Rutgers University
Math 152

Midterm 1 Practice Problems

1. Evaluate the following antiderivatives.

(a) $\displaystyle\int \frac{dx}{x^2+10x+29}$ (b) $\displaystyle\int \frac{dt}{t\sqrt{1-9\ln(t)^2}}$ (c) $\displaystyle\int \frac{\sqrt{1+\sqrt{z}}}{\sqrt{z}}dz$

2. Let $\mathcal{R}$ be the region bounded by the lines $y=x$, $y=2x$ and $y=-2x+12$.

   (a) Sketch the region $\mathcal{R}$. Label the curves, their intersection points and lightly shade the region.

   (b) Calculate the area of the region using (i) an x-integral and (ii) a y-integral.

   (c) The base of a solid is $\mathcal{R}$, and the cross-sections perpendicular to the x-axis are isosceles right triangles with height on the base. Set-up an expression with integrals that calculates the volume of the solid.

   (d) The base of a solid is $\mathcal{R}$, and the cross-sections perpendicular to the y-axis are circles with diameter in the base. Set-up an expression with integrals that calculates the volume of the solid.

   (e) We create a solid of revolution by revolving $\mathcal{R}$ about the line $x=-2$. Set up an expression with integrals that calculates the volume of the solid using (i) the method of disks/washers and (ii) the method of cylindrical shells.

   (f) We create a solid of revolution by revolving $\mathcal{R}$ about the line $y=8$. Set up an expression with integrals that calculates the volume of the solid using (i) the method of disks/washers and (ii) the method of cylindrical shells.

3. Consider the region $\mathcal{R}$ traversed by the y-axis and bounded by the line $y=2$ and the graph of $y=\sec(x)$.

   (a) Sketch the region. Clearly label the curves and their intersection points.

   (b) Calculate the area of the region.

   (c) Calculate the volume of a solid with base $\mathcal{R}$ if the cross-sections perpendicular to the x-axis are (i) squares and (ii) rectangles of perimeter 8.

   (d) We revolve the region $\mathcal{R}$ to obtain a solid of revolution. Calculate the volume of the solid if the axis of revolution is (i) the x-axis and (ii) the line $y=2$.

4. Consider the disk of equation $(y-2)^2+x^2 \leqslant 1$ centered at $(0,2)$ of radius 1. A torus (or informally, a donut) is created by revolving the disk about the x-axis.

   (a) Find the volume of the torus using the washer method.

   (b) Find the volume of the torus using the shell method.

5. The two parts of this problem are independent.

(a) Let $\mathcal{R}$ be the region under the graph of $y = \dfrac{1}{\sqrt{16 - x^2}}$ for $0 \leqslant x \leqslant 2$. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = -3$.

(b) Let $\mathcal{R}$ be the region bounded by the coordinate axes, the graph of $y = \ln(x - 3)$ and the line $y = 2$. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = 12$.

6. Calculate the arc length of the given curves.

   (a) $x = \dfrac{6}{7} y^{7/6}$, $0 \leqslant y \leqslant 1$.

   (b) $y = \cos\left(\dfrac{x}{2}\right) - \ln\left(\csc\left(\dfrac{x}{2}\right) + \cot\left(\dfrac{x}{2}\right)\right)$, $\dfrac{\pi}{2} \leqslant x \leqslant \pi$.

7. Consider the region below the graph $y = \cot(5x) + \csc(5x)$ for $\frac{\pi}{20} \leqslant x \leqslant \frac{\pi}{10}$.

   (a) Find the area of the region.

   (b) Find the volume of the solid obtained by revolving the region about the x-axis.

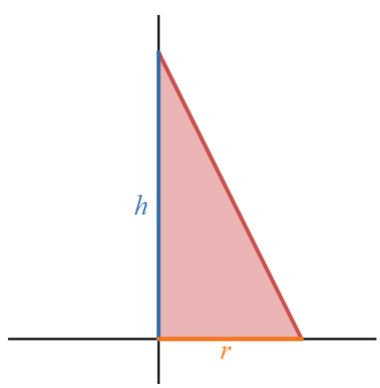
8. Calculate the surface area obtained by revolving the given curve about the given axis.

   (a) The curve $x = \dfrac{4}{3} y^{3/4}$, $0 \leqslant y \leqslant 1$, revolved about the y-axis.

   (b) The curve $y = 1 + \sqrt{2 - x^2}$, $0 \leqslant x \leqslant 1$, revolved about the x-axis.

9. Find the area of the surface obtained by revolving the curve $y = ax^2$, $0 \leqslant x \leqslant 1$, about the y-axis (where a is a positive constant).

10. A circular cone of height h and radius r is created by revolving a right triangle with base r, height h and right angle at the origin about the y-axis, see figure below.



   (a) The triangle is bounded by the coordinate axes and a line. Find an equation of the line bounding the triangle in terms of the constants r and h.

   (b) Calculate the volume of the cone using the disk/washer method.

   (c) Calculate the volume of the cone using the shell method.

   (d) Calculate the surface area of the side of the cone.