Final Exam Practice Session

1. Suppose $\tan(\theta) = -\frac{3}{2}$ and $\frac{\pi}{2} < \theta < \pi$. Evaluate the following.

   (a) $\cos(\theta)$ (b) $\sin(\theta)$ (c) $\cos(2\theta)$ (d) $\sin(2\theta)$

2. Calculate the following limits. You may use any valid method.

   (a) $\lim\limits_{x\to 0} \dfrac{x^3 - 5x^2}{3^{x^2} - 1}$ (b) $\lim\limits_{x\to\infty} xe^{-\sqrt{x}}$ (c) $\lim\limits_{x\to 4} \dfrac{\frac{2}{\sqrt{x}} - 1}{x - 4}$ (d) $\lim\limits_{x\to 0} (\cos(3x) + \tan(5x))^{1/x}$

3. Calculate $\dfrac{dy}{dx}$ for the following curves. You do not have to simplify your answers.

   (a) $y = \sqrt{9x^2 + 16x + 4}$

   (b) $y = \dfrac{\sin(13x)}{(2x+5)^{10}}$

   (c) $y = \sin^{-1}\left(\sqrt[4]{x}\right)$

   (d) $y = x^{\arctan(2x)}$

   (e) $y = \displaystyle\int_{\ln(x)}^{\pi} \cos(t^3)dt$

   (f) $y = \tan(xe^{7x})$

4. Calculate the following integrals.

   (a) $\displaystyle\int \left(\frac{9}{x^3} - \frac{1}{\sqrt{1-25x^2}}\right) dx$

   (b) $\displaystyle\int \tan^7(3\theta)\sec^2(3\theta)d\theta$

   (c) $\displaystyle\int_0^1 xe^{x^2}\cos\left(e^{x^2}\right) dx$

   (d) $\displaystyle\int \frac{6t+21}{t^2+7t+3}dt$

   (e) $\displaystyle\int_0^6 x\sqrt{36-x^2}dx$

   (f) $\displaystyle\int_0^6 \sqrt{36-x^2}dx$

5. The two parts of this problem are independent.

   (a) Alice walks at 4 ft/sec towards a building of height 600 ft. At what rate is the viewing angle between Alice and the top of the building changing when Alice is 200 ft away from the building?

   (b) A rectangular ice block with square base melts at a rate of 60 in^3/min. When the side length of the base is 4 in, the height is 6 in and decreases at a rate of 2 in/min. At what rate is the side length of the base decreasing at that time?

6. A closed cylindrical box has volume 250π ft^3. Find the dimensions of the box (height and radius) that give the minimal possible surface area.

7. Sketch the graph of a function f with the following features. Label all asymptotes, local extrema and inflection points.

   - The domain of f is $(-\infty,-3)\cup(-3,1)\cup(1,\infty)$ and the lines $x=-3$ and $x=1$ are asymptotes of f.
   - $\lim\limits_{x\to-\infty} f(x) = \infty$ and $\lim\limits_{x\to\infty} f(x) = 1$.
   - $f(-4) = -2$, $f(-1) = 2$, $f(3) = 4$ and $f(5) = 2$.
   - The signs of f' and f'' are given by the following charts.

x	$(-\infty,-4)$	$(-4,-3)$	$(-3,-1)$	$(-1,1)$	$(1,3)$	$(3,5)$	$(5,\infty)$
$f'(x)$	$-$	$+$	$-$	$-$	$+$	$-$	$-$
$f''(x)$	$+$	$+$	$+$	$-$	$-$	$-$	$+$

8. Find all horizontal and vertical asymptotes of the function $f(x) = \dfrac{\sqrt{4x^2+9}+5x}{x+3}$.

9. For each region described below, (i) sketch the region, then use (ii) integration with respect to x and (iii) integration with respect to y to set-up expression with integrals calculating the area of the region.

   (a) The region bounded by the parabola $x = 7 - y^2$ and the line $x = 3$.

   (b) The region bounded by the x-axis, the y-axis, the curve $y = 2^x$ and the line $y = 6 - x$. *(You can use fact that the curve and the line intersect at the point* $(2,4)$ *only.)*

10. Let $f(x) = \sin(x)\sqrt[3]{\cos(x)}$. Find the absolute maximum and minimum values of f on the interval $[0, \pi]$ and where they occur.

11. Suppose that f is a one-to-one differentiable function. The following table of values is given for f and f'.

x	-1	0	1	2
$f(x)$	2	3	6	11
$f'(x)$	7	2	8	5

   (a) Find an equation of the tangent line to the graph of $y = f(x)$ at the point $x = 1$.

   (b) Find an equation of the tangent line to the graph of $y = f^{-1}(x)$ at the point $x = 2$.

   (c) Let $G(x) = \arccos(2x)f(3x)$. Calculate $G'(0)$.

12. Let $f(x) = \frac{1}{2}x^{2/3}(x+5)$. Find the open intervals where f is increasing, decreasing, concave up, concave down, the x-coordinates of the local maxima, local minima and inflection points of f. Then sketch the graph of f.

13. Let $F(x) = \displaystyle\int_0^{x^2} e^{-t^2/4}dt$. Find the open intervals where F is concave up, concave down and the x-coordinates of the inflection points of F.

14. Find the points on the ellipse of equation $x^2 + xy + y^2 = 12$ where the tangent line is (a) horizontal and (b) vertical.

15. A particle is moving along an axis with acceleration $a(t) = \frac{6t}{(9+t^2)^2}$, initial velocity $v(0) = 1$ and initial position $s(0) = -2$. Find the position $s(t)$ of the particle.

16. Let $f(x) = \begin{cases} Ax + B & \text{if } x \le 0, \\ \arcsin\left(\frac{1}{x+2}\right) & \text{if } 0 < x. \end{cases}$

   (a) Find the value of the constant B for which f is continuous for all real numbers.

   (b) Find the values of the constants A, B for which f satisfies the conditions of the Mean Value Theorem on the interval $[-1, 1]$.