Section 10.2: Infinite Series - Worksheet

#60. Each of the series below is either geometric or telescoping. Determine if each series converges or diverges, and compute its sum if it converges.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-\pi)^n}{8}$

(b) $\displaystyle\sum_{n=4}^{\infty} 2^n 3^{-n}$

(c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{4}{2n+1} - \frac{4}{2n+5}\right)$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{1-3\cdot 4^{2n}}{5^{n-1}}$

(e) $\displaystyle\sum_{n=3}^{\infty} \ln\left(\frac{3n+1}{3n+4}\right)$

(f) $\displaystyle\sum_{n=1}^{\infty} 5\cdot 3^{1-2n}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{5^n+2^n}{6^n}$

(h) $\displaystyle\sum_{n=1}^{\infty} \left(\tan^{-1}(n+1) - \tan^{-1}(n)\right)$

(i) $\displaystyle\sum_{n=1}^{\infty} \left(5^{1/n} - 5^{1/(n+1)}\right)$

#61. Use geometric series to express the repeating decimals below as a fraction of two integers.

(a) $1.5222\cdots = 1.5\overline{2}$

(b) $0.126126\cdots = 0.\overline{126}$

#62. For each sequence $\{a_n\}_{n=n_0}^{\infty}$ given below, determine

(i) whether the **sequence** $\{a_n\}_{n=n_0}^{\infty}$ converges or diverges. If the sequence converges, find its limit.

(ii) whether the **series** $\sum\limits_{n=n_0}^{\infty} a_n$ converges or diverges. If the series converges, find its sum if possible.

(a) $\displaystyle\left\{\left(1+\frac{4}{n}\right)^n\right\}_{n=1}^{\infty}$

(b) $\left\{\sqrt{n+1}-\sqrt{n}\right\}_{n=0}^{\infty}$

(c) $\{e^{-n}\}_{n=0}^{\infty}$

(d) $\displaystyle\left\{\frac{e^{5n}}{n^{3/2}}\right\}_{n=1}^{\infty}$

(e) $\displaystyle\left\{\frac{3n+2\cos(n)}{5n}\right\}_{n=1}^{\infty}$

(f) $\displaystyle\left\{\cos\left(\frac{\pi}{n}\right) - \cos\left(\frac{\pi}{n+2}\right)\right\}_{n=3}^{\infty}$

#63. Consider the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{\sin(n)}{n} - \frac{\sin(n+1)}{n+1}\right)$.

(a) Find an explicit formula for the partial sum $S_N = \displaystyle\sum_{n=1}^{N}\left(\frac{\sin(n)}{n} - \frac{\sin(n+1)}{n+1}\right)$ of the series.

(b) Does the series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{\sin(n)}{n} - \frac{\sin(n+1)}{n+1}\right)$ converge or diverge? If it converges, find its sum. If it diverges, explain why.

#64. Suppose that $\{a_n\}_{n=1}^{\infty}$ is a sequence such that for all $n \geqslant 1$, we have

$$\frac{4n}{2n+1} \leqslant a_n \leqslant 1 + 5^{1/n}.$$

(a) Determine whether the **sequence** $\{a_n\}_{n=1}^{\infty}$ converges or diverges. If it converges, find $\lim_{n\to\infty} a_n$. If it diverges, explain why.

(b) Determine whether the **series** $\sum_{n=1}^{\infty} a_n$ converges or diverges. Justify your answer and name any test used.

#65. Let $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{2 \cdot 5^{n+1}}$. Find the values of x for which the series converges and find the sum of the series when it converges.

#66. Consider the series $\sum_{n=0}^{\infty} \left(\frac{5}{A-2}\right)^n$, where A is an unspecified positive constant.

(a) Find the values of the positive constant A for which the series converges.

(b) For the values of A you found in part (a), evaluate the sum of the series.