Rutgers University
Math 151

Section 5.3: Definite Integrals - Worksheet

1. Let $f(x) = 4 - 2x$. We are going to calculate $\int_0^2 f(x)dx$ using two methods.

   (a) Geometric method.

      (i) Sketch the graph of $y = f(x)$.

      (ii) Use your graph and a geometric formula to calculate $\int_0^2 f(x)dx$.

   (b) With Riemann sums.

      (i) Calculate R_n, the right-endpoint Riemann sum of f on $[0, 2]$ with n rectangles. Your answer should not contain the Σ or $\cdots$ symbols. *Hint: you will need to use the reference sum*

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

      (ii) Using your formula for R_n, calculate $\int_0^2 f(x)dx$.

2. Write each limit below as the integral of a function $f(x)$ on an interval $[0, b]$.

   (a) $\lim_{n\to\infty} \sum_{k=1}^{n} \sqrt{\frac{3k}{n} + 5}\ \frac{3}{n}$.

   (b) $\lim_{n\to\infty} \sum_{k=1}^{n} e^{12k/n} \frac{8}{n}$.

   (c) $\lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\frac{k^3}{n^3}\right) \frac{2}{n}$.

   (d) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2n + 5k}$.

3. Suppose that f and g are functions such that

$$\int_{-2}^{0} f(x)dx = 4, \quad \int_{-2}^{5} f(x)dx = -1, \quad \int_{-2}^{5} g(x)dx = 10.$$

   Evaluate the following integrals.

   (a) $\int_{-2}^{5} \frac{g(x)}{2} dx$

   (b) $\int_{-2}^{5} (2g(x) - 3f(x))dx$

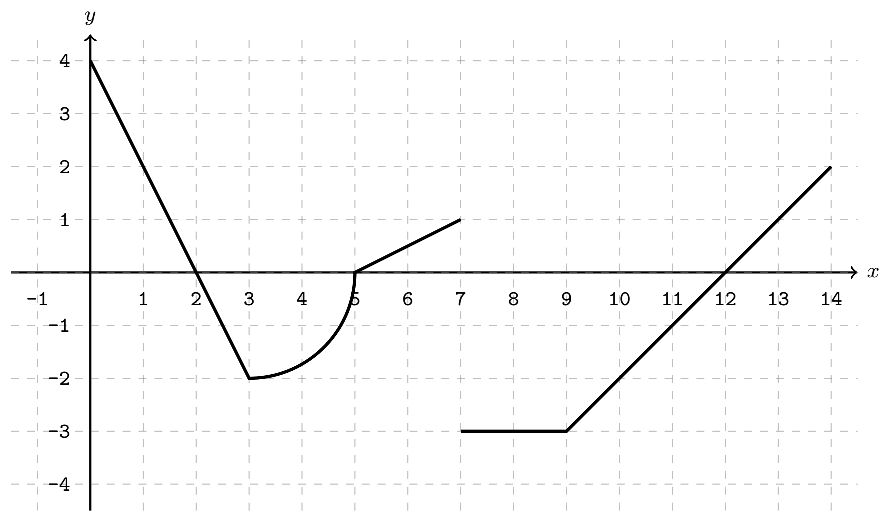
   (c) $\int_{0}^{5} 7f(x)dx$

   (d) $\int_{5}^{-2} (f(x) + 4g(x))\, dx$

   (e) $\int_{-2}^{0} (2x + f(x) - 1)\, dx$

   (f) $\int_{5}^{0} \left(f(x) - 4\sqrt{25 - x^2}\right) dx$

4. Let f be the function whose graph is sketched below. You can assume that each piece of the graph of f is either a straight line or a circle arc.



Calculate the following integrals.

(a) $\int_0^5 f(x)dx$ (b) $\int_3^9 (3 - f(x))dx$ (c) $\int_{12}^5 f(x)dx$ (d) $\int_7^{14} |f(x)|dx$