Rutgers University
Math 152

Sections 5.5, 5.6, 8.1: Review of Integration - Worksheet

1. Evaluate the following antiderivatives.

(a) $\displaystyle\int \frac{dx}{\sqrt{8x - x^2}}$

(b) $\displaystyle\int \frac{\tan^{-1}(t)^3}{1 + t^2} dt$

(c) $\displaystyle\int \frac{\tan(3\ln(x))}{x} dx$

2. For each of the regions described below (i) sketch the region, clearly labeling the curves and their inter-section points, (ii) calculate the area of the region using an x-integral and (iii) calculate the area of the region using a y-integral.

(a) The region to the right of the parabola $y = 1 - (x - 2)^2$, below the line $y = 1$ and to the left of the line $x - 2y = 3$.

(b) The region bounded by the curves $y = 2x$ and $y = \sqrt[3]{32x}$.

(c) The region bounded by the curves $y = \dfrac{4}{x + 2}$ and $y = 3 - x$.

3. Calculate the area of the regions described below.

(a) The region bounded by the parabola $x = (y + 3)^2 - 4$ and the line $x = 3y + 9$.

(b) The region bounded by $y = \dfrac{4}{3 + x^2}$ and $y = 1$.

(c) The region bounded by $y = 2\ln(x + 1)$, the x-axis and the line $x = 4$.

(d) The region to the right of the y-axis, above the graph of $y = \sec(x)^2$ and below the graph of $y = 2\sec(x)$.

4. Suppose that f is an **even** function such that

$$\int_{-9}^{5} f(x)dx = -13 \text{ and } \int_{0}^{9} f(x)dx = 4.$$

Evaluate the definite integrals below.

(a) $\displaystyle\int_{-9}^{9} f(x)dx$

(b) $\displaystyle\int_{0}^{5} (4x - 3f(x))dx$

(c) $\displaystyle\int_{-3}^{3} xf(x)dx$

(d) $\displaystyle\int_{0}^{3} xf(x^2)dx$

5. Find the average value of the following functions on the given interval.

(a) $f(x) = \dfrac{3}{\sqrt{100 - x^2}}$ on $[0, 5]$.

(b) $f(x) = x\sqrt[3]{3x - 7}$ on $[2, 5]$.