Rutgers University
Math 151

Midterm 3 Practice Session

1. The two parts of this problem are independent.

   (a) Suppose $f(-2) = 7$ and $f(1) = -4$. Fill in the blanks below. Your answer to the last blank must be a real number.

   If f is _____ on the interval $[-2, 1]$ and _____ on the interval $(-2, 1)$, then the Mean Value Theorem guarantees the existence of a number c in the interval _____ such that the the slope of the tangent line to the graph of f at $x = c$ is equal to _____.

   (b) Suppose that f is a differentiable function such that $f'(x) \geqslant -2$ and $f(3) = 4$. Find the maximum possible value of $f(-1)$ and the minimum possible value of $f(5)$.

2. Let $f(x) = \sqrt[3]{4\cos^2(x) - 1}$. Find the absolute extrema and where they occur for $f(x)$ on the interval $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right]$.

3. Let $f(x) = \ln(x^2 + 4)$. Find:

   (a) Find:
   (i) the critical points of f.
   (ii) the open intervals where f is increasing and decreasing.
   (iii) the open intervals where f is concave up and concave down.
   (iv) the x-coordinates of the local maxima and local minima of f.
   (v) the x-coordinates of the inflection points of f.

   (b) Sketch the graph of f.

4. Sketch the graph of a function f with the given properties.

   - $\lim_{x\to-\infty} f(x) = -\infty$ and $\lim_{x\to\infty} f(x) = -2$.
   - $\lim_{x\to-1^-} f(x) = -\infty$ and $\lim_{x\to-1^+} f(x) = \infty$.
   - $f(-3) = 2$, $f(2) = -5$, $f(4) = -3$.
   - The first two derivatives of f have the following sign chart.

x	$(-\infty, -3)$	$(-3, -1)$	$(-1, 2)$	$(2, 4)$	$(4, \infty)$
$f'(x)$	$+$	$-$	$-$	$+$	$+$
$f''(x)$	$-$	$-$	$+$	$+$	$-$

   Label all asymptotes, local extrema and inflection points.

5. A closed cylindrical box has total surface area 150π ft^2. Find the dimensions of the box (height and radius) that give the maximum possible volume.

6. A particle moving along an axis has acceleration $a(t) = \dfrac{8}{t^2} + 6$. Find the position $s(t)$ of the particle if $v(-1) = 3$ and $s(-1) = 5$.

7. Evaluate the following limits.

   (a) $\lim\limits_{x \to \frac{\pi}{6}} \sec^2(3x) \ln(\sin(3x))$

   (b) $\lim\limits_{x \to \infty} \left(\dfrac{2 \arctan(5x)}{\pi} \right)^x$