Rutgers University
Math 152

Calculus 1 Review Worksheet - Solutions

1. Simplify the following expressions. Your answer should not involve any trigonometric or inverse trigonometric functions.

   (a) $\cos^{-1}\left(\frac{1}{2}\right)$

   Solution. Recall that $\cos^{-1}(x)$ is the angle θ in $[0, \pi]$ such that $\cos(\theta) = x$. Therefore

   $$\boxed{\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}}.$$

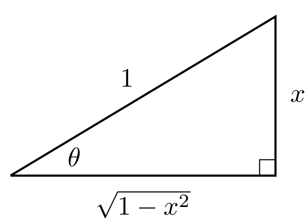
   (b) $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$

   Solution. Recall that $\sin^{-1}(x)$ is the angle θ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin(\theta) = x$. Since $\sin\left(\frac{7\pi}{4}\right) = -\frac{\sqrt{2}}{2}$, we deduce that

   $$\boxed{\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right) = -\frac{\pi}{4}}.$$

   (c) $\cos(\sin^{-1}(x))$

   Solution. Consider a right triangle with acute angle $\theta = \sin^{-1}(x)$. Then $\sin(\theta) = x = \frac{\text{opp}}{\text{hyp}}$. So we can choose the sides to be opp $= x$ and hyp $= 1$ as shown below.



   The Pythagoran identity then gives adj $= \sqrt{\text{hyp}^2 - \text{opp}^2} = \sqrt{1 - x^2}$. Therefore

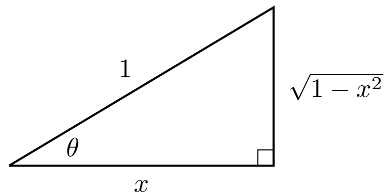
   $$\cos(\sin^{-1}(x)) = \frac{\text{adj}}{\text{hyp}} = \boxed{\sqrt{1 - x^2}}.$$

(d) $\sin(2\cos^{-1}(x))$

Solution. First, using the trigonometric identity $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, we have

$$\sin(2\cos^{-1}(x)) = 2\sin(\sin^{-1}(x))\sin(\cos^{-1}(x)) = 2x\sin(\cos^{-1}(x)).$$

To simplify $\sin(\cos^{-1}(x))$, we consider a right triangle with acute angle $\theta = \cos^{-1}(x)$. Then $\cos(\theta) = x = \frac{\text{adj}}{\text{hyp}}$. So we can choose the sides to be adj $= x$ and hyp $= 1$ as shown below.



The Pythagoran identity then gives opp $= \sqrt{\text{hyp}^2 - \text{adj}^2} = \sqrt{1-x^2}$. Therefore

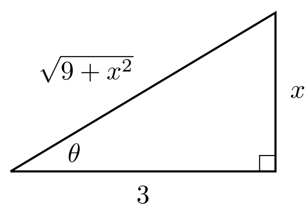
$$\sin(\cos^{-1}(x)) = \frac{\text{opp}}{\text{hyp}} = \sqrt{1-x^2}.$$

We conclude that

$$\boxed{\sin(2\cos^{-1}(x)) == 2x\sqrt{1-x^2}}.$$

(e) $\sec\left(\tan^{-1}\left(\frac{x}{3}\right)\right)$

Solution. Consider a right triangle with acute angle $\theta = \tan^{-1}\left(\frac{x}{3}\right)$. Then $\tan(\theta) = \frac{x}{3} = \frac{\text{opp}}{\text{adj}}$. So we can choose the sides to be opp $= x$ and adj $= 3$ as shown below.

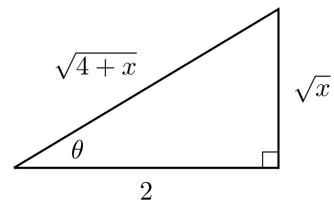


The Pythagoran identity then gives hyp $= \sqrt{\text{adj}^2 + \text{opp}^2} = \sqrt{9+x^2}$. Therefore

$$\sec\left(\tan^{-1}\left(\frac{x}{3}\right)\right) = \frac{\text{hyp}}{\text{adj}} = \boxed{\frac{\sqrt{9+x^2}}{3}}.$$

(f) $\sin\left(\cot^{-1}\left(\frac{2}{\sqrt{x}}\right)\right)$

Solution. Consider a right triangle with acute angle $\theta = \cot^{-1}\left(\frac{2}{\sqrt{x}}\right)$. Then $\cot(\theta) = \frac{2}{\sqrt{x}} = \frac{\text{adj}}{\text{opp}}$. So we can choose the sides to be adj $= 2$ and opp $= \sqrt{x}$ as shown below.



The Pythagorean identity then gives $\text{hyp} = \sqrt{\text{adj}^2 + \text{opp}^2} = \sqrt{4+x}$. Therefore

$$\sin\left(\cot^{-1}\left(\frac{2}{\sqrt{x}}\right)\right) = \frac{\text{opp}}{\text{hyp}} = \boxed{\frac{\sqrt{x}}{\sqrt{4+x}}}.$$

2. Calculate the following limits.

(a) $\displaystyle\lim_{x\to\infty} \frac{\ln(x)^2}{\sqrt{x}}$

Solution. We can compute this limit using L'Hôpital's Rule twice with the indeterminate form $\frac{\infty}{\infty}$. This gives

$$\begin{aligned}
\lim_{x\to\infty} \frac{\ln(x)^2}{\sqrt{x}} &\overset{\text{L'H}}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{2\ln(x)\frac{1}{x}}{\frac{1}{2\sqrt{x}}} \\
&= \lim_{x\to\infty} \frac{4\ln(x)}{\sqrt{x}} \\
&\overset{\text{L'H}}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{\frac{4}{x}}{\frac{1}{2\sqrt{x}}} \\
&= \lim_{x\to\infty} \frac{8}{\sqrt{x}} \\
&= \boxed{0}.
\end{aligned}$$

(b) $\displaystyle\lim_{x\to 0} \frac{5^x - 3^x}{\sin(2x)}$

Solution. This limit is an indeterminate form $\frac{0}{0}$. We can resolve the indeterminate form using L'Hôpital's Rule, remembering that for a positive constant a, we have

$$\frac{d}{dx}a^x = \ln(a)a^x.$$

We obtain

$$\begin{aligned}
\lim_{x\to 0} \frac{5^x - 3^x}{\sin(2x)} &\overset{\text{L'H}}{\underset{\frac{0}{0}}{=}} \lim_{x\to 0} \frac{\ln(5)5^x - \ln(3)3^x}{2\cos(2x)} \\
&= \frac{\ln(5)5^0 - \ln(3)^0}{2\cos(2\cdot 0)} \\
&= \boxed{\frac{\ln(5) - \ln(3)}{2}}.
\end{aligned}$$

(c) $\lim_{x\to\infty} \tan^{-1}(x^2 - x^3)$

Solution. We start by investigating the behavior of the "inside" $x^2 - x^3$ as $x \to \infty$. Since $\lim_{x\to\infty} \left(x^2 - x^3\right)$ is an indeterminate form $\infty - \infty$, we will need a bit of algebra to be able to compute the limit. We have

$$\lim_{x\to\infty} \left(x^2 - x^3\right) = \lim_{x\to\infty} x^3 \left(\frac{1}{x} - 1\right) = \text{“}\infty(0-1)\text{”} = -\infty.$$

Therefore, letting $u = x^2 - x^3$, we have $u \to -\infty$ as $x \to \infty$, so

$$\lim_{x\to\infty} \tan^{-1}(x^2 - x^3) = \lim_{u\to-\infty} \tan^{-1}(u) = \boxed{-\frac{\pi}{2}}.$$

(d) $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x$

Solution. This limit is an indeterminate power 1^∞. **Warning:** limits of the form 1^∞ need not be equal to 1! We can resolve the indeterminate form by rewriting the power with an exponential using the formula

$$a^b = e^{b\ln(a)}$$

and applying L'Hôpital's Rule in the resulting exponent. This gives

$$\begin{aligned}
\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^x &= \lim_{x\to\infty} e^{x\ln\left(1+\frac{2}{x}\right)} \\
&= e^{\lim_{x\to\infty} \frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}} \\
&\overset{\text{L'H}}{\underset{\frac{0}{0}}{=}} e^{\lim_{x\to\infty} \frac{-\frac{2}{x^2}\cdot\frac{1}{1+\frac{2}{x}}}{-\frac{1}{x^2}}} \\
&= e^{\lim_{x\to\infty} \frac{2}{1+\frac{2}{x}}} \\
&= \boxed{e^2}.
\end{aligned}$$

(e) $\lim_{x\to-\infty} \frac{2x + 3\cos(x)}{5x}$

Solution. This limit is an indeterminate form $\frac{\infty}{\infty}$. However, we **cannot use L'Hôpital's Rule** here. This is because L'Hôpital's Rule only applies if the resulting limit exists or is infinite, but here, the resulting limit

$$\lim_{x\to-\infty} \frac{2 - 3\sin(x)}{5}$$

does not exist. The Squeeze (or Sandwich) Theorem will work for this limit. Since $-1 \leqslant \cos(x) \leqslant 1$ for all x, we have

$$\frac{2x-3}{5x} \leqslant \frac{2x + 3\cos(x)}{5x} \leqslant \frac{2x+3}{5x}$$

for any $x \neq 0$. Furthermore, we have

$$\lim_{x\to-\infty} \frac{2x-3}{5x} = \lim_{x\to\infty} \frac{2}{5} - \frac{3}{5x} = \frac{2}{5},$$
$$\lim_{x\to-\infty} \frac{2x+3}{5x} = \lim_{x\to\infty} \frac{2}{5} + \frac{3}{5x} = \frac{2}{5}.$$

Since the two limits are equal, we conclude that

$$\boxed{\lim_{x\to-\infty} \frac{2x+3\cos(x)}{5x} = \frac{2}{5}}.$$

(f) $\lim_{x\to\infty} x^{1/x}$

Solution. This limit is an indeterminate power ∞^0. **Warning:** limits of the form ∞^0 need not be equal to 1! We can resolve the indeterminate form by rewriting the power with an exponential using the formula

$$a^b = e^{b\ln(a)}$$

and applying L'Hôpital's Rule in the resulting exponent. This gives

$$\begin{aligned}
\lim_{x\to\infty} x^{1/x} &= \lim_{x\to\infty} e^{\frac{\ln(x)}{x}} \\
&= e^{\lim_{x\to\infty} \frac{\ln(x)}{x}} \\
&\overset{\text{L'H}}{\underset{\frac{\infty}{\infty}}{=}} e^{\lim_{x\to\infty} \frac{1/x}{1}} \\
&= e^0 \\
&= \boxed{1}.
\end{aligned}$$

3. Find the horizontal asymptotes of the following functions.

(a) $f(x) = \dfrac{11x^3+2x-1}{2x^3-x^2+3}$

Solution. This function is rational. We will be able to compute its limits at infinity after dividing the numerator and denominator by the highest power of x appearing in the expression, here x^3. This gives

$$\lim_{x\to\infty} \frac{11x^3+2x-1}{2x^3-x^2+3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to\infty} \frac{11+\frac{2}{x^2}-\frac{1}{x^3}}{2-\frac{1}{x}+\frac{3}{x^3}} = \frac{11}{2},$$
$$\lim_{x\to-\infty} \frac{11x^3+2x-1}{2x^3-x^2+3} \cdot \frac{\frac{1}{x^3}}{\frac{1}{x^3}} = \lim_{x\to-\infty} \frac{11+\frac{2}{x^2}-\frac{1}{x^3}}{2-\frac{1}{x}+\frac{3}{x^3}} = \frac{11}{2}.$$

Therefore, the only horizontal asymptote of f is $\boxed{y = \frac{11}{2}}$.

(b) $f(x) = \dfrac{5x + \sqrt{16x^2 + 25}}{18x - 7}$

Solution. Note that

$$\sqrt{16x^2 + 25} = \sqrt{x^2}\sqrt{16 + \frac{25}{x^2}} = |x|\sqrt{16 + \frac{25}{x^2}}.$$

When $x \to \infty$, we have $|x| = x$, so

$$\begin{aligned}
\lim_{x\to\infty} \frac{5x + \sqrt{16x^2 + 25}}{18x - 7} &= \lim_{x\to\infty} \frac{5x + |x|\sqrt{16 + \frac{25}{x^2}}}{18x - 7} \\
&= \lim_{x\to\infty} \frac{5x + x\sqrt{16 + \frac{25}{x^2}}}{18x - 7} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to\infty} \frac{5 + \sqrt{16 + \frac{25}{x^2}}}{18 - \frac{7}{x}} \\
&= \frac{5 + \sqrt{16}}{18} \\
&= \frac{1}{2}.
\end{aligned}$$

When $x \to -\infty$, we have $|x| = -x$, so

$$\begin{aligned}
\lim_{x\to\infty} \frac{5x + \sqrt{16x^2 + 25}}{18x - 7} &= \lim_{x\to\infty} \frac{5x + |x|\sqrt{16 + \frac{25}{x^2}}}{18x - 7} \\
&= \lim_{x\to\infty} \frac{5x - x\sqrt{16 + \frac{25}{x^2}}}{18x - 7} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} \\
&= \lim_{x\to\infty} \frac{5 - \sqrt{16 + \frac{25}{x^2}}}{18 - \frac{7}{x}} \\
&= \frac{5 - \sqrt{16}}{18} \\
&= \frac{1}{18}.
\end{aligned}$$

Therefore the two horizontal asymptotes of f are $\boxed{y = \frac{1}{18},\ y = \frac{1}{2}}$.

(c) $f(x) = \dfrac{3e^{2x} - 2e^x + 4x^2}{x^2 - 6e^{2x}}$

Solution. When $x \to \infty$, the dominant term in the expression is e^{2x}, so we will divide numerator and denominator by e^{2x} to compute the limit when $x \to \infty$. We obtain

$$\begin{aligned}
\lim_{x\to\infty} \frac{3e^{2x} - 2e^x + 4x^2}{x^2 - 6e^{2x}} &= \lim_{x\to\infty} \frac{3e^{2x} - 2e^x + 4x^2}{x^2 - 6e^{2x}} \cdot \frac{\frac{1}{e^{2x}}}{\frac{1}{e^{2x}}} \\
&= \lim_{x\to\infty} \frac{3 - \frac{2}{e^x} + \frac{4x^2}{e^{2x}}}{\frac{x^2}{e^{2x}} - 6}.
\end{aligned}$$

Using L'Hôpital's Rule, we have

$$\lim_{x\to\infty} \frac{x^2}{e^{2x}} \overset{\text{L'H}}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{2x}{2e^{2x}} \overset{\text{L'H}}{\underset{\frac{\infty}{\infty}}{=}} \lim_{x\to\infty} \frac{2}{4e^{2x}} = 0.$$

Therefore,

$$\lim_{x\to\infty} \frac{3e^{2x} - 2e^x + 4x^2}{x^2 - 6e^{2x}} = \lim_{x\to\infty} \frac{3 - \frac{2}{e^x} + \frac{4x^2}{e^{2x}}}{\frac{x^2}{e^{2x}} - 6} = \frac{3}{-6} = -\frac{1}{2}.$$

When $x \to -\infty$, the dominant term is x^2 as $e^x, e^{2x} \to 0$. So we will divide the numerator and denominator by x^2 to compute the limit when $x \to -\infty$. This gives

$$\begin{aligned}\lim_{x\to-\infty} \frac{3e^{2x} - 2e^x + 4x^2}{x^2 - 6e^{2x}} &= \lim_{x\to-\infty} \frac{3e^{2x} - 2e^x + 4x^2}{x^2 - 6e^{2x}} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} \\ &= \lim_{x\to-\infty} \frac{\frac{3e^{2x}}{x^2} - \frac{2e^x}{x^2} + 4}{1 - \frac{6e^{2x}}{x^2}} \\ &= 4.\end{aligned}$$

Note that $\lim_{x\to-\infty} \frac{e^x}{x^2} = \lim_{x\to-\infty} \frac{e^{2x}}{x^2} = \text{“}\frac{0}{\infty}\text{”} = 0$. These are not indeterminate forms and we cannot apply L'Hôpital's Rule (nor would we need to).

Therefore, the two horizontal asymptotes of the function f are $\boxed{y = -\frac{1}{2},\ y = 4}$.

4. Calculate the following indefinite or definite integrals.

(a) $\displaystyle\int (3x+1)\left(x^2 - \frac{5}{x}\right) dx$

Solution. We fully distribute the integrand, then use the power rule. This gives

$$\begin{aligned}\int (3x+1)\left(x^2 - \frac{5}{x}\right) dx &= \int \left(3x^3 + x^2 - 15 - \frac{5}{x}\right) dx \\ &= \boxed{\frac{3}{4}x^4 + \frac{1}{3}x^3 - 15x - 5\ln|x| + C}.\end{aligned}$$

(b) $\displaystyle\int x^3 \sin(x^4 + 2) dx$

Solution. We use the substitution $u = x^4 + 2$, $du = 4x^3 dx$. Therefore, $x^3 dx = \frac{du}{4}$, and we obtain

$$\begin{aligned}\int x^3 \sin(x^4 + 2) dx &= \int \frac{1}{4} \sin(u) du \\ &= -\frac{1}{4}\cos(u) + C \\ &= \boxed{-\frac{1}{4}\cos(x^4 + 2) + C}.\end{aligned}$$

(c) $\displaystyle\int_0^1 \frac{x^3}{\sqrt{3+x^2}}dx$

Solution 1. We use the substitution $u = 3 + x^2$. So $du = 2xdx$, that is $xdx = \frac{du}{2}$. The extraneous factor x^2 in the numerator can be expressed in terms of u using $x^2 = u - 3$. Finally, the bounds become

$$\begin{aligned} x = 0 &\Rightarrow u = 3 + 0^2 = 3, \\ x = 1 &\Rightarrow u = 3 + 1^2 = 4. \end{aligned}$$

So the integral becomes

$$\begin{aligned} \int_0^1 \frac{x^3}{\sqrt{3+x^2}}dx &= \int_0^1 \frac{x^2}{\sqrt{3+x^2}}xdx \\ &= \int_3^4 \frac{u-3}{2\sqrt{u}}du \\ &= \int_3^4 \left(\frac{1}{2}\sqrt{u} - \frac{3}{2\sqrt{u}}\right) du \\ &= \left[\frac{1}{2} \cdot \frac{2}{3}u^{3/2} - 3\sqrt{u}\right]_3^4 \\ &= \left(\frac{1}{3}4^{3/2} - 3\sqrt{4}\right) - \left(\frac{1}{3}3^{3/2} - 3\sqrt{3}\right) \\ &= \boxed{2\sqrt{3} - \frac{10}{3}}. \end{aligned}$$

Solution 2. We use the substitution $u = \sqrt{3+x^2}$. So $du = \frac{xdx}{\sqrt{3+x^2}}$. The extraneous factor x in the numerator can be expressed in terms of u using $x^2 = u^2 - 3$. Finally, the bounds become

$$\begin{aligned} x = 0 &\Rightarrow u = \sqrt{3 + 0^2} = \sqrt{3}, \\ x = 1 &\Rightarrow u = \sqrt{3 + 1^2} = 2. \end{aligned}$$

So the integral becomes

$$\begin{aligned} \int_0^1 \frac{x^3}{\sqrt{3+x^2}}dx &= \int_0^1 x^2 \frac{xdx}{\sqrt{3+x^2}} \\ &= \int_{\sqrt{3}}^2 (u^2 - 3)du \\ &= \left[\frac{1}{3}u^3 - 3u\right]_{\sqrt{3}}^2 \\ &= \left(\frac{1}{3}2^3 - 3 \cdot 2\right) - \left(\frac{1}{3}\sqrt{3}^3 - 3\sqrt{3}\right) \\ &= \boxed{2\sqrt{3} - \frac{10}{3}}. \end{aligned}$$

(d) $\displaystyle\int t\sec^2\left(3t^2\right)e^{7\tan\left(3t^2\right)}dt$

Solution. We use the substitution $u = 7\tan\left(3t^2\right)$. This gives

$$du = 7\sec^2(3t^2)\cdot 6t dt = 42t\sec^2(3t^2)dt.$$

So

$$t\sec^2\left(3t^2\right)dt = \frac{du}{42}.$$

We get

$$\begin{aligned}\int t\sec^2\left(3t^2\right)e^{7\tan\left(3t^2\right)}dt &= \int \frac{1}{42}e^u du \\ &= \frac{1}{42}e^u + C \\ &= \boxed{\frac{1}{42}e^{7\tan\left(3t^2\right)} + C}.\end{aligned}$$

(e) $\displaystyle\int e^x(e^x - 2)^{2/3}dx$

Solution. We use the substitution $u = e^x - 2$, so that $du = e^x dx$. This gives

$$\begin{aligned}\int e^x(e^x-2)^{2/3}dx &= \int u^{2/3}du \\ &= \frac{3}{5}u^{5/3} + C \\ &= \boxed{\frac{3}{5}(e^x-2)^{5/3} + C}.\end{aligned}$$

(f) $\displaystyle\int e^{2x}(e^x - 2)^{2/3}dx$

Solution. We again use the substitution $u = e^x - 2$, $du = e^x dx$. But this time, we have an extraneous factor e^x since $e^{2x} = e^x e^x$. We can express this extraneous factor in terms of u as $e^x = u + 2$. Therefore

$$\begin{aligned}\int e^{2x}(e^x-2)^{2/3}dx &= \int e^x(e^x-2)^{2/3}e^x dx \\ &= \int (u+2)u^{2/3}du \\ &= \int \left(u^{5/3} + 2u^{2/3}\right)du \\ &= \frac{3}{8}u^{8/3} + \frac{6}{5}u^{5/3} + C \\ &= \boxed{\frac{3}{8}(e^x-2)^{8/3} + \frac{6}{5}(e^x-2)^{5/3} + C}.\end{aligned}$$

(g) $\displaystyle\int_{e^3}^{e^6} \frac{dt}{t\ln(t)}$

Solution. We use the substitution $u = \ln(t)$, so that $du = \frac{dt}{t}$. The bounds change as follows

$$\begin{aligned} t = e^3 &\Rightarrow u = \ln(e^3) = 3, \\ t = e^6 &\Rightarrow u = \ln(e^6) = 6. \end{aligned}$$

The integral becomes

$$\begin{aligned} \int_{e^3}^{e^6} \frac{dt}{t\ln(t)} &= \int_3^6 \frac{du}{u} \\ &= [\ln|u|]_3^6 \\ &= \ln(6) - \ln(3) \\ &= \ln\left(\frac{6}{3}\right) \\ &= \boxed{\ln(2)}. \end{aligned}$$

(h) $\displaystyle\int \frac{dx}{5x + 4\sqrt{x}}$

Solution. We can first factor out a $\sqrt{x}$ from the denominator, which gives

$$\int \frac{dx}{5x + 4\sqrt{x}} = \int \frac{dx}{\sqrt{x}\,(5\sqrt{x}+4)}.$$

We can then use the substitution $u = 5\sqrt{x} + 4$, which gives $du = \frac{5dx}{2\sqrt{x}}$. This gives $\frac{dx}{\sqrt{x}} = \frac{2du}{5}$, and the integral becomes

$$\begin{aligned} \int \frac{dx}{\sqrt{x}\,(5\sqrt{x}+4)} &= \int \frac{2du}{5u} \\ &= \frac{2}{5}\ln|u| + C \\ &= \boxed{\frac{2}{5}\ln|5\sqrt{x}+4| + C}. \end{aligned}$$

(i) $\displaystyle\int \frac{dx}{\sqrt{2 - x^2}}$

Solution. Recall the reference antiderivative

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C.$$

We can use this antiderivative after factoring out a 2 from the square root and letting $u = \frac{x}{\sqrt{2}}$. This gives

$$\begin{aligned}
\int \frac{dx}{\sqrt{2-x^2}} &= \int \frac{dx}{\sqrt{2\left(1-\frac{x^2}{2}\right)}} \\
&= \int \frac{dx}{\sqrt{2}\sqrt{1-\left(\frac{x}{\sqrt{2}}\right)^2}} \\
&= \int \frac{du}{\sqrt{1-u^2}} \\
&= \sin^{-1}(u) + C \\
&= \boxed{\sin^{-1}\left(\frac{x}{\sqrt{2}}\right) + C}.
\end{aligned}$$

(j) $\displaystyle\int_0^1 \frac{xdx}{\sqrt{2-x^2}}$

Solution 1. This time, the numerator is (up to a constant factor) the derivative of the inside of the square root. Therefore, we can compute this integral with the substitution $u = 2 - x^2$, $du = -2xdx$. Thus we have $xdx = -\frac{du}{2}$, and the bounds change to

$$\begin{aligned}
x = 0 &\Rightarrow u = 2 - 0^2 = 2, \\
x = 1 &\Rightarrow u = 2 - 1^2 = 1.
\end{aligned}$$

We obtain

$$\begin{aligned}
\int_0^1 \frac{xdx}{\sqrt{2-x^2}} &= \int_2^1 -\frac{du}{2\sqrt{u}} \\
&= \left[-\sqrt{u}\right]_2^1 \\
&= \boxed{1-\sqrt{2}}.
\end{aligned}$$

Solution 2. We can be more ambitious with the substitution and let $u = \sqrt{2-x^2}$. The bounds change to

$$\begin{aligned}
x = 0 &\Rightarrow u = \sqrt{2-0^2} = \sqrt{2}, \\
x = 1 &\Rightarrow u = \sqrt{2-1^2} = 1.
\end{aligned}$$

Differentiating gives $du = -\frac{xdx}{\sqrt{2-x^2}}$, which is the entire integrand up to a negative sign. So the integral becomes

$$\int_0^1 \frac{xdx}{\sqrt{2-x^2}} = \int_{\sqrt{2}}^1 -\,du = \boxed{\sqrt{2}-1}.$$

(k) $\displaystyle\int_0^{2/3} \frac{dz}{4+9z^2}$

Solution. This integral will make use of the reference antiderivative

$$\int \frac{du}{1+u^2} = \tan^{-1}(u) + C.$$

To get to this form, we factor out a 4 from the denominator to obtain

$$\int_0^{2/3} \frac{dz}{4+9z^2} = \int_0^{2/3} \frac{dz}{4\left(1+\frac{9z^2}{4}\right)} = \int_0^{2/3} \frac{dz}{4\left(1+\left(\frac{3z}{2}\right)^2\right)}$$

We can then use the substitution $u = \frac{3z}{2}$, which gives $du = \frac{3dz}{2}$, so $dz = \frac{2du}{3}$. The bounds change to

$$x = 0 \ \Rightarrow \ u = 0,$$
$$x = \frac{2}{3} \ \Rightarrow \ u = 1.$$

We obtain

$$\begin{aligned}
\int_0^{2/3} \frac{dz}{4\left(1+\left(\frac{3z}{2}\right)^2\right)} &= \int_0^1 \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{du}{1+u^2} \\
&= \left[\frac{1}{6}\tan^{-1}(u)\right]_0^1 \\
&= \frac{1}{6}\left(\tan^{-1}(1) - \tan^{-1}(0)\right) \\
&= \frac{1}{6}\left(\frac{\pi}{4} - 0\right) \\
&= \boxed{\frac{\pi}{24}}.
\end{aligned}$$

(1) $\displaystyle\int \frac{dx}{x^2+6x+34}$

Solution. We will need to complete the square first to see a sum or difference of squares in the denominator:

$$x^2 + 6x + 34 = (x^2 + 6x + 9) - 9 + 34 = (x+3)^2 + 25$$

Therefore the integral becomes

$$\begin{aligned}
\int \frac{dx}{x^2+6x+34} &= \int \frac{dx}{(x+3)^2+25} \\
&= \int \frac{dx}{25\left(\frac{(x+3)^2}{25}+1\right)} \\
&= \frac{1}{25}\int \frac{dx}{\left(\frac{x+3}{5}\right)^2+1} \\
&= \frac{1}{25}\int \frac{5du}{u^2+1} \qquad \left(u = \frac{x+3}{2}, \ dx = 5du\right) \\
&= \frac{1}{5}\tan^{-1}(u) + C \\
&= \boxed{\frac{1}{5}\tan^{-1}\left(\frac{x+3}{5}\right) + C}.
\end{aligned}$$