Rutgers University
Math 152

Section 8.4: Trigonometric Substitution - Worksheet

1. Calculate the following integrals.

   (a) $\displaystyle\int \frac{\sqrt{25x^2-4}}{x}dx$ for $x > \frac{2}{5}$.

   (b) $\displaystyle\int \frac{dt}{t\sqrt{9+\ln(t)^2}}$.

   (c) $\displaystyle\int \frac{dx}{(6x-x^2-5)^{5/2}}$.

   (d) $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{x(2x^2-1)^{3/2}}$.

   (e) $\displaystyle\int \frac{e^{6x}}{\sqrt{16-e^{4x}}}dx$.

   (f) $\displaystyle\int_5^{11} \frac{dx}{(x^2-10x+61)^{5/2}}$.

2. Calculate the average value of the function $f(x) = \dfrac{1}{x\sqrt{64-x^2}}$ on the interval $[4, 4\sqrt{2}]$.

3. (a) Evaluate $\displaystyle\int \sqrt{1+x^2}dx$.

   (b) Use your result from part (a) for the following applications.

   (i) Calculate the length of the curve $y = x^2$, $0 \leqslant x \leqslant 1$.

   (ii) Calculate the area of the surface obtained by revolving the curve $y = e^x$, $0 \leqslant x \leqslant \ln(2)$, about the x-axis.

   (iii) Calculate the area of the surface obtained by revolving the curve $y = \sin^{-1}(x)$, $0 \leqslant x \leqslant 1$ about the y-axis.

4. Calculate the area of the region inside the circle of equation $x^2 - 2x + y^2 = 3$ and above the line $y = \sqrt{3}$.

5. Consider the region $\mathcal{R}$ bounded between the graph of $y = \dfrac{1}{16-x^2}$ and the x-axis for $0 \leqslant x \leqslant 2$. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = -3$.