Rutgers University
Math 151

Section 3.4: Rates of Change - Worksheet Solutions

1. The position of a body moving an axis is given by $s(t) = \dfrac{t^4}{4} - 2t^3 + 8$.

   (a) Find the body's displacement and average velocity on the time interval $[0, 2]$.

   Solution. The displacement is $\Delta s = s(2) - s(0) = -12 - 0 = \boxed{-12}$. The average velocity is $\dfrac{\Delta s}{\Delta t} = \dfrac{s(2) - s(0)}{2 - 0} = \dfrac{-12}{2} = \boxed{-6}$.

   (b) Find the velocity and acceleration of the body.

   Solution. The velocity is $v(t) = \dfrac{ds}{dt} = \boxed{t^3 - 6t^2}$. The acceleration is $a(t) = \dfrac{dv}{dt} = \boxed{3t^2 - 12t}$.

   (c) When does the body change direction?

   Solution. The body changes direction when the velocity changes sign. Factoring the velocity, we get $v(t) = t^2(t - 6)$. This polynomial changes sign at $\boxed{t = 6}$.

2. A projectile is thrown at $t = 0$ straight up in the air from an altitude of 99 m at a speed of 24 m/sec. The projectile being subject to gravity only, physicists tell us that the elevation of the projectile is subject to a law of the form $h(t) = at^2 + bt + c$, where a, b, c are unspecified constants.

   (a) Find b and c using the information given.

   Solution. We know that $h(0) = 99$, so $a \cdot 0^2 + b \cdot 0 + c = 99$, giving us $\boxed{c = 99}$. Also, $v(0) = 24$, which gives $(2at + b)_{|t=0} = 24$, so $\boxed{b = 24}$.

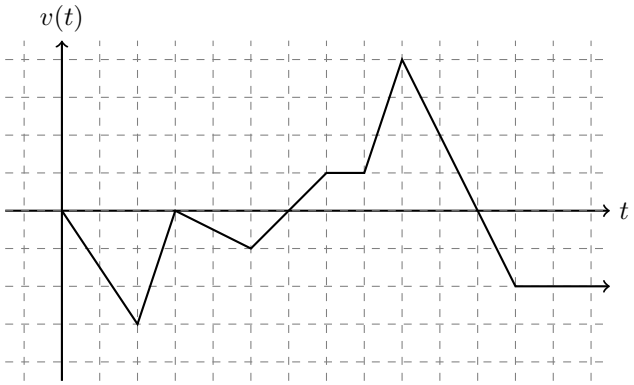
   (b) Suppose that the projectile reaches its maximum elevation 4 seconds after being thrown. Find the value of the constant a.

   Solution. We know that $v(4) = 0$ since the velocity is 0 at the instant the projectile reaches its maximum elevation. Therefore, $(2at + 24)_{|t=4} = 0$, that is $8a + 24 = 0$, so $\boxed{a = -3}$.

   (c) When will the projectile hit the ground?

   Solution. The projectile hits the ground when $h(t) = 0$, that is $-3t^2 + 24t + 99 = 0$. This gives the solutions $t = -3, 11$. Since the motion of the object is for $t \geqslant 0$, the only solution that makes sense is $\boxed{t = 11 \text{ sec}}$.

3. The graph below shows the velocity v of an object moving along an axis.



(a) When is the object moving forward? backward?

Solution. The object moves forward when $v(t) > 0$, which happens on the interval $\boxed{(6, 11)}$. The object moves backward when $v(t) < 0$, which happens on the intervals $\boxed{(0, 3), (3, 6), (11, \infty)}$.

(b) When does the object reverse direction?

Solution. The object reverses direction when the velocity changes sign, that is at $\boxed{t = 6, 11}$.

(c) Sketch the graph of the acceleration of the object.
Solution. Recall that the value of the acceleration gives the slope of the graph of the velocity