Rutgers University
Math 151

Section 5.4: Fundamental Theorem of Calculus - Worksheet

1. Evaluate the following definite integrals.

(a) $\displaystyle\int_1^3 \frac{3x^2-2x+1}{x}dx$

(b) $\displaystyle\int_0^{1/2} \frac{dt}{\sqrt{1-t^2}}$

(c) $\displaystyle\int_0^{\ln(2)} (e^x+1)^2\,dx$

(d) $\displaystyle\int_{\pi/30}^{\pi/20} \sec^2(5\theta)d\theta$

(e) $\displaystyle\int_{-3}^{\sqrt{3}} \frac{4}{x^2+3}dx$

(f) $\displaystyle\int_0^5 \frac{dz}{4z+7}$

(g) $\displaystyle\int_1^4 \sqrt{x}\left(x-\frac{4}{x}\right)dx$

(h) $\displaystyle\int_0^{2\pi} \left(\sin\left(\frac{x}{3}\right)+1\right)d\theta$

(i) $\displaystyle\int_{\sqrt{2}}^2 \frac{5}{3x\sqrt{x^2-1}}dx$

2. Evaluate the following derivatives.

(a) $\displaystyle\frac{d}{dx}\left(\int_4^x \sqrt{t^4+1}dt\right)$

(b) $\displaystyle\frac{d}{dx}\left(\int_x^0 \sec(5t^2)dt\right)$

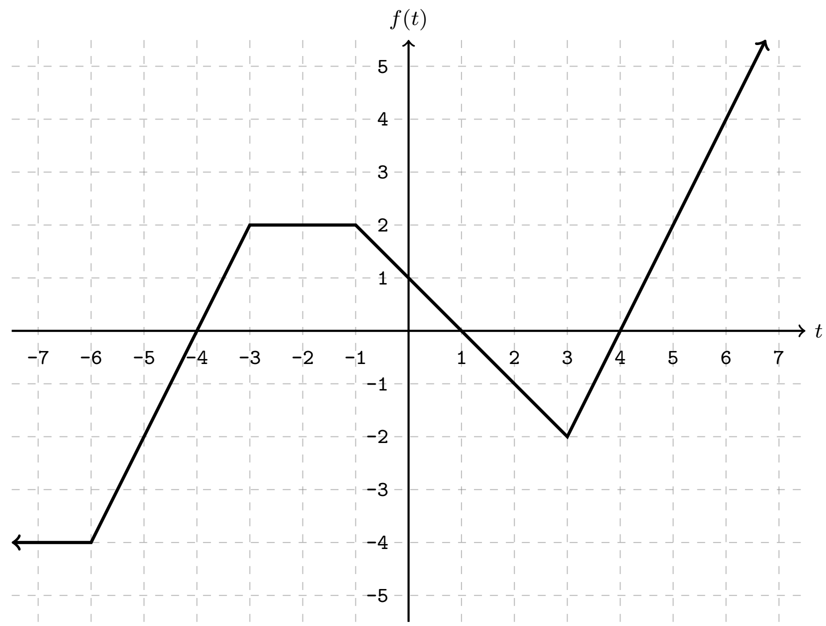
(c) $\displaystyle\frac{d}{dx}\left(\int_1^{2x} \frac{dt}{t^3+t+1}\right)$

(d) $\displaystyle\frac{d}{dx}\left(\int_{3x^2}^7 (t^4+2)^{3/4}dt\right)$

(e) $\displaystyle\frac{d}{dx}\left(\int_{\tan(2x)}^{\sec(2x)} \cos(\sqrt{t})dt\right)$

(f) $\displaystyle\frac{d}{dx}\left(\int_0^{\sin^{-1}(3x)} t^t dt\right)$

3. For the function $f(t)$ sketched below, let $F(x) = \displaystyle\int_{-3}^x f(t)dt$.



(a) Evaluate the following.

(i) $F(3)$ (ii) $F(-6)$ (iii) $F'(-2)$ (iv) $F'(4)$

(b) Find an equation of the tangent line to the graph of $y = F(x)$ at $x = 6$.

(c) Find the critical points of F.

(d) Find the intervals on which F is increasing and the intervals on which F is decreasing.

(e) Find the x-values at which $F(x)$ has a local maximum or a local minimum.

(f) Find the intervals on which F is concave up and the intervals on which F is concave down.

(g) Find the x-values at which $F(x)$ has an inflection point.

4. Let $f(x) = 7 + \int_{13}^{x} t(t-14)^{2/5} dt$.

(a) Find an equation of the tangent line to the graph of $y = f(x)$ at $x = 13$.

(b) Find the critical points of f.

(c) Find the intervals on which f is increasing and the intervals on which F is decreasing.

(d) Find the x-values at which $f(x)$ has a local maximum or a local minimum.

(e) Find the intervals on which f is concave up and the intervals on which F is concave down.

(f) Find the x-values at which $f(x)$ has an inflection point.