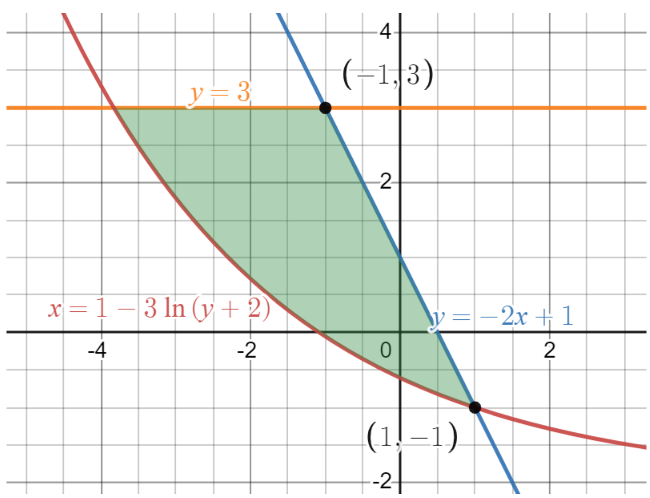
Rutgers University
Math 152

**Section 6.1: Volume by Cross-Sections - Worksheet**

1. Consider the region $\mathcal{R}$ in the first quadrant bounded by the curve $x = 4 - (y-1)^2$.
   (a) Sketch the region. Make sure to clearly label the curve and its intercepts.
   (b) A solid has base $\mathcal{R}$ and cross-sections perpendicular to the $y$-axis. Calculate the volume of the solid if the cross-sections are (i) semi-circles with diameter in the base and (ii) equilateral triangles with a side in the base.
   (c) A solid has base $\mathcal{R}$ and its cross-sections perpendicular to the $x$-axis are isosceles right triangles with hypotenuse in the base. Calculate the volume of the solid.
   (d) Calculate the volume of the solid of revolution obtained by revolving $\mathcal{R}$ about (i) the $y$-axis and (ii) the line $y = -2$.

2. Use the method of disks/washers to calculate the volume of the solids of revolutions obtained by revolving the regions described below about the given axis.
   (a) The region below the graph of $y = \ln(x)$ on $1 \leqslant x \leqslant 3$ revolved about the line $x = 3$.
   (b) The region below the graph of $y = \dfrac{1}{\sqrt{25+4x^2}}$ on $0 \leqslant x \leqslant \dfrac{5}{2}$ revolved about the $x$-axis.
   (c) The region bounded by $y = e^x$, $y = 4 - e^x$ and the coordinate axes revolved about the line $y = 4$.
   (d) The region below the graph of $y = 2\sin^{-1}\left(x^2\right)$ on $0 \leqslant x \leqslant 1$ revolved about the $y$-axis.

3. Consider the region $\mathcal{R}$ shaded in the figure below.




Use the method of washers to set-up integrals that compute the volume of the solid obtained by revolving $\mathcal{R}$ about the line

(a) $x = 2$, (b) $y = 3$, (c) $x = -4$, (d) $y = -2$.

You do not need to evaluate the integrals.