Rutgers University
Math 152

Section 8.8: Improper Integrals - Worksheet

1. Calculate the following integrals or determine if they diverge.

   (a) $\displaystyle\int_0^\infty e^{-5x}dx$

   (b) $\displaystyle\int_0^{\pi/4} \csc(x)dx$

   (c) $\displaystyle\int_{-\infty}^0 xe^{3x}dx$

   (d) $\displaystyle\int_{-\infty}^\infty \frac{dx}{(16+x^2)^{3/2}}$

   (e) $\displaystyle\int_0^1 \ln(x)dx$

   (f) $\displaystyle\int_{-2}^1 \frac{dx}{\sqrt[3]{3x-2}}$

   (g) $\displaystyle\int_0^{3/2} \frac{dx}{\sqrt{9-4x^2}}$

   (h) $\displaystyle\int_e^\infty \frac{dx}{x\ln(x)}$

   (i) $\displaystyle\int_0^\infty e^{-x}\sin(x)dx$

2. Use a convergence test to determine if the following improper integrals converge or diverge.

   (a) $\displaystyle\int_3^\infty \frac{dx}{xe^x}$

   (b) $\displaystyle\int_1^\infty \frac{dx}{x^2+3x+1}$

   (c) $\displaystyle\int_4^\infty \frac{\cos(x)+5}{x^{3/5}}dx$

   (d) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}+x^2}$

   (e) $\displaystyle\int_5^\infty \frac{xdx}{x^4-1}$

   (f) $\displaystyle\int_1^\infty \frac{x^3+5x^2+1}{\sqrt{x^7+4x+2}}dx$

3. Consider the unbounded region $\mathcal{R}$ between the graph of $y=\dfrac{\ln(x)}{x}$ and the x-axis for $x \geqslant 1$.

   (a) Find the area of the region $\mathcal{R}$ or determine if $\mathcal{R}$ has infinite area.

   (b) We now revolve the region $\mathcal{R}$ about the x-axis to form a solid of revolution. Calculate the volume of the solid or determine if the solid has infinite volume.

   (c) We now revolve the region $\mathcal{R}$ about the y-axis to form a solid of revolution. Calculate the volume of the solid or determine if the solid has infinite volume.