

Section 2.6: Limits Involving Infinity - Worksheet

1. Evaluate the following limits. If a limit does not exist, explain why. If a limit is infinite, specify it and determine if it is ∞ or $-\infty$.

(a) $\lim_{x \rightarrow -1^-} \frac{x^2 + 3x + 2}{(x + 1)^2}$.

(c) $\lim_{x \rightarrow 2\pi} \frac{x}{\cos(x) - 1}$.

(e) $\lim_{x \rightarrow -\infty} \frac{x^3 + 2}{\sqrt{16x^6 + 1}}$.

(b) $\lim_{x \rightarrow \infty} \frac{3x\sqrt{x} + 2}{\sqrt{4x^3 + 1}}$.

(d) $\lim_{x \rightarrow 2} \frac{x - 5}{x^2 - 2x}$.

(f) $\lim_{t \rightarrow \infty} \sqrt{9t^2 + 8t} - \sqrt{9t^2 - 5t}$.

[Advanced]

(g) $\lim_{\theta \rightarrow -\infty} \frac{2\theta + 5 \sin(3\theta)}{7\theta}$.

(h) $\lim_{x \rightarrow 0^+} \left(\frac{1}{\sqrt[3]{x}} - \frac{1}{\sqrt{x}} \right)$.

(i) $\lim_{t \rightarrow \infty} \frac{t \arctan(3t)}{\sqrt{t^2 + 1}}$.

2. Find the vertical and horizontal asymptotes of the following functions, if any. Also, determine the limit to the left and right of any vertical asymptote.

(a) $f(x) = \frac{x^2 - 3x - 4}{\sqrt{x} - 2}$.

(c) $f(x) = \frac{7 + 2e^x}{5e^x - 4}$.

(e) $f(x) = \frac{\sin(7x)}{x^2 + 3x}$.

(b) $f(x) = \frac{x^2 - 1}{|x + 1|^3}$.

(d) $f(x) = \frac{\sqrt{x^2 + 25} + 3x}{2x + 5}$.

(f) $f(x) = x^2 \cos\left(\frac{2}{x}\right)$.

[Advanced]

(g) $f(x) = \frac{3x \arctan(x) + 7}{x - 1}$.

(h) $f(x) = \frac{3e^{2x} - 5e^{-x}}{2e^{-x} + e^{4x}}$.

(i) $f(x) = \frac{1 - \cos(5x)}{x^2 + x^3}$.