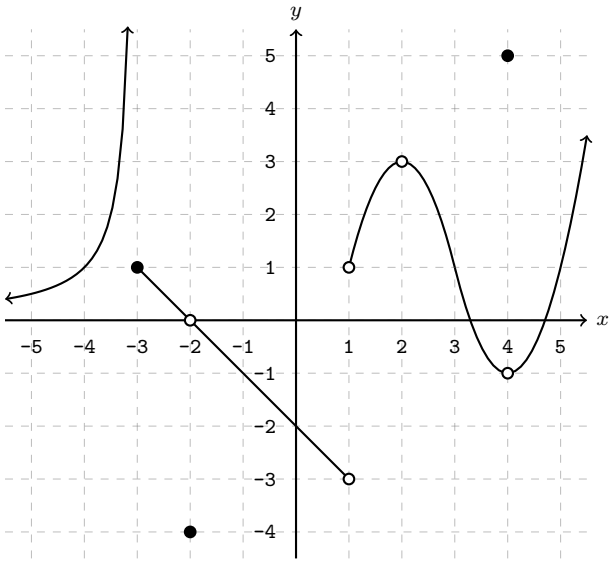
Rutgers University
Math 151

Section 2.1: Introduction to Limits - Worksheet

1. Calculate the average rate of change the following functions on the given intervals.
   (a) $f(x) = 2\ln(5x+1)$ on the interval $[0, 3]$.
   (b) $f(x) = \sin(4x)$ on the interval $\left[\frac{\pi}{24}, \frac{\pi}{12}\right]$.
   (c) $f(x) = \arctan(3x)$ on the interval $\left[-\frac{1}{3}, \frac{1}{3}\right]$.

2. The graph of the function $y = f(x)$ is given below.



   Evaluate $f(a)$ and $\lim_{x\to a} f(x)$ for the following values of a, or say if the quantity does not exist.

   (a) $a = -3$ (b) $a = -2$ (c) $a = 1$ (d) $a = 2$ (e) $a = 4$

3. The following table of values are given for the functions $f(x)$ and $g(x)$. Use these to estimate $\lim_{x\to 3} f(x)$ and $\lim_{x\to 3} g(x)$ or say if a limit does not exist.

x	2.9	3.01	2.999	3.0001	2.99999
$f(x)$	4.15	3.95	4.05	3.9993	4.0005
$g(x)$	7.98	1.001	7.997	1.0002	7.99992

4. Using a limit of average rates of change, find the instantaneous rate of change of the following functions at the given value of x.

   (a) $f(x) = x^2 - 3x + 7$ at $x = 0$.

   (b) $f(x) = \dfrac{x}{5 - x}$ at $x = -1$.

   (c) **[Advanced]** $f(x) = \dfrac{1}{\sqrt{2x + 1}}$ at $x = 4$.

5. The position of an object moving along an axis is given by the function $s(t) = 6\sqrt{x + 1}$.

   (a) Find the average velocity of the object between $t = 0$ and $t = 15$.

   (b) Find the position and instantaneous velocity of the object at $t = 3$.