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Math 152

Math 152 All Worksheets
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Calculus 1 Review Worksheet

1. Simplify the following expressions. Your answer should not involve any trigonometric or inverse trigonometric functions.

   (a) $\cos^{-1}\left(\frac{1}{2}\right)$

   (b) $\sin^{-1}\left(\sin(\frac{7\pi}{4})\right)$

   (c) $\cos(\sin^{-1}(x))$

   (d) $\sin(2\cos^{-1}(x))$

   (e) $\sec\left(\tan^{-1}\left(\frac{x}{3}\right)\right)$

   (f) $\sin\left(\cot^{-1}\left(\frac{2}{\sqrt{x}}\right)\right)$

2. Calculate the following limits.

   (a) $\lim_{x\to\infty} \frac{\ln(x)^2}{\sqrt{x}}$

   (b) $\lim_{x\to 0} \frac{5^x - 3^x}{\sin(2x)}$

   (c) $\lim_{x\to\infty} \tan^{-1}(x^2 - x^3)$

   (d) $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$

   (e) $\lim_{x\to-\infty} \frac{2x + 3\cos(x)}{5x}$

   (f) $\lim_{x\to\infty} x^{1/x}$

3. Find the horizontal asymptotes of the following functions.

   (a) $f(x) = \frac{11x^3 + 2x - 1}{2x^3 - x^2 + 3}$

   (b) $f(x) = \frac{5x + \sqrt{16x^2 + 25}}{18x - 7}$

   (c) $f(x) = \frac{3e^{2x} - 2e^x + 4x^2}{x^2 - 6e^{2x}}$

4. Calculate the following indefinite or definite integrals.

   (a) $\int (3x + 1)\left(x^2 - \frac{5}{x}\right) dx$

   (b) $\int x^3 \sin(x^4 + 2) dx$

   (c) $\int_0^1 \frac{x^3}{\sqrt{3 + x^2}} dx$

   (d) $\int t \sec^2\left(3t^2\right) e^{7\tan\left(3t^2\right)} dt$

   (e) $\int e^x (e^x - 2)^{2/3} dx$

   (f) $\int e^{2x} (e^x - 2)^{2/3} dx$

   (g) $\int_{e^3}^{e^6} \frac{dt}{t\ln(t)}$

   (h) $\int \frac{dx}{5x + 4\sqrt{x}}$

   (i) $\int \frac{dx}{\sqrt{2 - x^2}}$

   (j) $\int_0^1 \frac{x dx}{\sqrt{2 - x^2}}$

   (k) $\int_0^{2/3} \frac{dz}{4 + 9z^2}$

   (l) $\int \frac{dx}{x^2 + 6x + 34}$

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Sections 5.5, 5.6, 8.1: Review of Integration - Worksheet

1. Evaluate the following antiderivatives.

(a) $\displaystyle\int \frac{dx}{\sqrt{8x - x^2}}$ (b) $\displaystyle\int \frac{\tan^{-1}(t)^3}{1+t^2}dt$ (c) $\displaystyle\int \frac{\tan(3\ln(x))}{x}dx$

2. For each of the regions described below (i) sketch the region, clearly labeling the curves and their intersection points, (ii) calculate the area of the region using an x-integral and (iii) calculate the area of the region using a y-integral.

   (a) The region to the right of the parabola $y = 1 - (x-2)^2$, below the line $y = 1$ and to the left of the line $x - 2y = 3$.

   (b) The region bounded by the curves $y = 2x$ and $y = \sqrt[3]{32x}$.

   (c) The region bounded by the curves $y = \dfrac{4}{x+2}$ and $y = 3 - x$.

3. Calculate the area of the regions described below.

   (a) The region bounded by the parabola $x = (y+3)^2 - 4$ and the line $x = 3y + 9$.

   (b) The region bounded by $y = \dfrac{4}{3+x^2}$ and $y = 1$.

   (c) The region bounded by $y = 2\ln(x+1)$, the x-axis and the line $x = 4$.

   (d) The region to the right of the y-axis, above the graph of $y = \sec(x)^2$ and below the graph of $y = 2\sec(x)$.

4. Suppose that f is an **even** function such that

$$\int_{-9}^{5} f(x)dx = -13 \quad \text{and} \quad \int_{0}^{9} f(x)dx = 4.$$

   Evaluate the definite integrals below.

(a) $\displaystyle\int_{-9}^{9} f(x)dx$ (b) $\displaystyle\int_{0}^{5} (4x - 3f(x))dx$ (c) $\displaystyle\int_{-3}^{3} xf(x)dx$ (d) $\displaystyle\int_{0}^{3} xf(x^2)dx$

5. Find the average value of the following functions on the given interval.

(a) $f(x) = \dfrac{3}{\sqrt{100 - x^2}}$ on $[0, 5]$.

(b) $f(x) = x\sqrt[3]{3x - 7}$ on $[2, 5]$.

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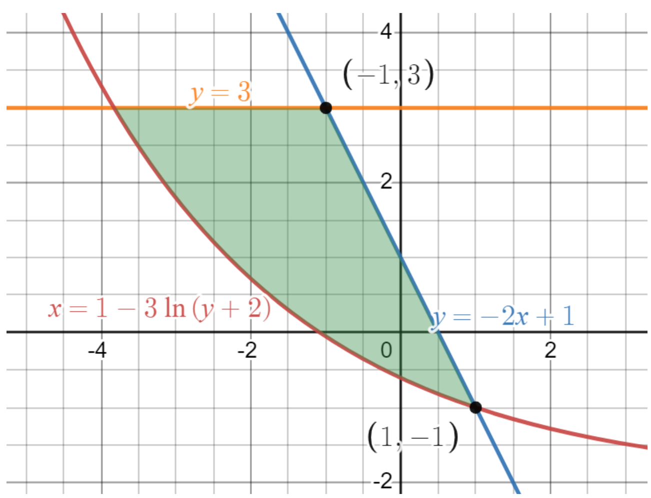
Section 6.1: Volume by Cross-Sections - Worksheet

1. Consider the region $\mathcal{R}$ in the first quadrant bounded by the curve $x = 4 - (y-1)^2$.
   (a) Sketch the region. Make sure to clearly label the curve and its intercepts.
   (b) A solid has base $\mathcal{R}$ and cross-sections perpendicular to the y-axis. Calculate the volume of the solid if the cross-sections are (i) semi-circles with diameter in the base and (ii) equilateral triangles with a side in the base.
   (c) A solid has base $\mathcal{R}$ and its cross-sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base. Calculate the volume of the solid.
   (d) Calculate the volume of the solid of revolution obtained by revolving $\mathcal{R}$ about (i) the y-axis and (ii) the line $y = -2$.

2. Use the method of disks/washers to calculate the volume of the solids of revolutions obtained by revolving the regions described below about the given axis.
   (a) The region below the graph of $y = \ln(x)$ on $1 \leqslant x \leqslant 3$ revolved about the line $x = 3$.
   (b) The region below the graph of $y = \dfrac{1}{\sqrt{25+4x^2}}$ on $0 \leqslant x \leqslant \dfrac{5}{2}$ revolved about the x-axis.
   (c) The region bounded by $y = e^x$, $y = 4 - e^x$ and the coordinate axes revolved about the line $y = 4$.
   (d) The region below the graph of $y = 2\sin^{-1}\left(x^2\right)$ on $0 \leqslant x \leqslant 1$ revolved about the y-axis.

3. Consider the region $\mathcal{R}$ shaded in the figure below.



Use the method of washers to set-up integrals that compute the volume of the solid obtained by revolving $\mathcal{R}$ about the line

(a) $x = 2$, (b) $y = 3$, (c) $x = -4$, (d) $y = -2$.

You do not need to evaluate the integrals.

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Section 6.2: Volume by Shells - Worksheet

1. Find the volume of the solid of revolution obtained by revolving the given region about the given axis using (i) the method of cylindrical shells and (ii) the method of disks/washers.

   (a) The region bounded by the y-axis, the curve $y = 5\sqrt{x}$ and the line $y = 10$ revolved about the line $x = -2$.

   (b) The region in the first quadrant bounded by the curves $y = 9 - x^2$ and $y = 1 - \frac{1}{3}x$ revolved about the line $y = 9$.

   (c) The region below the graph of $y = \dfrac{2}{\sqrt{x+1}}$ for $0 \leqslant x \leqslant 3$ revolved about the y-axis.

   (d) The region below the graph of $y = \dfrac{2}{\sqrt{x+1}}$ for $0 \leqslant x \leqslant 3$ revolved about the x-axis.

2. Find the volume of the solid of revolution obtained by revolving the given region about the given axis using the method of cylindrical shells.

   (a) The region bounded by the curve $y = 3\sqrt{\ln(x)}$, the line $y = 3$ and the line $x = 1$ revolved about the x-axis.

   (b) The region below the graph of $y = \dfrac{1}{16 + x^4}$ for $0 \leqslant x \leqslant 2$ revolved about the y-axis.

   (c) The region bounded by the curve $x = 4 - (y+1)^2$, the line $x = 4$ and the line $y = x + 7$ revolved about the line $y = -5$.

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Section 6.3: Arc Length - Worksheet

1. Calculate the arc length of the given curves.

   (a) $y = 11 - 2(x-5)^{3/2}$, $5 \leqslant x \leqslant 6$.

   (b) $x = \frac{1}{4}\sqrt[3]{y} - \frac{9}{5}\sqrt[3]{y^5}$, $1 \leqslant y \leqslant 2$.

   (c) $x = \sqrt{16y - y^2}$, $4 \leqslant y \leqslant 12$.

   (d) $y = \frac{1}{6}\ln\left(\sin(3x)\cos(3x)\right)$, $\frac{\pi}{18} \leqslant x \leqslant \frac{\pi}{9}$.

   (e) $y = \frac{e^{5x} + e^{-5x}}{10}$, $0 \leqslant x \leqslant \frac{1}{5}$.

   (f) $x = \frac{4}{5}y^{5/4}$, $0 \leqslant y \leqslant 9$.

2. Find a curve of the form $y = f(x)$ passing through $(4, 13)$, having negative derivative, and whose length integral on $1 \leqslant x \leqslant 7$ is given by

$$L = \int_1^7 \sqrt{1 + \frac{25}{x^3}}\,dx.$$

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Section 6.4: Areas of Surfaces of Revolution - Worksheet

1. Find the surface area obtained by revolving the given curve about the given axis.
   (a) The curve $y = \sqrt{3x-5}$, $2 \leqslant x \leqslant 3$, revolved about the x-axis.
   (b) The curve $x = \sqrt{16y - y^2}$, $0 \leqslant y \leqslant 8$, revolved about the y-axis.
   (c) The curve $x = 2\sqrt[3]{y}$, $0 \leqslant y \leqslant 1$, revolved about the x-axis.
   (d) The curve $x = \dfrac{3}{5}y^{5/3}$, $0 \leqslant y \leqslant 1$, revolved about the y-axis.
   (e) The curve $y = x^{3/2}$, $1 \leqslant x \leqslant 4$, revolved about the y-axis.

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Section 8.2: Integration by Parts - Worksheet

1. Evaluate the following antiderivatives.

(a) $\int x^3 \cos(5x)dx$

(b) $\int x^2 \sin^{-1}(x)dx$

(c) $\int \frac{\ln(x)}{x^5}dx$

(d) $\int x^3 e^{-x^2} dx$

(e) $\int e^{-2x} \sin(3x)dx$

(f) $\int x \sec(5x)^2 dx$

2. Calculate the volume of the solid obtained by revolving the given region about the given axis using (i) the method of disks/washers and (ii) the method of cylindrical shells.

(a) The region between the graph of $y = \sqrt{\tan^{-1}(x)}$ and the x-axis for $0 \leqslant x \leqslant 1$ revolved about the x-axis.

(b) The region bounded by the y-axis, the graph of $y = \sin(x)$ and the line $y = 1$ revolved about the y-axis.

(c) The region between the graph of $y = \ln(x)$ and the x-axis for $1 \leqslant x \leqslant e$ revolved about the line $x = -2$.

3. Find reduction formulas for the following integrals.

(a) $\int \cos(3x)^n dx$

(b) $\int \ln(x)^n dx$

(c) $\int \sec(5x)^n dx$

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Section 8.3: Trigonometric Integrals - Worksheet

1. Calculate the following integrals.

(a) $\displaystyle\int \sin(5x)^2 dx$

(b) $\displaystyle\int \sec(2x)^4 \tan(2x)^6 dx$

(c) $\displaystyle\int_0^{\pi/21} \tan(7\theta)^3 d\theta$

(d) $\displaystyle\int \sec(3x)^2 \ln(\sec(3x)) dx$

(e) $\displaystyle\int_\pi^{3\pi/2} \cos(z)^5 \sin(z)^8 dz$

(f) $\displaystyle\int_{\pi/3}^{\pi/2} \sqrt{\frac{1+\cos(t)}{1-\cos(t)}} dt$

2. Express $\displaystyle\int \sin(3x)^n dx$ in terms of $\displaystyle\int \sin(3x)^{n-2} dx$.

3. Consider the region bounded by the x-axis, the graph of $y = \sec(x)^2 \tan(x)$ and the lines $x = 0$, $x = \frac{\pi}{4}$. Calculate the volume of the solid obtained by revolving $\mathcal{R}$ about (a) the x-axis, (b) the y-axis.

4. Evaluate $\displaystyle\int \sec(\theta)^3 d\theta$ and $\displaystyle\int \sec(\theta) \tan(\theta)^2 d\theta$.

5. Calculate the arc length of the curve $y = x + \cos(x)\sin(x) - \dfrac{1}{8}\tan(x)$, $0 \leqslant x \leqslant \dfrac{\pi}{4}$.

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Section 8.4: Trigonometric Substitution - Worksheet

1. Calculate the following integrals.

   (a) $\displaystyle\int \frac{\sqrt{25x^2-4}}{x}dx$ for $x > \frac{2}{5}$.

   (b) $\displaystyle\int \frac{dt}{t\sqrt{9+\ln(t)^2}}$.

   (c) $\displaystyle\int \frac{dx}{(6x-x^2-5)^{5/2}}$.

   (d) $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{x(2x^2-1)^{3/2}}$.

   (e) $\displaystyle\int \frac{e^{6x}}{\sqrt{16-e^{4x}}}dx$.

   (f) $\displaystyle\int_5^{11} \frac{dx}{(x^2-10x+61)^{5/2}}$.

2. Calculate the average value of the function $f(x) = \dfrac{1}{x\sqrt{64-x^2}}$ on the interval $[4, 4\sqrt{2}]$.

3. (a) Evaluate $\displaystyle\int \sqrt{1+x^2}dx$.

   (b) Use your result from part (a) for the following applications.

   (i) Calculate the length of the curve $y = x^2$, $0 \leqslant x \leqslant 1$.

   (ii) Calculate the area of the surface obtained by revolving the curve $y = e^x$, $0 \leqslant x \leqslant \ln(2)$, about the x-axis.

   (iii) Calculate the area of the surface obtained by revolving the curve $y = \sin^{-1}(x)$, $0 \leqslant x \leqslant 1$ about the y-axis.

4. Calculate the area of the region inside the circle of equation $x^2 - 2x + y^2 = 3$ and above the line $y = \sqrt{3}$.

5. Consider the region $\mathcal{R}$ bounded between the graph of $y = \dfrac{1}{16-x^2}$ and the x-axis for $0 \leqslant x \leqslant 2$. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = -3$.

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Section 8.8: Improper Integrals - Worksheet

1. Calculate the following integrals or determine if they diverge.

(a) $\displaystyle\int_0^{\infty} e^{-5x}dx$

(b) $\displaystyle\int_0^{\pi/4} \csc(x)dx$

(c) $\displaystyle\int_{-\infty}^{0} xe^{3x}dx$

(d) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(16+x^2)^{3/2}}$

(e) $\displaystyle\int_0^1 \ln(x)dx$

(f) $\displaystyle\int_{-2}^{1} \frac{dx}{\sqrt[3]{3x-2}}$

(g) $\displaystyle\int_0^{3/2} \frac{dx}{\sqrt{9-4x^2}}$

(h) $\displaystyle\int_e^{\infty} \frac{dx}{x\ln(x)}$

(i) $\displaystyle\int_0^{\infty} e^{-x}\sin(x)dx$

2. Use a convergence test to determine if the following improper integrals converge or diverge.

(a) $\displaystyle\int_3^{\infty} \frac{dx}{xe^x}$

(b) $\displaystyle\int_1^{\infty} \frac{dx}{x^2+3x+1}$

(c) $\displaystyle\int_4^{\infty} \frac{\cos(x)+5}{x^{3/5}}dx$

(d) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}+x^2}$

(e) $\displaystyle\int_5^{\infty} \frac{xdx}{x^4-1}$

(f) $\displaystyle\int_1^{\infty} \frac{x^3+5x^2+1}{\sqrt{x^7+4x+2}}dx$

3. Consider the unbounded region $\mathcal{R}$ between the graph of $y = \dfrac{\ln(x)}{x}$ and the x-axis for $x \geqslant 1$.

(a) Find the area of the region $\mathcal{R}$ or determine if $\mathcal{R}$ has infinite area.

(b) We now revolve the region $\mathcal{R}$ about the x-axis to form a solid of revolution. Calculate the volume of the solid or determine if the solid has infinite volume.

(c) We now revolve the region $\mathcal{R}$ about the y-axis to form a solid of revolution. Calculate the volume of the solid or determine if the solid has infinite volume.

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Section 10.1: Sequences - Worksheet

1. Determine if the sequences below converge or diverge. In case of convergence, find the limit.

   (a) $a_n = \dfrac{\sqrt{1+16n^4}}{n^2+1}$

   (b) $a_n = \dfrac{5n+4}{2\cos(n)^2+3n}$

   (c) $a_n = \tan^{-1}(1-\sqrt{n})$

   (d) $a_n = \dfrac{n+(-1)^n}{n^3+1}$

   (e) $a_n = \dfrac{4^n-5^{2n}}{7^n}$

   (f) $a_n = \cos\left(\dfrac{5}{\sqrt{n}}\right)^n$

   (g) $a_n = \left(\dfrac{n+5}{n+3}\right)^{4n}$

   (h) $a_n = (2n+1)^{3/n}$

   (i) $a_n = \sin(n\pi)e^n$

2. Suppose that a_n is a sequence defined inductively by

$$\begin{cases} a_1 = 2, \\ a_{n+1} = \dfrac{5}{a_n+4} \text{ for } n \geqslant 1. \end{cases}$$

   (a) Find the first 4 terms of the sequence $\{a_n\}$.

   (b) The sequence $\{a_n\}$ converges. Find its limit.

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Section 10.2: Infinite Series - Worksheet

1. Each of the series $\sum_{n=n_0}^{\infty} a_n$ below is either geometric or telescoping. For each series, find a formula for the partial sum $S_N = \sum_{n=n_0}^{N} a_n$, then determine if the series converges or diverges, and compute its sum if it does.

   (a) $\sum_{n=4}^{\infty} 2^n 3^{-n}$

   (b) $\sum_{n=0}^{\infty} \left(\frac{4}{2n+1} - \frac{4}{2n+5}\right)$

   (c) $\sum_{n=0}^{\infty} \frac{1-3\cdot 4^{2n}}{5^{n-1}}$

   (d) $\sum_{n=3}^{\infty} \ln\left(\frac{3n+1}{3n+4}\right)$

   (e) $\sum_{n=0}^{\infty} 5\cdot 3^{1-2n}$

   (f) $\sum_{n=1}^{\infty} \left(\tan^{-1}(n+1) - \tan^{-1}(n)\right)$

2. Use geometric series to express the repeating decimals below as a fraction of two integers.

   (a) $1.5222\cdots = 1.5\overline{2}$

   (b) $0.126126\cdots = 0.\overline{126}$

3. For each sequence $\{a_n\}_{n=n_0}^{\infty}$ given below, determine

   (i) whether the **sequence** $\{a_n\}_{n=n_0}^{\infty}$ converges or diverges. If the sequence converges, find its limit.

   (ii) whether the **series** $\sum_{n=n_0}^{\infty} a_n$ converges or diverges. If the series converges, find its sum if possible.

   (a) $\left\{\left(1+\frac{4}{n}\right)^n\right\}_{n=1}^{\infty}$

   (b) $\left\{\sqrt{n+1}-\sqrt{n}\right\}_{n=0}^{\infty}$

   (c) $\{e^{-n}\}_{n=0}^{\infty}$

   (d) $\left\{\frac{e^{5n}}{n^{3/2}}\right\}_{n=1}^{\infty}$

4. Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{2\cdot 5^{n+1}}$. Find the values of x for which the series converges and find the sum of the series when it converges.

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Section 10.3: The Integral Test - Worksheet

1. For each sequence $\{a_n\}_{n=n_0}^{\infty}$ given below, determine
   (i) whether the **sequence** $\{a_n\}_{n=n_0}^{\infty}$ converges or diverges. If the sequence converges, find its limit.
   (ii) whether the **series** $\sum_{n=n_0}^{\infty} a_n$ converges or diverges. If the series converges, find its sum if possible.

   Note: the integral test is not possible/necessary for all the series.

   (a) $\{n5^{-n}\}_{n=0}^{\infty}$

   (b) $\left\{\dfrac{1}{n\left(1+\ln(n)^2\right)}\right\}_{n=2}^{\infty}$

   (c) $\left\{\dfrac{1}{n^{\log_5(3)}}\right\}_{n=1}^{\infty}$

   (d) $\left\{\cos\left(n^{1/n}\right)\right\}_{n=1}^{\infty}$

   (e) $\left\{\dfrac{1}{(n^2+9)^{3/2}}\right\}_{n=0}^{\infty}$

   (f) $\left\{\sec\left(\frac{\pi}{n}\right)-\sec\left(\frac{\pi}{n+1}\right)\right\}_{n=3}^{\infty}$

   (g) $\left\{2^{2n+1}5^{-n}\right\}_{n=0}^{\infty}$

   (h) $\left\{\left(1+\dfrac{1}{2n}\right)^{n}\right\}_{n=1}^{\infty}$

   (i) $\left\{\dfrac{1}{n\ln(n)\ln(\ln(n))}\right\}_{n=4}^{\infty}$

2. (a) Determine for which values of p the series $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln(n)^p}$ converges or diverges.

   (b) Determine for which values of p the series $\displaystyle\sum_{n=1}^{\infty}\frac{\ln(n)}{n^p}$ converges or diverges.

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Section 10.4: Comparison Tests - Worksheet

Determine if the series below converge or diverge. Make sure to clearly label and justify the use of any convergence test used. **Note:** some of these problems require convergence tests from previous sections.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{(5\sqrt{n}-2)^3}{3n^2-2n+4}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{3^n}{n5^n}$

3. $\displaystyle\sum_{n=0}^{\infty} \frac{2^{2n}}{3^n+11n^2}$

4. $\displaystyle\sum_{n=3}^{\infty} \frac{\ln(n)^2}{\sqrt{n}}$

5. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}\ln(n)^2}$

6. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)^2}$

7. $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n}{n+3}\right)^n$

8. $\displaystyle\sum_{n=1}^{\infty} \frac{7-3\cos\left(n^2\right)}{n^5+3}$

9. $\displaystyle\sum_{n=2}^{\infty} n\sin\left(\frac{5}{n^3}\right)$

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Section 10.5: Absolute Convergence, Ratio & Root Tests - Worksheet

1. Determine if the series below converge or diverge. Make sure to clearly label and justify the use of any convergence test used. **Note:** some of these problems require convergence tests from previous sections.

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^n}{3^{2n}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt[3]{8n^6+7n+11}}{3n^7+8n^5-1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{(2n+1)!}{e^n n!(n+1)!}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(8n)+3}{4^n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{\ln(\ln(n))}$

(f) $\displaystyle\sum_{n=1}^{\infty} 4^n \left(\frac{n-2}{n}\right)^{n^2}$

(g) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{((2n)!)^2}{(4n)!}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{3^n(n+2)!}$

(i) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n+5\sin(n)}{3n}\right)^n$

2. Let a_n be the sequence defined recursively by

$$a_1 = 7, \quad a_{n+1} = a_n \left(\frac{n}{n+3}\right)^n \text{ for } n \geqslant 1.$$

Determine whether the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges or diverges. Make sure to clearly label and justify the use of any convergence test used.

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Section 10.6: Alternating Series & Conditional Convergence - Worksheet

1. Determine if the series below converge absolutely, converge conditionally or diverge. Make sure to clearly label and justify the use of any convergence test used. **Note:** some of these problems require convergence tests from previous sections.

   (a) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{n\log_2(n)}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n\arctan(n)}{\sqrt[3]{8n^6+1}}$

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{3^n+\cos(n)}$

   (e) $\displaystyle\sum_{n=2}^{\infty} \frac{\sec(\pi n)}{\sqrt{n}}$

   (f) $\displaystyle\sum_{n=2}^{\infty} (-1)^n \ln\left(\frac{n+1}{n}\right)$

   (g) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{e^{\sqrt{n}}}$

   (h) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{n}{2n+1}$

   (i) $\displaystyle\sum_{n=3}^{\infty} \cos\left(\frac{\pi}{n}\right)^{n^2}$

2. Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{7n+4}}$.

   (a) Show that this series meets the conditions of the Alternating Series Estimation Theorem.

   (b) Find the smallest integer N for which the partial sum $S_N = \displaystyle\sum_{n=1}^{N} \frac{(-1)^n}{\sqrt[3]{7n+4}}$ approximates the sum of the series with an error of at most 0.1.

3. Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2^{3n-7}+9}$.

   (a) Show that this series meets the conditions of the Alternating Series Estimation Theorem.

   (b) Find the smallest integer N for which the partial sum $S_N = \displaystyle\sum_{n=0}^{N} \frac{(-1)^{n+1}}{2^{3n-7}+9}$ approximates the sum of the series with an error of at most 10^{-3}.

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Section 10.7: Power Series - Worksheet

1. Find the radius and interval of convergence of the power series below. Specify for which values of x in the interval of convergence the series converges absolutely and for which it converges conditionally.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{\sqrt[3]{n}5^n}$.

   (b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-9)^{3n}}{8^n (n+1)}$.

   (c) $\displaystyle\sum_{n=0}^{\infty} n3^n (2x+1)^n$.

   (d) $\displaystyle\sum_{n=0}^{\infty} \frac{n^n (x+2)^n}{6^n}$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n (x-4)^{2n}}{36^n \sqrt{n}}$.

   (f) $\displaystyle\sum_{n=0}^{\infty} \frac{(3x+2)^n}{n^2+4}$.

2. Find the radius of convergence of the following power series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^{2n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \left(1-\frac{3}{n}\right)^{n^2} (x+5)^n$.

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$.

3. Suppose that a power series converges absolutely at $x = 5$, converges conditionally at $x = -3$ and diverges at $x = 11$. What can you say, if anything, about the convergence or divergence of the power series at the following values of x?

   (a) $x = -4$.

   (b) $x = 2$.

   (c) $x = 15$.

   (d) $x = 7$.

4. Let $f(x) = \dfrac{3}{2+7x}$. Use the power series representation of $\dfrac{1}{1-x}$ and power series operations to find a power series representation of $f(x)$ centered at $a = 0$. What are the radius and interval of convergence of the resulting power series?

5. Consider the power series $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n (n+1)}$.

   (a) Find the radius and interval of convergence of f.

   (b) Find a power series representation of $f'(x)$ centered at $a = -1$. What are its radius and interval of convergence?

   (c) Let $g(x)$ be the antiderivative of $f(x)$ such that $g(-1) = -8$. Find a power series representation of $g(x)$ centered at $a = -1$. What are its radius and interval of convergence?

6. (a) Use term-by-term differentiation to find a power series representation of $\dfrac{1}{(1-x)^2}$. What is its raidus of convergence?

   (b) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{5^n}$.

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Section 10.8: Taylor and Maclaurin Series - Worksheet

1. Find the Taylor polynomials for the following functions at the order and center indicated.

   (a) $f(x) = 2\cos\left(\frac{\pi}{3} - 5x\right)$, $T_4(x)$ at $a = 0$.

   (b) $f(x) = \sqrt[3]{4+2x}$, $T_3(x)$ at $a = 2$.

   (c) $f(x) = 2^{3-x}$, $T_4(x)$ at $a = 1$.

   (d) $f(x) = \ln(\cos(x))$, $T_3(x)$ at $a = \frac{\pi}{4}$.

   (e) $f(x) = \frac{6}{5-3x}$, $T_4(x)$ at $a = 1$.

   (f) $f(x) = \ln(5+x)$, $T_3(x)$ at $a = -4$.

2. In 1.(b), you found the third degree Taylor polynomial of $f(x) = \sqrt[3]{4+2x}$ centered at $a = 2$. Use this Taylor polynomial to estimate $\sqrt[3]{8.6}$.

3. Consider the function $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{9^n(n+1)}(x-4)^{2n+1}$.

   (a) Find the radius and interval of convergence of f.

   (b) Find $f^{(7)}(4)$, $f^{(8)}(4)$ and $f^{(9)}(4)$.

4. Use the reference Maclaurin series to calculate the Maclaurin series of the following functions.

   (a) $f(x) = x^7\cos(4x^5)$.

   (b) $f(x) = e^{-x^3} - 1 + x^3$.

   (c) $f(x) = \sin(2x) - 2\tan^{-1}(x)$.

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Section 10.9: Convergence of Taylor Series - Worksheet

1. Use the Remainder Estimation Theorem to estimate the error made when approximating $f(x) = \sqrt{1+3x}$ by its 2^{nd} degree Maclaurin polynomial $T_2(x)$ on the interval $[0, 0.1]$.

2. Consider the function $f(x) = \dfrac{1}{1+2x}$.
   (a) Find the Maclaurin series of f using geometric series. What are its radius and interval of convergence?
   (b) Find the Maclaurin series of f using its definition. (Hint: compute the first few derivatives of f and identify a pattern to find a formula for $f^{(n)}(x)$.)
   (c) Find is the smallest integer N for which the Maclaurin polynomial $T_N(x)$ of $f(x)$ approximates $f(x)$ with an error of at most 10^{-4} on the the interval $[-0.1, 0.1]$.

3. Find the Maclaurin series of the following functions.
   (a) $f(x) = x^4 \ln(1 - 5x^3)$.
   (b) $f(x) = e^{-x^2/2} - \cos(x)$.

4. Find the first three non-zero terms of the Maclaurin series of $\sin(2x)e^{3x}$.

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Section 10.10: Applications of Taylor Series - Worksheet

1. Use Maclaurin series to compute the following limits.

   (a) $\lim_{x\to 0} \frac{e^{-2x^2} - \cos(2x)}{x^2 \ln(1+5x) - 5x^3}$.

   (b) $\lim_{x\to\infty} x^3 \left(\tan^{-1}\left(\frac{4}{x}\right) - 2\sin\left(\frac{2}{x}\right)\right)$.

   (c) $\lim_{x\to 0} \frac{\sin(x^6)}{\cos(x^3) - 1}$.

   (d) $\lim_{x\to\infty} x^2 \left(5\ln\left(1+\frac{3}{x}\right) - 3\ln\left(1+\frac{5}{x}\right)\right)$.

2. Use Maclaurin series to write each integral below as the sum of an infinite series of numbers (your series should not contain x).

   (a) $\int_0^{1/2} \cos(5x^2)dx$.

   (b) $\int_0^1 x^3 e^{-4x^3} dx$.

   (c) $\int_0^{1/3} x^7 \sin(2x^5)dx$.

3. (a) Use Maclaurin series to write the integral $I = \int_0^1 e^{-x^2} dx$ as the sum of an infinite series of numbers.

   (b) Use the Alternating Series Estimation Theorem to find how many terms of the series found in (a) need to be summed in order to obtain an approximation of I with an error of less than 10^{-5}.

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Sections 11.1, 11.2: Parametric Curves - Worksheet

1. Find an equation of the tangent line to the given parametric curve at the point defined by the given value of t.

   (a) $\begin{cases} x = 5t^2 - 7 \\ y = t^4 - 3t \end{cases}$, $t = -1$. (b) $\begin{cases} x = e^{4t} - e^t + 2 \\ y = t - 3e^{2t} \end{cases}$, $t = 0$. (c) $\begin{cases} x = \sec(3t) \\ y = \cot(2t - \pi) \end{cases}$, $t = \frac{\pi}{12}$.

2. Find all points on the following parametric curves where the tangent line is (i) horizontal, and (ii) vertical.

   (a) $\begin{cases} x = \sin(2t) + 1 \\ y = \cos(t) \end{cases}$, $0 \leqslant t < 2\pi$.

   (b) $\begin{cases} x = 3t - t^3 \\ y = t^2 + 4t + 3 \end{cases}$

   (c) $\begin{cases} x = 4t - e^{2t} \\ y = t^2 - 18 \ln |t| \end{cases}$

3. Consider the ellipse of equation $x^2 + 4y^2 = 4$.

   (a) Find a parametrization of the ellipse.

   (b) Find the area enclosed by the ellipse.

   (c) Find the area of the surface obtained by revolving the top-half of the ellipse about the x-axis.

4. For each of the following parametric curves: (i) find the arc length, (ii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the x-axis and (iii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the y-axis.

   (a) $\begin{cases} x = e^{4t} \\ y = e^{5t} \end{cases}$, $0 \leqslant t \leqslant 1$. (b) $\begin{cases} x = \ln(t) \\ y = \sin^{-1}(t) \end{cases}$, $\frac{1}{2} \leqslant t \leqslant \frac{1}{\sqrt{2}}$. (c) $\begin{cases} x = t^3 - t \\ y = \sqrt{3}t^2 \end{cases}$, $0 \leqslant t \leqslant 1$.

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Sections 11.3, 11.4: Polar Coordinates - Worksheet

1. Convert the following Cartesian equations to polar.

   (a) $y = 11$.
   (b) $x + y = 0$.
   (c) $(x-3)^2 + y^2 = 9$.
   (d) $y = 7 + 2x$.
   (e) $x^2 + y^2 + xy = 2$.
   (f) $y^2 = 3x^2$.

2. Convert the following polar equations to Cartesian. Then describe the graph.

   (a) $r = -7\sec(\theta)$.
   (b) $r = \dfrac{5}{3\sin(\theta) - 4\cos(\theta)}$.
   (c) $\theta = \dfrac{\pi}{6}$.
   (d) $r = 3\cot(\theta)\csc(\theta)$.
   (e) $r = 5\sin(\theta)$.
   (f) $r = 2\cos(\theta) + 6\sin(\theta)$.

3. For each the following polar curves, identify the symmetries and sketch the graph.

   (a) $r = 9$.
   (b) $r = 8\cos(\theta)$.
   (c) $r = 3\sin(2\theta)$.
   (d) $r = 5\cos(5\theta)$.
   (e) $r = 1 - \cos(\theta)$.
   (f) $r = \sqrt{3} + 2\sin(\theta)$.

4. Find an equation of the tangent line to the following polar curves at the given value of θ.

   (a) $r = \cos(3\theta)$, $\theta = \dfrac{\pi}{4}$.
   (b) $r = 1 + 2\sin(\theta)$, $\theta = \dfrac{\pi}{6}$.

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Sections 11.5: Areas and Lengths in Polar Coordinates - Worksheet

1. Find the areas of the given regions.
   (a) The region shared by the circles $r = 2\sin(\theta)$ and $r = 2\cos(\theta)$.
   (b) The region contained inside the leaves of the rose $r = 6\sin(2\theta)$ and outside the circle $r = 3$.
   (c) The region inside the cardioid $r = 1 + \sin(\theta)$ and below the line $x = \sqrt{3}y$.
   (d) The region inside the circle $r = \cos(\theta)$ and outside the cardioid $r = 1 - \cos(\theta)$.
   (e) The region shared by one leaf of the rose $r = 2\cos(3\theta)$ and the circle $r = 1$.

2. Consider the region $\mathcal{R}$ contained in the circle $r = 4\cos(\theta)$ to the right of the line $x = 3$.
   (a) Find the area of the region $\mathcal{R}$ using integration with respect to x.
   (b) Find the area of the region $\mathcal{R}$ using integration with respect to y.
   (c) Find the area of the region $\mathcal{R}$ using integration with respect to θ.

3. Find the lengths of the given polar curves.
   (a) $r = \sqrt{1 + \cos(2\theta)}$, $0 \leqslant \theta \leqslant \frac{\pi}{2}$.
   (b) $r = \dfrac{2}{1 - \cos(\theta)}$, $\frac{\pi}{2} \leqslant \theta \leqslant \pi$.
   (c) $r = e^{3\theta}$, $0 \leqslant \theta \leqslant \pi$.