Rutgers University
FA23 Math 152

Midterm Exam 3 Practice Problems

1. Determine if the sequences below converge or diverge, and find their limit in case of convergence.
Basic:

(a) $\left\{\dfrac{e^{n^2}}{n^3}\right\}$ (b) $\left\{n\ln\left(1-\dfrac{2}{5n}\right)\right\}$

Advanced:

(c) $\left\{\dfrac{2^n+\arctan(n)}{2^{n+5}}\right\}$ (d) $\left\{\cos(e^{-n})^{e^{2n}}\right\}$

2. Determine if the series below converge absolutely, converge conditionally or diverge. Name any test used and show all work to justify its use. In case of convergence, evaluate the sum when possible.
Basic:

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{(-5)^n+1}{3^{2n+1}}$ (d) $\displaystyle\sum_{n=0}^{\infty}\frac{3^n+2}{7\sqrt{25^n+1}}$ (g) $\displaystyle\sum_{n=1}^{\infty}\frac{(n+3)!}{(-5)^n}$

(b) $\displaystyle\sum_{n=1}^{\infty}\frac{e^{-\sqrt{n}}}{\sqrt{n}}$ (e) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{\pi}}$ (h) $\displaystyle\sum_{n=1}^{\infty}\frac{n^n}{3^{n^2}}$

(c) $\displaystyle\sum_{n=1}^{\infty}\left(5+\frac{3}{n}\right)^2$ (f) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{2n+5\sqrt{n}}$ (i) $\displaystyle\sum_{n=1}^{\infty}\frac{5\cos(n)-2}{n^2}$

Advanced:

(j) $\displaystyle\sum_{n=3}^{\infty}\left(\frac{n+3}{n}\right)^{n^2}$ (l) $\displaystyle\sum_{n=2}^{\infty}\frac{(-1)^n}{n\ln(n)}$ (n) $\displaystyle\sum_{n=3}^{\infty}\left(\tan\left(\frac{\pi}{n}\right)-\tan\left(\frac{\pi}{n+2}\right)\right)$

(k) $\displaystyle\sum_{n=1}^{\infty}\frac{2n^2+3\cos(n)}{n^4+1}$ (m) $\displaystyle\sum_{n=1}^{\infty}\frac{(n!)^2e^n}{(2n)!}$ (o) $\displaystyle\sum_{n=1}^{\infty}\frac{\ln(n)^2}{n}$

3. Consider the series $S=\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{(3n+11)^2}$.

(a) *(Basic)* Show that this series meets the conditions of the Alternating Series Estimation Theorem.

(b) *(Advanced)* Find the smallest integer N for which the partial sum $S_N=\displaystyle\sum_{n=1}^{N}\frac{(-1)^n}{(3n+11)^2}$ approximates the sum of the series S with an error of at most 0.0001.