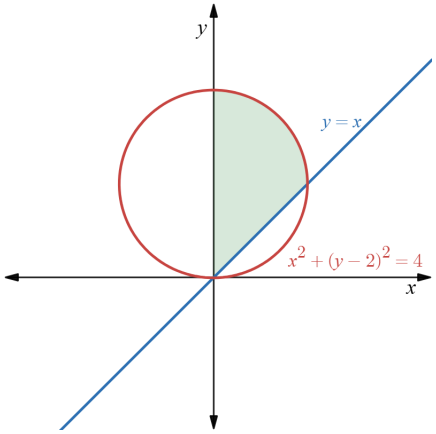
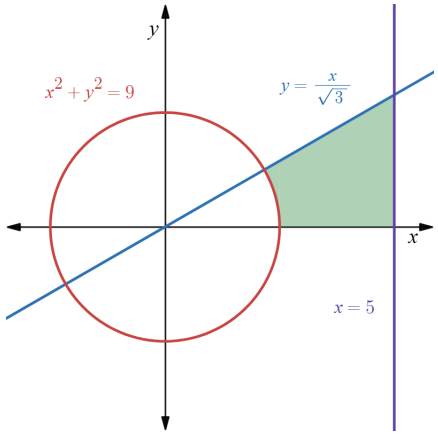
Section 11.5: Areas and Lengths in Polar - Worksheet

#115. Find the areas of the regions sketched below using polar coordinates.

(a) The region in the first quadrant contained in the circle of equation $x^2 + (y-2)^2 = 4$ and above the line $y = x$.



(b) The region in the first quadrant bounded by the circle $x^2 + y^2 = 9$ and the lines $x = 5$, $y = \frac{x}{\sqrt{3}}$.



#116. Find the areas of the given regions.

(a) The region shared by the circles $r = 2\sin(\theta)$ and $r = 2\cos(\theta)$.

(b) The region contained inside the leaves of the rose $r = 6\sin(2\theta)$ and outside the circle $r = 3$.

(c) The region inside the cardioid $r = 1 + \sin(\theta)$ and below the line $x = \sqrt{3}y$.

(d) The region inside the circle $r = \cos(\theta)$ and outside the cardioid $r = 1 - \cos(\theta)$.

(e) The region shared by one leaf of the rose $r = 2\cos(3\theta)$ and the circle $r = 1$.

#117. Consider the region $\mathcal{R}$ contained in the circle $r = 4\cos(\theta)$ to the right of the line $x = 3$.

(a) Find the area of the region $\mathcal{R}$ using integration with respect to x.

(b) Find the area of the region $\mathcal{R}$ using integration with respect to y.

(c) Find the area of the region $\mathcal{R}$ using integration with respect to θ.

#118. Find the lengths of the given polar curves.

(a) $r = \theta^2$, $0 \leqslant \theta \leqslant 2$

(b) $r = \sqrt{1 + \cos(2\theta)}$, $0 \leqslant \theta \leqslant \frac{\pi}{2}$.

(c) $r = \dfrac{2}{1 - \cos(\theta)}$, $\frac{\pi}{2} \leqslant \theta \leqslant \pi$.

(d) $r = e^{3\theta}$, $0 \leqslant \theta \leqslant \pi$.