Rutgers University
Math 151

Section 3.6: Chain Rule - Worksheet Solutions

1. Calculate the derivatives of the following functions.

(a) $f(x) = 2\sec(4x^3 + 7)$

Solution.

$$f'(x) = 2\sec(4x^3+7)\tan(4x^3+7)(12x^2) = \boxed{24x^2\sec(4x^3+7)\tan(4x^3+7)(12x^2)}.$$

(b) $f(x) = 14\sqrt[7]{4x - \sin(5x)}$

Solution.

$$f'(x) = 14\frac{1}{7}(4x - \sin(5x))^{1/7-1}(4 - 5\cos(5x)) = \boxed{\frac{2(4 - 5\cos(5x))}{(4x - \sin(5x))^{6/7}}}.$$

(c) $f(x) = \cos(x^2) - \cos(x)^2$

Solution.

$$f'(x) = -\sin(x^2)(2x) - 2\cos(x)(-\sin(x)) = \boxed{-2x\sin(x^2) + 2\cos(x)\sin(x)}.$$

(d) $f(x) = 3\left(\tan\left(\frac{x}{7}\right) + 1\right)^{21}$

Solution.

$$f'(x) = 3 \cdot 21\left(\tan\left(\frac{x}{7}\right) + 1\right)^{20}\sec\left(\frac{x}{7}\right)^2\frac{1}{7} = \boxed{9\left(\tan\left(\frac{x}{7}\right) + 1\right)^{20}\sec\left(\frac{x}{7}\right)^2}.$$

(e) $f(x) = \sqrt{25 - 4x^2}$

Solution.

$$f'(x) = \frac{1}{2}(25 - 4x^2)^{1/2-1}(-8x) = \boxed{-\frac{4x}{\sqrt{25 - 4x^2}}}.$$

(f) $f(x) = e^{5\cos(3x)}$

Solution.

$$f'(x) = e^{5\cos(3x)}5(-3\sin(3x)) = \boxed{-15e^{5\cos(3x)}\sin(3x)}.$$

(g) $f(x) = x5^{3x^2}$

Solution.

$$f'(x) = 1 \cdot 5^{3x^2} + x \cdot \ln(5)5^{3x^2}(6x) = \boxed{5^{3x^2}\left(1 + 6\ln(5)x^2\right)}.$$

(h) $f(x) = 6\cos(x^3 \sin(1-2x))$

Solution.

$$\begin{aligned} f'(x) &= -6\sin(x^3\sin(1-2x))\left(3x^2\sin(1-2x) + x^3\cos(1-2x)(-2)\right) \\ &= \boxed{-6\sin(x^3\sin(1-2x))\left(3x^2\sin(1-2x) - 2x^3\cos(1-2x)\right)}. \end{aligned}$$

(i) $f(x) = \frac{2x}{\sqrt{\cos(3x)}}$

Solution.

$$f'(x) = \frac{2\sqrt{\cos(3x)} - 2x\frac{1}{2}(\cos(3x))^{-1/2}(-3\sin(3x))}{(\sqrt{\cos(3x)})^2} = \boxed{\frac{2\cos(3x) + 3x\sin(3x)}{\cos(3x)^{3/2}}}.$$

2. Find the x-values of the points on the graph of $f(x) = (2x+1)e^{-x^2}$ where the tangent line is horizontal.

Solution. We have

$$f'(x) = 2e^{-x^2} + (2x+1)e^{-x^2}(-2x) = 2e^{-x^2}(1 + (2x+1)(-x)) = 2e^{-x^2}(1 - x - 2x^2).$$

The tangent line to the graph of f is horizontal when $f'(x) = 0$. The equation $2e^{-x^2}(1-x-2x^2) = 0$ gives $1 - x - 2x^2 = 0$ since $2e^{-x^2}$ is never zero. The solutions of this quadratic equation are

$$x = \frac{-1 \pm \sqrt{1 + 4 \cdot 2}}{4} = \frac{-1 \pm 3}{4} = \boxed{-1, \frac{1}{2}}.$$

3. **[Advanced]** Suppose that f is a differentiable function such that

$$\begin{aligned} &f(0) = -1, \quad f(1) = 3, \quad f(2) = -5, \quad f(4) = 7, \\ &f'(0) = -2, \quad f'(1) = 4, \quad f(2) = 3, \quad f'(4) = -1. \end{aligned}$$

Find an equation of the tangent lines to each of the following functions at the given point.

(a) $g(x) = f(-2x)$ at $x = -1$.

Solution. We have $g(-1) = f(2) = -5$ and

$$g'(-1) = \frac{d}{dx}\left(f(-2x)\right)_{|x=-1} = -2f'(-2x)_{|x=-1} = -2f'(2) = -6.$$

So the tangent line has equation $\boxed{y = -6(x+1) - 5}$.

(b) $g(x) = f(x^2)$ at $x = 2$.

Solution. We have $g(2) = f(2^2) = f(4) = 7$ and

$$g'(2) = -\frac{d}{dx}\left(f(x^2)\right)_{|x=2} = 2xf'(x^2)_{|x=2} = 4f'(4) = -4.$$

So the tangent line has equation $\boxed{y = -4(x-2) - 7}$.

(c) $g(x) = \sec\left(\frac{\pi f(x)}{12}\right)$ at $x = 1$.

Solution. We have $g(1) = \sec\left(\frac{\pi f(1)}{12}\right) = \sec\left(\frac{3\pi}{12}\right) = \sec\left(\frac{\pi}{4}\right) = \sqrt{2}$ and

$$g'(1) = -\frac{d}{dx}\left(\sec\left(\frac{\pi f(x)}{12}\right)\right)_{|x=1} = \frac{\pi f'(x)}{12}\sec\left(\frac{\pi f(x)}{12}\right)\tan\left(\frac{\pi f(x)}{12}\right)_{|x=1} = \frac{4\pi}{12}\sec\left(\frac{\pi}{4}\right)\tan\left(\frac{\pi}{4}\right) = \frac{\pi\sqrt{2}}{3}.$$

So the tangent line has equation $\boxed{y = \frac{\pi\sqrt{2}}{3}(x-1) - \sqrt{2}}$.

(d) $g(x) = f(4x)e^{3x}$ at $x = 0$.

Solution. We have $g(0) = f(4 \cdot 0)e^{3 \cdot 0} = -1$ and

$$g'(0) = -\frac{d}{dx}\left(f(4x)e^{3x}\right)_{|x=0} = \left(4f'(4x)e^{3x} + 3f(4x)e^{3x}\right)_{|x=0} = 4f'(0) + 3f(0) = -11.$$

So the tangent line has equation $\boxed{y = -11x - 1}$.