Rutgers University
Math 152

Midterm 2 Practice Problems

1. Let $\mathcal{R}$ the region between the graph of $y = \cos(3x)\sin(3x)$ and the x-axis on $0 \leqslant x \leqslant \frac{\pi}{6}$. Calculate the volume of the solid obtained by revolving $\mathcal{R}$ about (a) the x-axis, (b) the y-axis.

2. Evaluate the following definite or indefinite integrals.

   (a) $\displaystyle\int x^5 \ln(x)^2 dx.$

   (b) $\displaystyle\int \tan(\theta)^3 \sec(\theta)^5 d\theta.$

   (c) $\displaystyle\int \frac{dx}{\sqrt{x^2-6x}}$, $x > 6$.

   (d) $\displaystyle\int_{\sqrt{2}}^{2} \sec^{-1}(t) dt.$

   (e) $\displaystyle\int x^2 \tan^{-1}(x) dx.$

   (f) $\displaystyle\int \sin(5x)^4 dx.$

   (g) $\displaystyle\int \sin^{-1}(x)^2 dx.$

   (h) $\displaystyle\int \frac{x^2}{(1+9x^2)^{5/2}} dx.$

   (i) $\displaystyle\int_0^8 \sqrt{64+x^2} dx.$

3. Calculate the surface area obtained by revolving the curve $y = \dfrac{1}{2}x^2$, $0 \leqslant x \leqslant 1$, about the x-axis.

4. Evaluate $\displaystyle\int \frac{\sqrt{9-4x^2}}{x^2} dx$ two ways.

5. Use a comparison test to determine if the following improper integrals converge or diverge.

   (a) $\displaystyle\int_0^1 \frac{dx}{\sqrt{x}e^{3x}}.$

   (b) $\displaystyle\int_3^\infty \frac{2\cos(x)+5x^3}{13x^5+6x+7} dx.$

   (c) $\displaystyle\int_1^\infty \frac{\sqrt{x^3-1}}{x^2} dx.$

6. Evaluate the following improper integrals.

   (a) $\displaystyle\int_0^\infty \frac{dx}{(x^2+16)^2}.$

   (b) $\displaystyle\int_0^1 \frac{\ln(x)}{\sqrt[3]{x}} dx.$

   (c) $\displaystyle\int_0^\infty e^{-\sqrt{z}} dz.$

7. Find the sums of the following infinite series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-5)^n}{3^{2n+1}}.$

   (b) $\displaystyle\sum_{n=3}^{\infty} \left(\tan\left(\frac{\pi}{n}\right) - \tan\left(\frac{\pi}{n+2}\right)\right).$

8. Use geometric series to write the number $3.41232323\cdots$ as a quotient of two integers.

9. Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{3 \cdot 4^n}{(A-5)^{2n}}$, where A is an unspecified constant. Use this series in parts (a)-(b).

   (a) For which values of the constant A does the series converge?

   (b) For the values of A that you found in part (a), find the sum of the series.

10. Determine if the following series converge or diverge.

   (a) $\displaystyle\sum_{n=0}^{\infty} n^2 e^{-n}$.

   (b) $\displaystyle\sum_{n=3}^{\infty} \left(\frac{n+3}{n}\right)^n$.

   (c) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{3n-2}}{5^{n+1}}$.