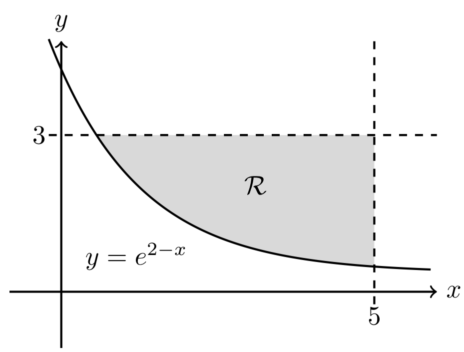
Rutgers University
FA23 Math 152

Final Exam Practice Problems

1. (a) Find a function $f(x)$ and an interval $[a,b]$ so that the right endpoint Riemann sum of $f(x)$ on the interval $[a,b]$ is
$$\sum_{k=1}^{n} \tan^3\left(\frac{\pi k}{4n}\right)\frac{\pi}{8n}.$$
   (b) Evaluate $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n} \tan^3\left(\frac{\pi k}{4n}\right)\frac{\pi}{8n}$.

2. Consider the region $\mathcal{R}$ bounded by the curve $y = e^{2-x}$, the line $x = 5$ and the line $y = 3$. The region $\mathcal{R}$ is sketched below.



Set-up integrals computing the volume of the solid obtained by revolving $\mathcal{R}$ about each axis given below using (i) the disk/washer method, and (ii) the shell method.

(a) x-axis (b) y-axis (c) $y = -1$ (d) $x = 7$ (e) $y = 3$ (f) $x = -4$

3. Evaluate the following integrals. If an integral diverges, explain why.

(a) $\displaystyle\int \frac{dx}{(7+6x-x^2)^{3/2}}$

(b) $\displaystyle\int_0^{\infty} x^2 e^{-3x}\,dx$

(c) $\displaystyle\int \cos^2(5\theta)\sin^2(5\theta)\,d\theta$

(d) $\displaystyle\int_1^{\sqrt{2}} \frac{dx}{\sqrt{4x^2-2}}$

(e) $\displaystyle\int \sqrt{x}\cos(3\sqrt{x})\,dx$

(f) $\displaystyle\int_{\pi/4}^{3\pi/4} \tan(x)\sec^3(x)\,dx$

4. Find the area inside the circle $x^2 + y^2 = 3$ and outside the circle $x^2 + y^2 + 2y = 0$. *(Hint: use polar coordinates.)*

5. Find the length of the polar curve $r = \theta^4$, $0 \leqslant \theta \leqslant \sqrt{2}$.

6. Determine if the sequences below converge or diverge. If a sequence converges, find its limit.

   (a) $a_n = n^2\left(1 - \sec\left(\dfrac{5}{n}\right)\right)$ (b) $a_n = \dfrac{\ln(2^n+1)}{\ln(n)}$

7. Determine if the series below converge absolutely, converge conditionally or diverge. If a series converges, find its sum when possible.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{\cos(n) - 5^n}{3^{2n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{3 \cdot 2^{2n}}{5^{n+1}}$ (c) $\displaystyle\sum_{n=3}^{\infty} \frac{(-1)^n}{n\sqrt{\ln(n)^2+1}}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{9n^2+2}}{2n^4}$

8. Find the radius and interval of convergence of the following power series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n(x+3)^n}{n}$ (b) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2x^n}{5^{n^2}}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+5)^{3n}}{\sqrt{64^n n+1}}$

9. Consider the curve with parametric equations $x = 2t, y = 3\ln(t) + 2$, $\dfrac{3}{2} \leqslant t \leqslant 3$.

   (a) Calculate the length of the curve.
   (b) Calculate the area of the surface of revolution obtained by revolving the curve about the y-axis.
   (c) Set-up (but do not evaluate) an integral that computes the area of the surface of revolution obtained by revolving the curve about the x-axis.

10. Consider the region $\mathcal{R}$ in the xy-plane bounded by the lines $y = 2x$, $y = 1$ and the graph $y = \sin\left(\dfrac{x}{2}\right)$.

    (a) Sketch the region $\mathcal{R}$.
    (b) Calculate the area of the region using (i) an x-integral, and (i) a y-integral.
    (c) A solid is obtained by revolving the region $\mathcal{R}$ about the line $y = 2$. Set-up integrals that calculate the volume of the solid using (i) the disk/washer method, and (ii) the shell method. Then evaluate one of the integrals to find the volume of the solid.