Rutgers University
Math 152

Section 10.7: Power Series - Worksheet

1. Find the radius and interval of convergence of the power series below. Specify for which values of x in the interval of convergence the series converges absolutely and for which it converges conditionally.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{\sqrt[3]{n}5^n}$.

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n(x-9)^{3n}}{8^n(n+1)}$.

   (c) $\displaystyle\sum_{n=0}^{\infty} n3^n(2x+1)^n$.

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{n^n(x+2)^n}{6^n}$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n(x-4)^{2n}}{36^n\sqrt{n}}$.

   (f) $\displaystyle\sum_{n=0}^{\infty} \frac{(3x+2)^n}{n^2+4}$.

2. Find the radius of convergence of the following power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!}x^{2n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{3}{n}\right)^{n^2}(x+5)^n$.

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{n^n}x^n$.

3. Suppose that a power series converges absolutely at $x=5$, converges conditionally at $x=-3$ and diverges at $x=11$. What can you say, if anything, about the convergence or divergence of the power series at the following values of x?

   (a) $x=-4$.

   (b) $x=2$.

   (c) $x=15$.

   (d) $x=7$.

4. Let $f(x)=\dfrac{3}{2+7x}$. Use the power series representation of $\dfrac{1}{1-x}$ and power series operations to find a power series representation of $f(x)$ centered at $a=0$. What are the radius and interval of convergence of the resulting power series?

5. Consider the power series $f(x)=\displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n(n+1)}$.

   (a) Find the radius and interval of convergence of f.

   (b) Find a power series representation of $f'(x)$ centered at $a=-1$. What are its radius and interval of convergence?

   (c) Let $g(x)$ be the antiderivative of $f(x)$ such that $g(-1)=-8$. Find a power series representation of $g(x)$ centered at $a=-1$. What are its radius and interval of convergence?

6. (a) Use term-by-term differentiation to find a power series representation of $\dfrac{1}{(1-x)^2}$. What is its raidus of convergence?

   (b) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{5^n}$.