Rutgers University
Math 152

Section 10.2: Infinite Series - Worksheet

1. Each of the series $\sum_{n=n_0}^{\infty} a_n$ below is either geometric or telescoping. For each series, find a formula for the partial sum $S_N = \sum_{n=n_0}^{N} a_n$, then determine if the series converges or diverges, and compute its sum if it does.

   (a) $\sum_{n=4}^{\infty} 2^n 3^{-n}$

   (b) $\sum_{n=0}^{\infty} \left(\frac{4}{2n+1} - \frac{4}{2n+5} \right)$

   (c) $\sum_{n=0}^{\infty} \frac{1 - 3 \cdot 4^{2n}}{5^{n-1}}$

   (d) $\sum_{n=3}^{\infty} \ln\left(\frac{3n+1}{3n+4} \right)$

   (e) $\sum_{n=0}^{\infty} 5 \cdot 3^{1-2n}$

   (f) $\sum_{n=1}^{\infty} \left(\tan^{-1}(n+1) - \tan^{-1}(n) \right)$

2. Use geometric series to express the repeating decimals below as a fraction of two integers.

   (a) $1.5222\cdots = 1.5\overline{2}$

   (b) $0.126126\cdots = 0.\overline{126}$

3. For each sequence $\{a_n\}_{n=n_0}^{\infty}$ given below, determine

   (i) whether the **sequence** $\{a_n\}_{n=n_0}^{\infty}$ converges or diverges. If the sequence converges, find its limit.

   (ii) whether the **series** $\sum_{n=n_0}^{\infty} a_n$ converges or diverges. If the series converges, find its sum if possible.

   (a) $\left\{ \left(1 + \frac{4}{n}\right)^n \right\}_{n=1}^{\infty}$

   (b) $\left\{ \sqrt{n+1} - \sqrt{n} \right\}_{n=0}^{\infty}$

   (c) $\{e^{-n}\}_{n=0}^{\infty}$

   (d) $\left\{ \frac{e^{5n}}{n^{3/2}} \right\}_{n=1}^{\infty}$

4. Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{2 \cdot 5^{n+1}}$. Find the values of x for which the series converges and find the sum of the series when it converges.