Section 10.3: The Integral Test - Worksheet

#67. Determine if the series below converge or diverge. In case of convergence, find the sum of the series if possible. **Note:** the Integral Test is not possible/necessary for all the series. Some of these use tests from earlier sections.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n-1)!}{n!}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{7}{3^{n+1}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{(n^2+8)^4}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{n^8}{(n^2+8)^4}$

(e) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{5}{\sqrt{n}} - \frac{5}{\sqrt{n+2}}\right)$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln(n)+4}}$

#68. For each sequence $\{a_n\}_{n=n_0}^{\infty}$ given below, determine

(i) whether the **sequence** $\{a_n\}_{n=n_0}^{\infty}$ converges or diverges. If the sequence converges, find its limit.

(ii) whether the **series** $\sum\limits_{n=n_0}^{\infty} a_n$ converges or diverges. If the series converges, find its sum if possible.

Note: the Integral Test is not possible/necessary for all the series. Some of these use tests from earlier sections.

(a) $\{n5^{-n}\}_{n=0}^{\infty}$

(b) $\displaystyle\left\{\frac{1}{n\left(1+\ln(n)^2\right)}\right\}_{n=2}^{\infty}$

(c) $\displaystyle\left\{\frac{1}{n^{\log_5(3)}}\right\}_{n=1}^{\infty}$

(d) $\left\{\cos\left(n^{1/n}\right)\right\}_{n=1}^{\infty}$

(e) $\displaystyle\left\{\frac{1}{(n^2+9)^{3/2}}\right\}_{n=0}^{\infty}$

(f) $\left\{\sec\left(\frac{\pi}{n}\right) - \sec\left(\frac{\pi}{n+1}\right)\right\}_{n=3}^{\infty}$

(g) $\left\{2^{2n+1}5^{-n}\right\}_{n=0}^{\infty}$

(h) $\displaystyle\left\{\left(1+\frac{1}{2n}\right)^n\right\}_{n=1}^{\infty}$

(i) $\displaystyle\left\{\frac{1}{n\ln(n)\ln(\ln(n))}\right\}_{n=4}^{\infty}$

#69. (a) Determine for which values of p the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)^p}$ converges or diverges.

(b) Determine for which values of p the series $\displaystyle\sum_{n=1}^{\infty} \frac{\ln(n)}{n^p}$ converges or diverges.