Rutgers University
Math 151

Sections 3.8-9: Derivatives of Inverse Functions - Worksheet

1. Calculate the derivatives of the following functions.

   (a) $f(x) = \sin^{-1}(4x)$
   (b) $f(x) = \ln(2\arctan(5x) + 1)$
   (c) $f(x) = x\sec^{-1}(7x)$
   (d) $f(x) = \ln(x)^2 + 8\arccos(-x)$
   (e) $f(x) = \cot^{-1}(e^{3x})$
   (f) $f(x) = \cos(x)\log_7(\sec(x))$
   (g) $f(x) = x^{3\tan^{-1}(2x)}$
   (h) $f(x) = \cos(x)^{\ln(x)}$
   (i) $f(x) = (1-5x)^{x^2}$

2. Simplify each of the following. Your answer should not contain any trigonometric or inverse trigonometric functions.

   (a) $\cos\left(\sin^{-1}(x+1)\right)$
   (b) $\sin(2\cos^{-1}(3x))$
   (c) $\csc\left(\tan^{-1}\left(\frac{2x}{3}\right)\right)$
   (d) $\sec(\theta)$ given that $\cot(\theta) = 5$ and $\sin(\theta) < 0$

3. Suppose that f is a one-to-one function and that the tangent line to the graph of $y = f(x)$ at $x = 3$ is $y = -4x + 5$. Find an equation of the tangent line to the graph of $y = f^{-1}(x)$ at $x = f(3)$.

4. Consider the one-to-one function $f(x) = 3xe^{x^2-4}$. Calculate $f(2)$ and find an equation of the tangent line to the graph of $y = f^{-1}(x)$ at $x = f(2)$.

5. Suppose that f and g are differentiable functions such that

$$\begin{array}{lll} f(-1) = 4, & f(0) = 2, & f(1) = 4, \\ f'(-1) = 3, & f'(0) = -5, & f'(1) = 8, \\ g(-1) = 2, & g(0) = 3, & g(1) = -2, \\ g'(-1) = 7, & g'(0) = -4, & g'(1) = 6. \end{array}$$

   (a) For $F(x) = \ln\left(f(x^2) + g(x)\right)$, evaluate $F'(-1)$.
   (b) For $G(x) = \arctan\left(3\sqrt{f(x)}\right)$, evaluate $G'(1)$.
   (c) For $H(x) = 2^{f(x)}g(3x+1)$, evaluate $H'(0)$.
   (d) **[Advanced]** For $K(x) = f(2x)^{g(x)}$, evaluate $K'(0)$.