

**Sections 11.5: Areas and Lengths in Polar Coordinates - Worksheet**

1. Find the areas of the given regions.
  - (a) The region shared by the circles  $r = 2 \sin(\theta)$  and  $r = 2 \cos(\theta)$ .
  - (b) The region contained inside the leaves of the rose  $r = 6 \sin(2\theta)$  and outside the circle  $r = 3$ .
  - (c) The region inside the cardioid  $r = 1 + \sin(\theta)$  and below the line  $x = \sqrt{3}y$ .
  - (d) The region inside the circle  $r = \cos(\theta)$  and outside the cardioid  $r = 1 - \cos(\theta)$ .
  - (e) The region shared by one leaf of the rose  $r = 2 \cos(3\theta)$  and the circle  $r = 1$ .
  
2. Consider the region  $\mathcal{R}$  contained in the circle  $r = 4 \cos(\theta)$  to the right of the line  $x = 3$ .
  - (a) Find the area of the region  $\mathcal{R}$  using integration with respect to  $x$ .
  - (b) Find the area of the region  $\mathcal{R}$  using integration with respect to  $y$ .
  - (c) Find the area of the region  $\mathcal{R}$  using integration with respect to  $\theta$ .
  
3. Find the lengths of the given polar curves.
  - (a)  $r = \sqrt{1 + \cos(2\theta)}$ ,  $0 \leq \theta \leq \frac{\pi}{2}$ .
  - (b)  $r = \frac{2}{1 - \cos(\theta)}$ ,  $\frac{\pi}{2} \leq \theta \leq \pi$ .
  - (c)  $r = e^{3\theta}$ ,  $0 \leq \theta \leq \pi$ .