Rutgers University
Math 151

## Midterm 2 Practice Session

1. Find $\dfrac{dy}{dx}$ for the following equations.

   (a) $y = \cos(7) + 4e^{3x} - \dfrac{5}{\sqrt[8]{x^3}}$

   (b) $y = \arctan(7\ln(x))$

   (c) $y = x^2 \sin^{-1}(2x)e^{-x}$

   (d) $y = \sec^3\left(\dfrac{2}{x} - e^{-4x}\right)$

   (e) $y = (1 - 3x)^{8\cot(5x^2)}$

   (f) $y = \sqrt{4 - 9x^2} - \sec^{-1}(3x)$

   (g) $y = \dfrac{\sqrt[7]{x^2}(x^2 + 6x + 1)^{32}}{(2x + 1)^{10}(x + 2)^3}$

   (h) $y = \dfrac{5^x}{\cos(2x) + 3x}$

   (i) $y = \sin(3x)^{x^2}$

2. Find the values of the constants $A, B$ for which the following function is differentiable at $x = 1$.

$$f(x) = \begin{cases} 10 - 3x & \text{if } x < 1, \\ x^A + Bx + 3 & \text{if } x \geqslant 1. \end{cases}$$

3. Suppose that $f$ is a one-to-one differentiable function. The following table of values is given for $f$ and $f'$.

| $x$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|
| $f(x)$ | $2$ | $3$ | $6$ | $11$ |
| $f'(x)$ | $7$ | $2$ | $8$ | $5$ |

   (a) Find an equation of the tangent line to the graph of $y = f(x)$ at the point $x = -1$.

   (b) Find an equation of the tangent line to the graph of $y = f^{-1}(x)$ at the point $x = 2$.

   (c) Let $G(x) = 2^{7x}f(3x)$. Calculate $G'(0)$.

   (d) Let $H(x) = \tan^{-1}\left(f(x^2)\right)$. Calculate $H'(-1)$.

   (e) Let $K(x) = \sqrt{f(2 - x)^2 + e^{16x}}$. Calculate $K'(0)$.

   (f) Let $M(x) = \cos(\pi x) f(2x)$. Calculate $M''\left(\frac{1}{2}\right)$.

4. Consider the curve of equation $x^2 + 6xy - y^2 = 40$. Find the points on the curve, if any, where the tangent line is (a) horizontal, (b) vertical, (c) perpendicular to $y = 2x + 9$.

5. The graph below shows the velocity $v$ of an object moving along an axis.
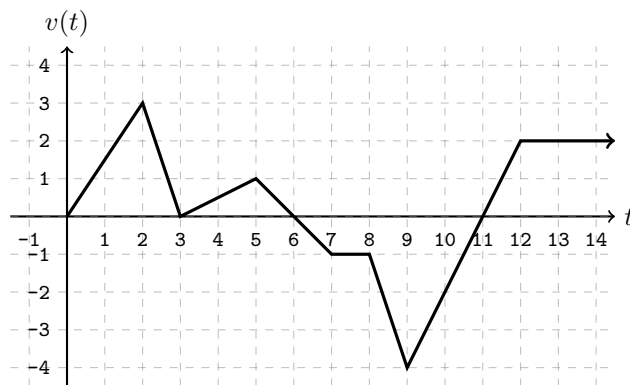
(a) When is the object moving forward? backward? standing still?

(b) When does the object reverse direction?

(c) When does the object move at greatest speed?

(d) When is the acceleration positive?

(e) What is the average acceleration on the interval $5 \leqslant t \leqslant 8$?

(f) What is the exact value of the acceleration at $t = 1$?

(g) Sketch the graph of the acceleration of the object.

6. A snow ball in the shape of a perfect sphere melts at a rate of 4 cm³/min. How fast is the surface area changing when the radius of the sphere is 7 cm? *[Hint: the volume and surface area of a sphere of radius $R$ are given by the formulas $V = \frac{4}{3}\pi R^3$, $S = 4\pi R^2$].*

7. Find the value of the constant $A$ such that the tangent line to $y = 2e^{Ax} + \tan^{-1}(7x)$ at $x = 0$ passes through the point $(-3, 11)$.