

Section 2.5: Continuity - Worksheet

1. For each function, find the values of the constants a, b that make it continuous.

$$(a) \ f(x) = \begin{cases} 3x - b & \text{if } x \leq 1 \\ ax + 4 & \text{if } 1 < x \leq 3 \\ bx - 2a & \text{if } x > 3 \end{cases}$$

$$(b) \ f(x) = \begin{cases} bx + 4 & \text{if } x < 1 \\ a & \text{if } x = 1 \\ \frac{x^{-1} - 1}{x^2 - 1} & \text{if } x > 1 \end{cases}$$

$$(c) \ [\text{Advanced}] \ f(x) = \begin{cases} \frac{\sin(ax)}{3x} & \text{if } x < 0 \\ b & \text{if } x = 0 \\ \frac{x^2 + 5x}{\sqrt{x+4} - 2} & \text{if } x > 0 \end{cases}$$

2. Consider the function $f(x) = \begin{cases} x^2 + 4x + 5 & \text{if } x < -2 \\ 3 & \text{if } x = -2 \\ \cos(\pi x) & \text{if } -2 < x < 3 \\ x + 2 & \text{if } 3 \leq x \leq 4 \\ 6 - \ln(x - 3) & \text{if } x > 4 \end{cases}$

- (a) Find the values of a for which $\lim_{x \rightarrow a} f(x)$ does not exist.
 (b) Find the values of x where f is discontinuous.
3. Show that each equation has a solution in the given interval.

- (a) $x^3 = 14 + 2\sqrt{x}$ in $[0, 4]$.
 (b) $\ln(x) = 2 - x$ in $[1, e]$.
 (c) **[Advanced]** $\cos(x) = \arcsin(x)$ in $[0, 1]$.