Rutgers University
Math 151

Section 2.4: One-Sided Limits - Worksheet

1. Evaluate the following limits. If a limit does not exist, explain why.

   (a) $\lim\limits_{x\to 3^-} \dfrac{x^2-4x+3}{|x-3|}$.

   (b) $\lim\limits_{x\to 3} \dfrac{x^2-4x+3}{|x-3|}$.

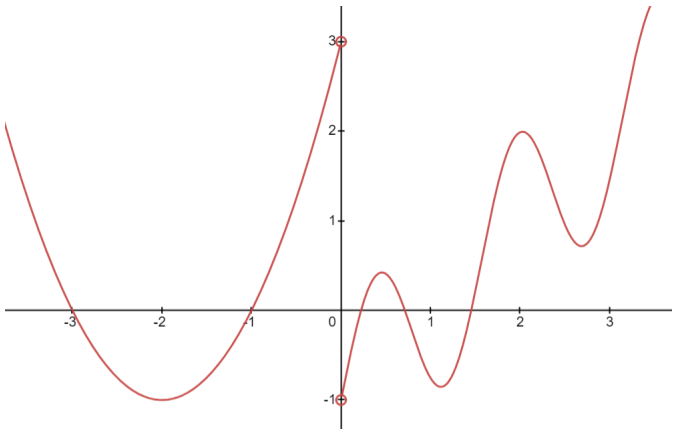
   (c) $\lim\limits_{h\to 0^-} \dfrac{1-(1-|h|)^3}{h}$.

   (d) $\lim\limits_{t\to 1} \dfrac{t^3-2t^2+t}{|t-1|}$.

   (e) $\lim\limits_{x\to -2} f(x)$ where $f(x) = \begin{cases} 3x+8 & \text{if } x<-2 \\ 8 & \text{if } x=-2 \\ \dfrac{x+2}{\sqrt{x+3}-1} & \text{if } x>-2 \end{cases}$.

   (f) $\lim\limits_{x\to 0} f(x)$ where $f(x) = \begin{cases} \dfrac{\sin(3x)}{x} & \text{if } x<0 \\ 2e^{\cos(x)-1} & \text{if } x\geqslant 0 \end{cases}$.

2. Consider the function $f(x) = \dfrac{\tan(8x)}{|2x|}$ and suppose that the graph of another function g is given below.



   (a) Find $\lim\limits_{x\to 0} f(x)$ and $\lim\limits_{x\to 0} g(x)$ or explain why it does not exist.

   (b) Find $\lim\limits_{x\to 0} |f(x)|$ and $\lim\limits_{x\to 0} |g(x)|$ or explain why it does not exist.

   (c) Find $\lim\limits_{x\to 0} f(x)+2g(x)$ or explain why it does not exist.

   (d) **[Advanced]** Find the value of the constant a for which $\lim\limits_{x\to 0} \dfrac{g(x)}{f(x)+a}$ exists. For this value of a, find the value of the limit.