Rutgers University
Math 152

Sections 11.1, 11.2: Parametric Curves - Worksheet

1. Find an equation of the tangent line to the given parametric curve at the point defined by the given value of t.

(a) $\begin{cases} x = 5t^2 - 7 \\ y = t^4 - 3t \end{cases}$, $t = -1$. (b) $\begin{cases} x = e^{4t} - e^t + 2 \\ y = t - 3e^{2t} \end{cases}$, $t = 0$. (c) $\begin{cases} x = \sec(3t) \\ y = \cot(2t - \pi) \end{cases}$, $t = \frac{\pi}{12}$.

2. Find all points on the following parametric curves where the tangent line is (i) horizontal, and (ii) vertical.

(a) $\begin{cases} x = \sin(2t) + 1 \\ y = \cos(t) \end{cases}$, $0 \leqslant t < 2\pi$.

(b) $\begin{cases} x = 3t - t^3 \\ y = t^2 + 4t + 3 \end{cases}$

(c) $\begin{cases} x = 4t - e^{2t} \\ y = t^2 - 18\ln|t| \end{cases}$

3. Consider the ellipse of equation $x^2 + 4y^2 = 4$.
   (a) Find a parametrization of the ellipse.
   (b) Find the area enclosed by the ellipse.
   (c) Find the area of the surface obtained by revolving the top-half of the ellipse about the x-axis.

4. For each of the following parametric curves: (i) find the arc length, (ii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the x-axis and (iii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the y-axis.

(a) $\begin{cases} x = e^{4t} \\ y = e^{5t} \end{cases}$, $0 \leqslant t \leqslant 1$. (b) $\begin{cases} x = \ln(t) \\ y = \sin^{-1}(t) \end{cases}$, $\frac{1}{2} \leqslant t \leqslant \frac{1}{\sqrt{2}}$. (c) $\begin{cases} x = t^3 - t \\ y = \sqrt{3}t^2 \end{cases}$, $0 \leqslant t \leqslant 1$.