Rutgers University
Math 151

**Midterm 1 Practice Session**

1. Suppose that $\theta$ is an angle such that $\cot(\theta) = \dfrac{8}{3}$ and $\csc(\theta) < 0$. Find $\sec(\theta)$, $\sin(2\theta)$ and $\cos(2\theta)$.

2. Evaluate the following limits.

   (a) $\lim\limits_{x\to 1} \dfrac{\sqrt{4x^2+7} - \sqrt{x+10}}{x-1}$

   (b) $\lim\limits_{h\to 0} \dfrac{\dfrac{6}{3+7h} - 2}{h}$

   (c) $\lim\limits_{x\to 1^+} \dfrac{x^2-6x}{x^2-5x+4}$

   (d) $\lim\limits_{x\to -\infty} \dfrac{4x+3}{\sqrt{9x^2+5x+2}}$

   (e) $\lim\limits_{x\to 8} \dfrac{|3x-24|}{x^2-64}$.

   (f) $\lim\limits_{\theta\to \frac{\pi}{3}^+} \dfrac{1}{2\cos(\theta)-1}$

   (g) $\lim\limits_{x\to 0} \dfrac{x\sin(5x)}{\tan^2(3x)}$

   (h) $\lim\limits_{x\to \infty} \dfrac{4e^{2x}+e^{3x}}{5e^{3x}+2e^x}$

   (i) $\lim\limits_{x\to -2} \dfrac{3x}{x^2+4x+4}$

3. Suppose that $f(x)$ is a function such that $\cos(\pi x) \leqslant f(x) \leqslant x^4 - 8x^2 + 17$ for $-5 < x < 6$. Find the following limits, if possible. If there is not enough information to find a limit, explain why.

   (a) $\lim\limits_{x\to -2} f(x)$

   (b) $\lim\limits_{x\to 1} f(x)$

   (c) $\lim\limits_{x\to 2} f(x)$

4. Find horizontal asymptotes of $f(x) = \dfrac{|6x+1|+7x}{3x+8}$.

5. Find the vertical asymptotes of $f(x) = \dfrac{\sqrt{x^2+8}-3}{x^2-3x-4}$. Also find the limits to the left and right of any vertical asymptote.

6. Find the values of the constants $A$ and $B$ making the following function continuous on $\mathbb{R}$.

$$f(x) = \begin{cases} \arctan(Ax) & \text{if } x > 3, \\ \dfrac{\pi(x+2)}{20} & \text{if } 0 \leqslant x \leqslant 3, \\ \dfrac{x^2+Bx}{|x|} & \text{if } x < 0. \end{cases}$$

7. Use the Intermediate Value Theorem to show that the equation $\sec^{-1}(x) = 3 - x$ has a solution in the interval $[1, 2]$.

8. Let $f(x) = \dfrac{3x}{\sqrt{x}+5}$. Use the limit definition of derivatives to compute $f'(1)$, then find an equation of the tangent line to the graph of $f$ at $x = 1$.