Rutgers University
Math 151

Sections 5.1-2: Areas Estimations and Riemann Sums - Worksheet

1. (a) Approximate the net area between the graph of $f(x) = 9 - x^2$ and the x-axis on $[-1, 3]$ using 4 rectangles of equal width and (i) left endpoints, (ii) right endpoints.

   (b) Approximate the net area between the graph of $f(x) = 2\cos(x)$ and the x-axis on $\left[0, \frac{\pi}{2}\right]$ using 3 rectangles of equal width and (i) left endpoints, (ii) right endpoints.

2. Suppose that the function f has the following values.

$$f(0) = 3,\ f(1) = 7,\ f(2) = 5,\ f(3) = 1,\ f(4) = 2,\ f(5) = 8,$$
$$f(6) = 0,\ f(7) = 1,\ f(8) = 5,\ f(9) = 3,\ f(10) = 1.$$

   Approximate the net area between the graph of $g(x) = f(8x + 2)$ and the x-axis on the interval $[0, 1]$ using a midpoint sum with 4 rectangles of equal width.

3. Evaluate the following sums.

   (a) $\displaystyle\sum_{k=0}^{5} \frac{k(k-1)}{2}$. (b) $\displaystyle\sum_{j=1}^{4} \cos(j\pi)j$. (c) $\displaystyle\sum_{n=1}^{5} \left(\frac{1}{n} - \frac{1}{n+1}\right)$.

4. Consider the sum $2 + 4 + 8 + 16 + 32 + 64$.

   (a) Write the sum in sigma notation with the index starting at the value 1.

   (b) Write the sum in sigma notation with the index starting at the value 0.

   (c) Write the sum in sigma notation with the index starting at the value 3.

5. Use the common sum formulas

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4},$$

   to evaluate the following sums.

   (a) $\displaystyle\sum_{k=1}^{136} (2k - 3)$. (b) $\displaystyle\sum_{j=2}^{20} j^2(j - 4)$.