 set, and lower than their Yes-responses to the 3 set. In other words, for adults, the 3.5 set does not seem to satisfy, the ‘exact’ reading of the numeral.

In order to gain further insight into how exactly adults quantify partial objects, we are currently conducting a follow-up experiment in which we employ the existential modal *allowed to*, which induces an upper bound on modals in its scope, as in (8) and (4) above.

(8) Employees are allowed to take three snacks from the break room.

In this example, the numeral *three* has an upper bounded, ‘at most’, interpretation. While taking up to two snacks would be acceptable, taking more would exceed the numerical condition. Using this modal in a follow-up experiment of a similar design will allow us to determine if a partial object can exceed an upper bound, even if it can’t meet a lower bound. If this is the case, adults may use a more complex system than children, understanding a partial object as a fractional portion, rather than as 0 or 1.

References

Brooks, Neon, Amanda Pogue & David Barner. 2011. Piecing together numerical language: children's use of default units in early counting and quantification. *Developmental Science* 14(1). 44–57. https://doi.org/10.1111/j.1467-7687.2010.00954.x.

Huang, Yi Ting & Jesse Snedeker. 2009. Semantic meaning and pragmatic interpretation in 5-yearolds: evidence from real-time spoken language comprehension. *Developmental Psychology* 45(6). 1723–1739. https://doi.org/10.1037/a0016704.

Huang, Yi Ting, Elizabeth Spelke & Jesse Snedeker. 2013. What exactly do numbers mean? *Language Learning and Development* 9(2). 105–129. https://doi.org/10.1080/15475441.2012.658731.

Kennedy, Christopher & Kristen Syrett. 2022. Numerals denote degree quantifiers: Evidence from child language. In Nicole Gotzner & Eli Sauerland (eds.), *Measurements, numerals and scales*, 135–162. Cham: Palgrave Macmillan.

Krifka, Manfred. 1989. Nominal reference, temporal constitution and quantification in event semantics. In Renate Bartsch, Johan van Benthem & Peter van Emde Boas (eds.), *Semantics and contextual expression*, 75–116. Mouton: De Gruyter.

Musolino, Julien. 2004. The semantics and acquisition of number words: Integrating linguistic and developmental perspectives. *Cognition* 93(1). 1–41. https://doi.org/10.1016/j.cognition.2003.10.002.

Papafragou, Anna & Julien Musolino. 2003. Scalar implicatures: experiments at the semantics–pragmatics interface. *Cognition* 86(3). 253–282. https://doi.org/10.1016/S0010-0277(02)00179-8.

Rothstein, Susan. 2010. Counting and the mass/count distinction. *Journal of Semantics* 27(3). 343–397. https://doi.org/10.1093/jos/ffq007.

Shipley, Elizabeth F. & Barbara Shepperson. 1990. Countable entities: Developmental changes. *Cognition* 34(2). 109–136. https://doi.org/10.1016/0010-0277(90)90041-H.

Srinivasan, Mahesh, Eleanor Chestnut, Peggy Li & David Barner. 2013. Sortal concepts and pragmatic inference in children's early quantification of objects. *Cognitive Psychology* 66(3). 302–326. https://doi.org/10.1016/j.cogpsych.2013.01.003.

Syrett, Kristen & Athulya Aravind. 2022. Context sensitivity and the semantics of count nouns in the evaluation of partial objects by children and adults. *Journal of Child Language* 49(2). 239–265. https://doi.org/10.1017/S0305000921000027.