

**Math 152 All Worksheets**  
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## Calculus I Review - Worksheet

#1. Simplify the following expressions. Your answer should not involve any trigonometric or inverse trigonometric functions.

(a) $\cos^{-1}\left(\frac{1}{2}\right)$	(c) $\cos(\sin^{-1}(x))$	(e) $\sec\left(\tan^{-1}\left(\frac{x}{3}\right)\right)$
(b) $\sin^{-1}\left(\sin\left(\frac{7\pi}{4}\right)\right)$	(d) $\sin(2\cos^{-1}(x))$	(f) $\sin\left(\cot^{-1}\left(\frac{2}{\sqrt{x}}\right)\right)$

#2. Calculate the following limits.

(a) $\lim_{x \rightarrow \infty} \frac{(\ln(x))^2}{\sqrt{x}}$	(c) $\lim_{x \rightarrow \infty} \tan^{-1}(x^2 - x^3)$	(e) $\lim_{x \rightarrow -\infty} \frac{2x + 3\cos(x)}{5x}$
(b) $\lim_{x \rightarrow 0} \frac{5^x - 3^x}{\sin(2x)}$	(d) $\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x$	(f) $\lim_{x \rightarrow \infty} x^{1/x}$

#3. Find the horizontal asymptotes of the following functions.

(a) $f(x) = \frac{11x^3 + 2x - 1}{2x^3 - x^2 + 3}$	(b) $f(x) = \frac{5x + \sqrt{16x^2 + 25}}{18x - 7}$	(c) $f(x) = \frac{3e^x + 4}{x - 5e^x}$
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#4. Calculate the following indefinite or definite integrals.

(a) $\int (3x + 1) \left(x^2 - \frac{5}{x}\right) dx$	(e) $\int e^x (e^x - 2)^{2/3} dx$	(i) $\int \frac{dx}{\sqrt{2 - x^2}}$
(b) $\int x^3 \sin(x^4 + 2) dx$	(f) $\int e^{2x} (e^x - 2)^{2/3} dx$	(j) $\int_0^1 \frac{x dx}{\sqrt{2 - x^2}}$
(c) $\int_0^1 \frac{x^3}{\sqrt{3 + x^2}} dx$	(g) $\int_{e^3}^{e^6} \frac{dt}{t \ln(t)}$	(k) $\int_0^{2/3} \frac{dz}{4 + 9z^2}$
(d) $\int t \sec^2(3t^2) e^{7 \tan(3t^2)} dt$	(h) $\int \frac{dx}{5x + 4\sqrt{x}}$	(l) $\int \frac{dx}{x^2 + 6x + 34}$

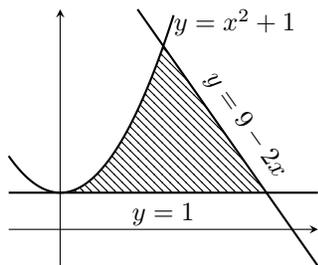
## Sections 5.5-6: Substitution Method and Area Between Curves - Worksheet

#5. Evaluate the following antiderivatives.

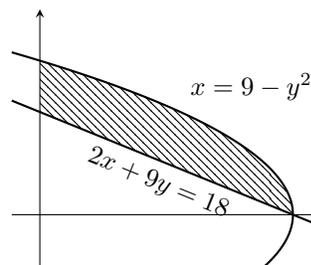
(a) $\int \frac{dx}{\sqrt{8x - x^2}}$	(d) $\int \frac{dx}{x^2 + 10x + 29}$	(g) $\int_0^{\pi/9} \frac{\sec(3x) \tan(3x)}{5 \sec(3x) - 4} dx$
(b) $\int \frac{[\tan^{-1}(t)]^3}{1 + t^2} dt$	(e) $\int \frac{dt}{t\sqrt{1 - 9[\ln(t)]^2}}$	(h) $\int e^{3x} \cos(5 - 2e^{3x}) dx$
(c) $\int \frac{\tan(3 \ln(x))}{x} dx$	(f) $\int \frac{\sqrt{1 + \sqrt{z}}}{\sqrt{z}} dz$	(i) $\int_{-2}^2 \sin(3x) \sqrt{x^2 + 4} dx$

#6. For the regions sketched below, set-up an expression that calculates the area of the region using (i) a  $dx$ -integral, and (ii) a  $dy$ -integral. You do not need to evaluate the integrals.

(a) The region to the right of the curve  $y = x^2 + 1$ , above the line  $y = 1$  and to the left of the line  $y = 9 - 2x$ .



(b) The region to the right of the  $y$ -axis, below the curve  $x = 9 - y^2$  and above the line  $2x + 9y = 18$ .



#7. For each of the regions described below (i) sketch the region, clearly labeling the curves and their intersection points, (ii) calculate the area of the region using an  $x$ -integral and (iii) calculate the area of the region using a  $y$ -integral.

(a) The region to the right of the parabola  $y = 1 - (x - 2)^2$ , below the line  $y = 1$  and to the left of the line  $x - 2y = 3$ .

(b) The region bounded by the curves  $y = 2x$  and  $y = \sqrt[3]{32x}$ .

(c) The region bounded by the curves  $y = \frac{4}{x + 2}$  and  $y = 3 - x$ .

#8. Calculate the area of the regions described below.

(a) The region bounded by the parabola  $x = (y + 3)^2 - 4$  and the line  $x = 3y + 9$ .

- (b) The region bounded by  $y = \frac{4}{3+x^2}$  and  $y = 1$ .
- (c) The region bounded by  $y = 2 \ln(x+1)$ , the  $x$ -axis and the line  $x = 4$ .
- (d) The region to the right of the  $y$ -axis, above the graph of  $y = \sec^2(x)$  and below the graph of  $y = 2 \sec(x)$ .

**#9.** Suppose that  $f$  is an **even** function such that

$$\int_{-9}^5 f(x) dx = -13 \quad \text{and} \quad \int_0^9 f(x) dx = 4.$$

Evaluate the definite integrals below.

(a)  $\int_{-9}^9 f(x) dx$

(c)  $\int_{-3}^3 xf(x) dx$

(b)  $\int_0^5 (4x - 3f(x)) dx$

(d)  $\int_0^3 xf(x^2) dx$

**#10.** Find the average value of the following functions on the given interval.

(a)  $f(x) = \frac{3}{\sqrt{100-x^2}}$  on  $[0, 5]$ .

(c)  $f(x) = \frac{e^x}{(e^x+1)^3}$  on  $[0, \ln(2)]$ .

(b)  $f(x) = \cos(5x)\sqrt{\sin(5x)}$  on  $[\frac{\pi}{30}, \frac{\pi}{10}]$ .

(d)  $f(x) = x\sqrt[3]{3x-7}$  on  $[2, 5]$ .

## Section 6.1: Volumes by Cross-Sections - Worksheet

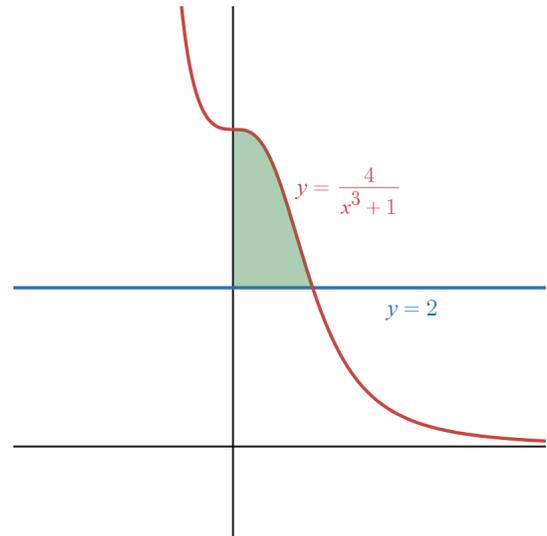
#11. Calculate the volume of each solid described below.

- (a) The base of the solid is the region bounded by the curves  $y = x^2$  and  $y = 8 - x^2$  and the cross-sections perpendicular to the  $x$ -axis are rectangles of height 5.
- (b) The base of the solid is the region inside the circle of equation  $x^2 + y^2 = 16$  and the cross-sections perpendicular to the  $y$ -axis are squares.
- (c) The base of the solid is the region between the graph of  $y = \sec(3x)$  and the  $x$ -axis for  $0 \leq x \leq \frac{\pi}{12}$  and the cross sections perpendicular to the  $x$ -axis are semi-circles with diameter in the base.
- (d) The base of the solid is the region bounded by the curves  $y = 2x$ ,  $y = \sqrt{x} + 1$  and the  $y$ -axis, and the cross-sections perpendicular to the  $y$ -axis are isosceles triangles of height 3.

#12. Consider the region bounded by the curve  $y = \frac{4}{x^3 + 1}$ , the  $y$ -axis and the line  $y = 2$ .

This region is revolved around an axis to form a solid of revolution. Use the disk/washer method to set-up an integral that calculates the volume of the solid for each of the axes of revolution given below. You do not need to evaluate the integrals.

- |                   |              |
|-------------------|--------------|
| (a) the $x$ -axis | (d) $y = 2$  |
| (b) the $y$ -axis | (e) $x = 3$  |
| (c) $y = -5$      | (f) $x = -2$ |



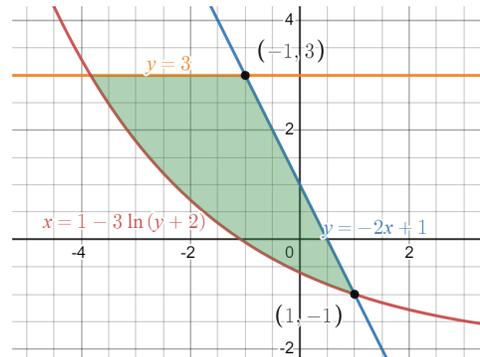
#13. Use the method of disks/washers to calculate the volume of the solids of revolutions obtained by revolving the regions described below about the given axis.

- (a) The region bounded by the curve  $y = \frac{15}{\sqrt{2x+1}}$ , the line  $y = 5$  and the  $y$ -axis revolved about the  $x$ -axis.
- (b) The region below the graph of  $y = \ln(x)$  on  $1 \leq x \leq 3$  revolved about the line  $x = 3$ .
- (c) The region below the graph of  $y = \frac{1}{\sqrt{25 + 4x^2}}$  on  $0 \leq x \leq \frac{5}{2}$  revolved about the  $x$ -axis.
- (d) The region bounded by  $y = e^x$ ,  $y = 4 - e^x$  and the coordinate axes revolved about the line  $y = 4$ .
- (e) The region below the graph of  $y = 2 \sin^{-1}(x^2)$  on  $0 \leq x \leq 1$  revolved about the  $y$ -axis.

#14. Consider the region  $\mathcal{R}$  in the first quadrant bounded by the curve  $x = 4 - (y - 1)^2$ .

- Sketch the region. Make sure to clearly label the curve and its intercepts.
- A solid has base  $\mathcal{R}$  and cross-sections perpendicular to the  $y$ -axis. Calculate the volume of the solid if the cross-sections are (i) semi-circles with diameter in the base and (ii) equilateral triangles with a side in the base.
- A solid has base  $\mathcal{R}$  and its cross-sections perpendicular to the  $x$ -axis are isosceles right triangles with hypotenuse in the base. Calculate the volume of the solid.
- Calculate the volume of the solid of revolution obtained by revolving  $\mathcal{R}$  about (i) the  $y$ -axis and (ii) the line  $y = -2$ .

#15. Consider the region  $\mathcal{R}$  bounded by the curves  $x = 1 - 3\ln(y + 2)$ ,  $y = 3$  and  $y = -2x + 1$ . The region  $\mathcal{R}$  is shaded in the figure to the right. Use the disk/washer method to set-up integrals that compute the volume of the solid obtained by revolving  $\mathcal{R}$  about each of the lines given below.



- $x = 2$
- $y = 3$
- $x = -4$
- $y = -2$

#16. Consider the region below the graph  $y = \cot(5x) + \csc(5x)$  for  $\frac{\pi}{20} \leq x \leq \frac{\pi}{10}$ .

- Find the area of the region.
- Find the volume of the solid obtained by revolving the region about the  $x$ -axis.

## Section 6.2: Volumes by Shells - Worksheet

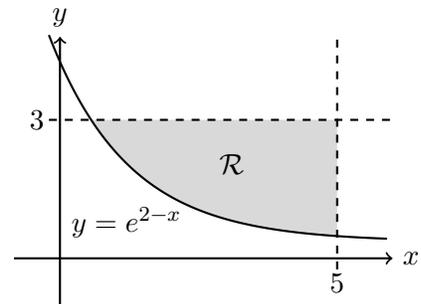
**#17.** Find the volume of the solid of revolution obtained by revolving the given region about the given axis using (i) the method of cylindrical shells and (ii) the method of disks/washers.

- (a) The region bounded by the curve  $y = 2x^3$  and the  $x$ -axis for  $0 \leq x \leq 1$  revolved about the  $x$ -axis.
- (b) The region bounded by the  $y$ -axis, the curve  $y = 5\sqrt{x}$  and the line  $y = 10$  revolved about the line  $x = -2$ .
- (c) The region in the first quadrant bounded by the curves  $y = 9 - x^2$  and  $y = 1 - \frac{1}{3}x$  revolved about the line  $y = 9$ .
- (d) The region below the graph of  $y = \frac{2}{\sqrt{x+1}}$  for  $0 \leq x \leq 3$  revolved about the  $y$ -axis.
- (e) The region below the graph of  $y = \frac{2}{\sqrt{x+1}}$  for  $0 \leq x \leq 3$  revolved about the  $x$ -axis.

**#18.** Find the volume of the solid of revolution obtained by revolving the given region about the given axis using the method of cylindrical shells.

- (a) The region bounded by the curve  $y = 3\sqrt{\ln(x)}$ , the line  $y = 3$  and the line  $x = 1$  revolved about the  $x$ -axis.
- (b) The region below the graph of  $y = \frac{1}{16 + x^4}$  for  $0 \leq x \leq 2$  revolved about the  $y$ -axis.
- (c) The region bounded by the curve  $x = 4 - (y + 1)^2$ , the line  $x = 4$  and the line  $y = x + 7$  revolved about the line  $y = -5$ .

**#19.** Consider the region  $\mathcal{R}$  bounded by the curve  $y = e^{2-x}$ , the line  $x = 5$  and the line  $y = 3$ . The region  $\mathcal{R}$  is sketched to the right. Set-up integrals computing the volume of the solid obtained by revolving  $\mathcal{R}$  about each axis given below using (i) the disk/washer method, and (ii) the shell method.



- (a)  $x$ -axis      (b)  $y$ -axis      (c)  $y = -1$       (d)  $x = 7$       (e)  $y = 3$       (f)  $x = -4$

**#20.** Let  $\mathcal{R}$  be the region bounded by the curve  $y = \frac{\cot(x)}{x}$ , the  $x$ -axis, the line  $x = \frac{\pi}{6}$  and the line  $x = \frac{\pi}{4}$ . Find the volume of the solid obtained by rotating  $\mathcal{R}$  about the  $y$ -axis.

**#21.** Let  $\mathcal{R}$  be the region bounded by the lines  $y = x$ ,  $y = 2x$  and  $y = -2x + 12$ .

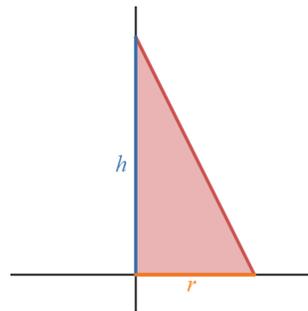
- Sketch the region  $\mathcal{R}$ . Label the curves, their intersection points and lightly shade the region.
- Calculate the area of the region using (i) an  $x$ -integral and (ii) a  $y$ -integral.
- The base of a solid is  $\mathcal{R}$ , and the cross-sections perpendicular to the  $x$ -axis are isosceles right triangles with height on the base. Set-up an expression with integrals that calculates the volume of the solid.
- The base of a solid is  $\mathcal{R}$ , and the cross-sections perpendicular to the  $y$ -axis are circles with diameter in the base. Set-up an expression with integrals that calculates the volume of the solid.
- We create a solid of revolution by revolving  $\mathcal{R}$  about the line  $x = -2$ . Set up an expression with integrals that calculates the volume of the solid using (i) the method of disks/washers and (ii) the method of cylindrical shells.
- We create a solid of revolution by revolving  $\mathcal{R}$  about the line  $y = 8$ . Set up an expression with integrals that calculates the volume of the solid using (i) the method of disks/washers and (ii) the method of cylindrical shells.

**#22.** Consider the disk of equation  $(y - 2)^2 + x^2 \leq 1$  centered at  $(0, 2)$  of radius 1. A torus (or informally, a donut) is created by revolving the disk about the  $x$ -axis.

- Find the volume of the torus using the washer method.
- Find the volume of the torus using the shell method.

**#23.** Let  $\mathcal{R}$  be the region under the graph of  $y = \frac{1}{\sqrt{16 - x^2}}$  for  $0 \leq x \leq 2$ . Find the volume of the solid obtained by revolving  $\mathcal{R}$  about the line  $x = -3$ .

**#24.** A circular cone of height  $h$  and radius  $r$  is created by revolving a right triangle with base  $r$ , height  $h$  and right angle at the origin about the  $y$ -axis, see figure to the right.



- The triangle is bounded by the coordinate axes and a line. Find an equation of the line bounding the triangle in terms of the constants  $r$  and  $h$ .
- Calculate the volume of the cone using the disk/washer method.
- Calculate the volume of the cone using the shell method.

### Section 6.3: Arc Length - Worksheet

#25. Calculate the arc length of the given curves.

(a)  $y = 11 - 2(x - 5)^{3/2}$ ,  $5 \leq x \leq 6$ .

(b)  $y = \frac{1}{3} \ln(\cos(3x)) + 4$ ,  $\frac{\pi}{4} \leq x \leq \frac{\pi}{3}$ .

(c)  $x = \frac{1}{4} \sqrt[3]{y} - \frac{9}{5} \sqrt[3]{y^5}$ ,  $1 \leq y \leq 2$ .

(d)  $y = \sqrt{36 - x^2}$ ,  $0 \leq x \leq 3$ .

(e)  $y = \frac{1}{3}(2x + 1)^{3/2}$ ,  $0 \leq x \leq 1$ .

(f)  $x = \sqrt{16y - y^2}$ ,  $4 \leq y \leq 12$ .

(g)  $y = \frac{1}{6} \ln(\sin(3x) \cos(3x))$ ,  $\frac{\pi}{18} \leq x \leq \frac{\pi}{9}$ .

(h)  $y = \frac{e^{5x} + e^{-5x}}{10}$ ,  $0 \leq x \leq \frac{1}{5}$ .

(i)  $x = \frac{4}{5}y^{5/4}$ ,  $0 \leq y \leq 9$ .

(j)  $x = \frac{6}{7}y^{7/6}$ ,  $0 \leq y \leq 1$ .

(k)  $y = \cos\left(\frac{x}{2}\right) - \ln\left(\csc\left(\frac{x}{2}\right) + \cot\left(\frac{x}{2}\right)\right)$ ,  $\frac{\pi}{2} \leq x \leq \pi$ .

#26. Find a curve of the form  $y = f(x)$  passing through  $(4, 13)$ , having negative derivative, and whose length integral on  $1 \leq x \leq 7$  is given by

$$L = \int_1^7 \sqrt{1 + \frac{25}{x^3}} dx.$$

#27. Consider the ellipse of equation  $9x^2 + 4y^2 = 36$ . Set-up an integral that calculates the perimeter of the ellipse.

## Section 6.4: Areas of Surfaces of Revolution - Worksheet

**#28.** Find the surface area obtained by revolving the given curve about the given axis.

- (a) The curve  $y = 2x^3$ ,  $0 \leq x \leq 1$ , revolved about the  $x$ -axis.
- (b) The curve  $y = \sqrt{3x-5}$ ,  $2 \leq x \leq 3$ , revolved about the  $x$ -axis.
- (c) The curve  $x = \sqrt{16y-y^2}$ ,  $0 \leq y \leq 8$ , revolved about the  $y$ -axis.
- (d) The curve  $x = 2\sqrt[3]{y}$ ,  $0 \leq y \leq 1$ , revolved about the  $x$ -axis.
- (e) The curve  $y = \sqrt{9+8x-x^2}$ ,  $3 \leq x \leq 7$ , revolved about the  $x$ -axis.
- (f) The curve  $x = \frac{3}{5}y^{5/3}$ ,  $0 \leq y \leq 1$ , revolved about the  $y$ -axis.
- (g) The curve  $y = x^{3/2}$ ,  $1 \leq x \leq 4$ , revolved about the  $y$ -axis.

**#29.** Let  $L$  be the line segment joining the points  $(0,7)$  and  $(2,1)$ . Find the areas of the surfaces obtained when  $L$  is revolved about the following lines.

- (a)  $x$ -axis
- (b)  $y$ -axis
- (c)  $y = -1$
- (d)  $x = -3$

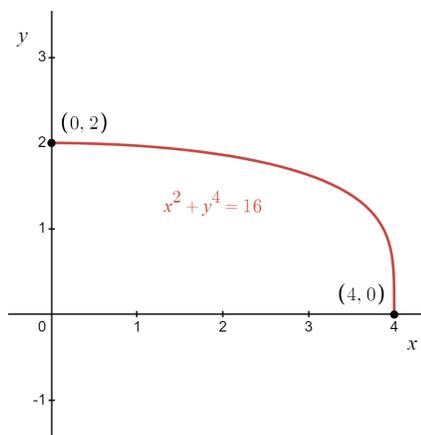
**#30.** Consider the curve of equation  $y = \frac{e^x + e^{-x}}{2}$ ,  $0 \leq x \leq 1$ .

- (a) Find the length of the curve.
- (b) Find the area of the surface obtained by revolving the curve about the  $x$ -axis.

**#31.** Consider the curve of equation  $x^2 + y^4 = 16$  for  $x, y \geq 0$ .

- (a) Set-up an integral computing the length of the curve using (i) integration with respect to  $x$  and (ii) integration with respect to  $y$ .
- (b) Set-up integrals computing the areas of the surfaces obtained by revolving the curve about the following axes.

- i.  $x$ -axis.
- ii.  $y$ -axis.
- iii.  $y = -2$ .
- iv.  $x = 6$ .



**#32.** Use integration to find the surface area of a sphere of radius  $R$ .

## Section 8.2: Integration by Parts - Worksheet

**#33.** Evaluate the following integrals. **Note:** some of these problems use integration techniques from earlier sections.

(a) $\int 5xe^{8x+1} dx$	(f) $\int x^2 \sin^{-1}(x) dx$	(k) $\int e^{-2x} \sin(3x) dx$
(b) $\int \arctan(7x) dx$	(g) $\int_0^{\pi/12} x \sec(4x) \tan(4x) dx$	(l) $\int_{-2}^{-1} x \sec^{-1}(x) dx$
(c) $\int x^3 \cos(5x) dx$	(h) $\int \frac{\ln(x)}{x^5} dx$	(m) $\int \theta \sec^2(7\theta) d\theta$
(d) $\int x^3 \cos(5x^4) dx$	(i) $\int x^3 e^{-x^2} dx$	(n) $\int \sin(\sqrt{3x+5}) dx$
(e) $\int_1^e (\ln(x))^2 dx$	(j) $\int_3^6 \frac{dx}{\sqrt{12x-x^2}}$	(o) $\int \frac{x}{\sqrt{5x+1}} dx$

**#34.** Calculate the volume of the solid obtained by revolving the given region about the given axis using (i) the method of disks/washers and (ii) the method of cylindrical shells.

- The region between the graph of  $y = \sqrt{\tan^{-1}(x)}$  and the  $x$ -axis for  $0 \leq x \leq 1$  revolved about the  $x$ -axis.
- The region bounded by the  $y$ -axis, the graph of  $y = \sin(x)$  and the line  $y = 1$  revolved about the  $y$ -axis.
- The region between the graph of  $y = \ln(x)$  and the  $x$ -axis for  $1 \leq x \leq e$  revolved about the line  $x = -2$ .

**#35.** Express  $\int \sin^7(2x) dx$  in terms of  $\int \sin^5(2x) dx$ .

**#36.** Express  $\int \sec^9(4x) dx$  in terms of  $\int \sec^7(4x) dx$ .

**#37.** Find reduction formulas for the following integrals.

(a) $\int \cos^n(3x) dx$	(b) $\int (\ln(x))^n dx$	(c) $\int \sec^n(5x) dx$
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### Section 8.3: Trigonometric Integrals - Worksheet

**#38.** Calculate the following integrals. **Note:** some of these problems use integration techniques from earlier sections.

(a) $\int \sin^2(5x)dx$	(g) $\int_{\pi/3}^{2\pi/3} \sqrt{1 - \cos(6x)}dx$	(m) $\int \cos^{42}(x) \sin^3(x)dx$
(b) $\int \cos^3(7\theta)d\theta$	(h) $\int \tan^3(\theta) \sec^5(\theta)d\theta$	(n) $\int_{\pi/3}^{\pi/2} \sqrt{\frac{1 + \cos(t)}{1 - \cos(t)}}dt$
(c) $\int \sec^4(2x) \tan^6(2x)dx$	(i) $\int \sin(5x)\sqrt{1 + \cos(5x)}dx$	(o) $\int \cos^2(8x) \sin^2(8x)dx$
(d) $\int (x^3 + 8x) \sin(3x)dx$	(j) $\int \sec^2(3x) \ln(\sec(3x))dx$	(p) $\int_{\pi/2}^{\pi} \sqrt{1 + \sin(\theta)}d\theta$
(e) $\int_0^{\pi/21} \tan^3(7\theta)d\theta$	(k) $\int_{\pi}^{3\pi/2} \cos^5(z) \sin^8(z)dz$	(q) $\int_{\pi}^{2\pi} \frac{\sqrt{1 + \cos(x)}}{e^x}dx$
(f) $\int_0^{\pi/6} x \cos(3x) \sin(3x)dx$	(l) $\int \sin^4(3x)dx$	

**#39.** Express  $\int \sin^n(3x)dx$  in terms of  $\int \sin^{n-2}(3x)dx$ .

**#40.** Calculate the volume of the solid obtained by revolving each region described below about (i) the  $x$ -axis and (ii) the  $y$ -axis.

- (a) The region bounded by  $x$ -axis, the graph of  $y = \cos^2(4x)$  on  $0 \leq x \leq \frac{\pi}{8}$ .
- (b) The region bounded by the  $x$ -axis, the graph of  $y = \sec^2(x) \tan(x)$  and the lines  $x = 0$ ,  $x = \frac{\pi}{4}$ .

**#41.** Evaluate  $\int \sec^3(\theta)d\theta$  and  $\int \sec(\theta) \tan^2(\theta)d\theta$ .

**#42.** Calculate the arc length of the curve  $y = x + \cos(x) \sin(x) - \frac{1}{8} \tan(x)$ ,  $0 \leq x \leq \frac{\pi}{4}$ .

## Section 8.4: Trigonometric Substitution - Worksheet

**#43.** Calculate the following integrals. **Note:** some of these problems use integration techniques from earlier sections.

(a) $\int \sqrt{25 - 9x^2} dx$	(h) $\int \frac{dx}{(6x - x^2 - 5)^{5/2}}$	(o) $\int \sin^4(5x) dx$
(b) $\int \frac{1}{(16 - x^2)^{3/2}} dx$	(i) $\int_1^{\sqrt{2}} \frac{dx}{x(2x^2 - 1)^{3/2}}$	(p) $\int \frac{dx}{\sqrt{x^2 - 6x}}, x > 6$
(c) $\int_0^2 \frac{1}{\sqrt{x^2 + 4}} dx$	(j) $\int_0^{5/2} \sqrt{25 - 4x^2} dx$	(q) $\int_{\sqrt{2}}^2 \sec^{-1}(t) dt$
(d) $\int \frac{\sqrt{25x^2 - 4}}{x} dx, x > \frac{2}{5}$	(k) $\int \frac{e^{6x}}{\sqrt{16 - e^{4x}}} dx$	(r) $\int x^2 \tan^{-1}(x) dx$
(e) $\int x \sqrt{81 - x^2} dx$	(l) $\int_0^{\pi/18} \cos^5(3x) dx$	(s) $\int \frac{dx}{(36 + x^2)^2}$
(f) $\int x^5 (\ln(x))^2 dx.$	(m) $\int \frac{x^2}{(1 + 9x^2)^{5/2}} dx$	(t) $\int \sin^{-1}(x)^2 dx$
(g) $\int \frac{dt}{t \sqrt{9 + (\ln(t))^2}}$	(n) $\int_5^{11} \frac{dx}{(x^2 - 10x + 61)^{5/2}}$	(u) $\int_0^8 \sqrt{64 + x^2} dx$

**#44.** Calculate the average value of the function  $f(x) = \frac{1}{x\sqrt{64 - x^2}}$  on the interval  $[4, 4\sqrt{2}]$

**#45.** (a) Evaluate  $\int \sqrt{1 + x^2} dx.$

(b) Use your result from part (a) for the following applications.

- i. Calculate the length of the curve  $y = x^2, 0 \leq x \leq 1.$
- ii. Calculate the area of the surface obtained by revolving the curve  $y = e^x, 0 \leq x \leq \ln(2),$  about the  $x$ -axis.
- iii. Calculate the area of the surface obtained by revolving the curve  $y = \sin^{-1}(x), 0 \leq x \leq 1$  about the  $y$ -axis.

**#46.** Calculate the area of the region between the semi-circle  $y = \sqrt{2 - x^2}$  and the  $x$ -axis for  $\frac{\sqrt{2}}{2} \leq x \leq 1.$

**#47.** Calculate the area of the region inside the circle of equation  $x^2 - 2x + y^2 = 3$  and above the line  $y = \sqrt{3}$

**#48.** Consider the region  $\mathcal{R}$  bounded between the graph of  $y = \frac{1}{16 - x^2}$  and the  $x$ -axis for  $0 \leq x \leq 2$ . Find the volume of the solid obtained by revolving  $\mathcal{R}$  about the line  $x = -3$ .

**#49.** (a) Evaluate  $\int \arcsin(x) dx$ .

(b) Using your answer to part (a), evaluate  $\int \ln(x) \arcsin(x) dx$ . (*Hint: LIATE.*)

**#50.** Calculate the surface area obtained by revolving the curve  $y = \frac{1}{2}x^2$ ,  $0 \leq x \leq 1$ , about the  $x$ -axis.

**#51.** Let  $\mathcal{R}$  be the region between the graph  $y = \frac{1}{x\sqrt{1+x^2}}$  and the  $x$ -axis for  $1 \leq x \leq \sqrt{3}$ .

(a) Find the area of  $\mathcal{R}$ .

(b) Find the volume of the solid obtained by revolving  $\mathcal{R}$  about the  $x$ -axis.

(c) Find the volume of the solid obtained by revolving  $\mathcal{R}$  about the  $y$ -axis.

## Section 8.8: Improper Integrals - Worksheet

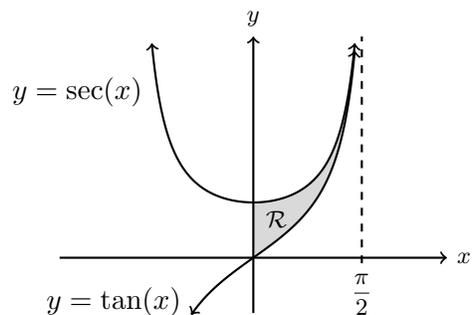
**#52.** Calculate the following integrals or determine if they diverge.

(a) $\int_0^{\infty} e^{-5x} dx$	(g) $\int_1^5 \frac{dx}{\sqrt[3]{10-2x}}$	(m) $\int_e^{\infty} \frac{dx}{x \ln(x)}$
(b) $\int_0^{\pi/4} \csc(x) dx$	(h) $\int_{-\pi/12}^{\pi/12} \csc(3\theta) d\theta$	(n) $\int_7^{14} \frac{dx}{\sqrt{14x-x^2}}$
(c) $\int_0^{\infty} \sin(5x) dx$	(i) $\int_{-\infty}^{\infty} \frac{dx}{(16+x^2)^{3/2}}$	(o) $\int_0^{\infty} e^{-x} \sin(x) dx$
(d) $\int_{-\infty}^0 xe^{3x} dx$	(j) $\int_0^1 \ln(x) dx$	(p) $\int_0^{\infty} \frac{dx}{(x^2+16)^2}$
(e) $\int_{-7}^{\infty} \frac{dx}{x^2+4x+29}$	(k) $\int_{-2}^1 \frac{dx}{\sqrt[3]{3x-2}}$	(q) $\int_0^1 \frac{\ln(x)}{\sqrt[3]{x}} dx$
(f) $\int_{-\infty}^0 x^4 e^{3x^5} dx$	(l) $\int_0^{3/2} \frac{dx}{\sqrt{9-4x^2}}$	(r) $\int_0^{\infty} e^{-\sqrt{z}} dz$

**#53.** Consider the unbounded region  $\mathcal{R}$  between the graph of  $y = \frac{\ln(x)}{x}$  and the  $x$ -axis for  $x \geq 1$ .

- (a) Find the area of the region  $\mathcal{R}$  or determine if  $\mathcal{R}$  has infinite area.
- (b) We now revolve the region  $\mathcal{R}$  about the  $x$ -axis to form a solid of revolution. Calculate the volume of the solid or determine if the solid has infinite volume.
- (c) We now revolve the region  $\mathcal{R}$  about the  $y$ -axis to form a solid of revolution. Calculate the volume of the solid or determine if the solid has infinite volume.

**#54.** Find the area of the unbounded region  $\mathcal{R}$  trapped between the graphs of  $y = \sec(x)$  and  $y = \tan(x)$  for  $0 \leq x < \frac{\pi}{2}$  (see graph to the right) or show that  $\mathcal{R}$  has infinite area.



**#55.** For the region  $\mathcal{R}$  defined in the previous problem, we consider the solid obtained by revolving  $\mathcal{R}$  about the  $x$ -axis. Find the volume of the solid or determine if the solid has infinite volume.

**#56.** Use a convergence test to determine if the following improper integrals converge or diverge.

$$(a) \int_3^{\infty} \frac{dx}{xe^x}$$

$$(b) \int_1^{\infty} \frac{dx}{x^2 + 3x + 1}$$

$$(c) \int_4^{\infty} \frac{\cos(x) + 5}{x^{3/5}} dx$$

$$(d) \int_0^1 \frac{dx}{\sqrt{x} + x^2}$$

$$(e) \int_5^{\infty} \frac{xdx}{x^4 - 1}$$

$$(f) \int_1^{\infty} \frac{x^3 + 5x^2 + 1}{\sqrt{x^7 + 4x + 2}} dx$$

## Section 10.1: Sequences - Worksheet

#57. Determine if the sequences below converge or diverge. In case of convergence, find the limit.

(a) $a_n = \frac{\sqrt{1+16n^4}}{n^2+1}$	(h) $a_n = \frac{e^{n^2}}{n^3}$	(o) $a_n = \cos\left(\frac{5}{\sqrt{n}}\right)^n$
(b) $a_n = n \ln\left(1 - \frac{2}{5n}\right)$	(i) $a_n = \frac{3n^2(n+1)!}{(n+3)!}$	(p) $a_n = \left(\frac{n+5}{n+3}\right)^{4n}$
(c) $a_n = \frac{2n + (-1)^n}{7n}$	(j) $a_n = \sqrt{n^2 + 5n} - n$	(q) $a_n = (2n+1)^{3/n}$
(d) $a_n = \frac{\ln(2^n + 1)}{\ln(n)}$	(k) $a_n = \frac{5^n}{n!}$	(r) $a_n = \sin(n\pi)e^n$
(e) $a_n = n^2 \left(1 - \sec\left(\frac{5}{n}\right)\right)$	(l) $a_n = \frac{n + (-1)^n}{n^3 + 1}$	(s) $a_n = \frac{2^n + \arctan(n)}{2^{n+5}}$
(f) $a_n = \frac{5n+4}{2\cos^2(n) + 3n}$	(m) $a_n = \frac{4^n - 5^{2n}}{7^n}$	(t) $a_n = \cos(e^{-n})e^{2n}$
(g) $a_n = \tan^{-1}(1 - \sqrt{n})$	(n) $a_n = n \tan\left(\pi + \frac{3}{n}\right)$	(u) $a_n = \left(\frac{2 \arctan(5n)}{\pi}\right)^n$

#58. Let  $\{a_n\}$  be the sequence 5, 11, 5, 11, 5, 11, ...

- (a) Does the sequence  $\{a_n\}$  converge or diverge? If it converges, find its limit. If it diverges, explain why.
- (b) Does the sequence  $\left\{\frac{3n + a_n}{2n + 5}\right\}$  converge or diverge? If it converges, find its limit. If it diverges, explain why.
- (c) Does the sequence  $\{\cos(\pi a_n)\}$  converge or diverge? If it converges, find its limit. If it diverges, explain why.

#59. Suppose that  $a_n$  is a sequence defined inductively by

$$\begin{cases} a_1 = 2, \\ a_{n+1} = \frac{5}{a_n + 4} \text{ for } n \geq 1. \end{cases}$$

- (a) Find the first 4 terms of the sequence  $\{a_n\}$ .
- (b) Suppose that the sequence  $\{a_n\}$  converges. Under this assumption, find the limit of the sequence.

## Section 10.2: Infinite Series - Worksheet

**#60.** Each of the series below is either geometric or telescoping. Determine if each series converges or diverges, and compute its sum if it converges.

$$\begin{array}{lll}
 \text{(a)} \sum_{n=0}^{\infty} \frac{(-\pi)^n}{8} & \text{(d)} \sum_{n=0}^{\infty} \frac{1 - 3 \cdot 4^{2n}}{5^{n-1}} & \text{(g)} \sum_{n=1}^{\infty} \frac{5^n + 2^n}{6^n} \\
 \text{(b)} \sum_{n=4}^{\infty} 2^n 3^{-n} & \text{(e)} \sum_{n=3}^{\infty} \ln \left( \frac{3n+1}{3n+4} \right) & \text{(h)} \sum_{n=1}^{\infty} (\tan^{-1}(n+1) - \tan^{-1}(n)) \\
 \text{(c)} \sum_{n=0}^{\infty} \left( \frac{4}{2n+1} - \frac{4}{2n+5} \right) & \text{(f)} \sum_{n=1}^{\infty} 5 \cdot 3^{1-2n} & \text{(i)} \sum_{n=1}^{\infty} (5^{1/n} - 5^{1/(n+1)})
 \end{array}$$

**#61.** Use geometric series to express the repeating decimals below as a fraction of two integers.

$$\text{(a)} \quad 1.5222 \dots = 1.5\overline{2} \qquad \qquad \text{(b)} \quad 0.126126 \dots = 0.\overline{126}$$

**#62.** For each sequence  $\{a_n\}_{n=n_0}^{\infty}$  given below, determine

- (i) whether the **sequence**  $\{a_n\}_{n=n_0}^{\infty}$  converges or diverges. If the sequence converges, find its limit.
- (ii) whether the **series**  $\sum_{n=n_0}^{\infty} a_n$  converges or diverges. If the series converges, find its sum if possible.

$$\begin{array}{lll}
 \text{(a)} \left\{ \left( 1 + \frac{4}{n} \right)^n \right\}_{n=1}^{\infty} & & \text{(e)} \left\{ \frac{3n + 2 \cos(n)}{5n} \right\}_{n=1}^{\infty} \\
 \text{(b)} \left\{ \sqrt{n+1} - \sqrt{n} \right\}_{n=0}^{\infty} & \text{(d)} \left\{ \frac{e^{5n}}{n^{3/2}} \right\}_{n=1}^{\infty} & \text{(f)} \left\{ \cos \left( \frac{\pi}{n} \right) - \cos \left( \frac{\pi}{n+2} \right) \right\}_{n=3}^{\infty} \\
 \text{(c)} \{e^{-n}\}_{n=0}^{\infty} & & 
 \end{array}$$

**#63.** Consider the series  $\sum_{n=1}^{\infty} \left( \frac{\sin(n)}{n} - \frac{\sin(n+1)}{n+1} \right)$ .

- (a) Find an explicit formula for the partial sum  $S_N = \sum_{n=1}^N \left( \frac{\sin(n)}{n} - \frac{\sin(n+1)}{n+1} \right)$  of the series.
- (b) Does the series  $\sum_{n=1}^{\infty} \left( \frac{\sin(n)}{n} - \frac{\sin(n+1)}{n+1} \right)$  converge or diverge? If it converges, find its sum. If it diverges, explain why.

**#64.** Suppose that  $\{a_n\}_{n=1}^{\infty}$  is a sequence such that for all  $n \geq 1$ , we have

$$\frac{4n}{2n+1} \leq a_n \leq 1 + 5^{1/n}.$$

- (a) Determine whether the **sequence**  $\{a_n\}_{n=1}^{\infty}$  converges or diverges. If it converges, find  $\lim_{n \rightarrow \infty} a_n$ . If it diverges, explain why.
- (b) Determine whether the **series**  $\sum_{n=1}^{\infty} a_n$  converges or diverges. Justify your answer and name any test used.

**#65.** Let  $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{2 \cdot 5^{n+1}}$ . Find the values of  $x$  for which the series converges and find the sum of the series when it converges.

**#66.** Consider the series  $\sum_{n=0}^{\infty} \left(\frac{5}{A-2}\right)^n$ , where  $A$  is an unspecified positive constant.

- (a) Find the values of the positive constant  $A$  for which the series converges.
- (b) For the values of  $A$  you found in part (a), evaluate the sum of the series.

### Section 10.3: The Integral Test - Worksheet

**#67.** Determine if the series below converge or diverge. In case of convergence, find the sum of the series if possible. **Note:** the Integral Test is not possible/necessary for all the series. Some of these use tests from earlier sections.

$$\begin{array}{lll}
 \text{(a)} \sum_{n=1}^{\infty} \frac{(n-1)!}{n!} & \text{(c)} \sum_{n=2}^{\infty} \frac{n}{(n^2+8)^4} & \text{(e)} \sum_{n=1}^{\infty} \left( \frac{5}{\sqrt{n}} - \frac{5}{\sqrt{n+2}} \right) \\
 \text{(b)} \sum_{n=0}^{\infty} \frac{7}{3^{n+1}} & \text{(d)} \sum_{n=2}^{\infty} \frac{n^8}{(n^2+8)^4} & \text{(f)} \sum_{n=1}^{\infty} \frac{1}{n\sqrt{\ln(n)+4}}
 \end{array}$$

**#68.** For each sequence  $\{a_n\}_{n=n_0}^{\infty}$  given below, determine

- (i) whether the **sequence**  $\{a_n\}_{n=n_0}^{\infty}$  converges or diverges. If the sequence converges, find its limit.
- (ii) whether the **series**  $\sum_{n=n_0}^{\infty} a_n$  converges or diverges. If the series converges, find its sum if possible.

*Note:* the Integral Test is not possible/necessary for all the series. Some of these use tests from earlier sections.

$$\begin{array}{lll}
 \text{(a)} \{n5^{-n}\}_{n=0}^{\infty} & \text{(d)} \{\cos(n^{1/n})\}_{n=1}^{\infty} & \text{(g)} \{2^{2n+1}5^{-n}\}_{n=0}^{\infty} \\
 \text{(b)} \left\{ \frac{1}{n(1+\ln(n)^2)} \right\}_{n=2}^{\infty} & \text{(e)} \left\{ \frac{1}{(n^2+9)^{3/2}} \right\}_{n=0}^{\infty} & \text{(h)} \left\{ \left(1 + \frac{1}{2n}\right)^n \right\}_{n=1}^{\infty} \\
 \text{(c)} \left\{ \frac{1}{n^{\log_5(3)}} \right\}_{n=1}^{\infty} & \text{(f)} \left\{ \sec\left(\frac{\pi}{n}\right) - \sec\left(\frac{\pi}{n+1}\right) \right\}_{n=3}^{\infty} & \text{(i)} \left\{ \frac{1}{n \ln(n) \ln(\ln(n))} \right\}_{n=4}^{\infty}
 \end{array}$$

- #69.** (a) Determine for which values of  $p$  the series  $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)^p}$  converges or diverges.
- (b) Determine for which values of  $p$  the series  $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^p}$  converges or diverges.

## Section 10.4: Comparison Tests - Worksheet

#70. Determine if the series below converge or diverge. Make sure to clearly label and justify the use of any convergence test used. **Note:** some of these problems require convergence tests from previous sections.

$$(a) \sum_{n=1}^{\infty} \frac{n}{\sqrt{25n^6 - 1}}$$

$$(b) \sum_{n=0}^{\infty} (2n + 3)^{1/n}$$

$$(c) \sum_{n=1}^{\infty} \frac{\cos^2(n!)}{2^n}$$

$$(d) \sum_{n=2}^{\infty} \frac{(5\sqrt{n} - 2)^3}{3n^2 - 2n + 4}$$

$$(e) \sum_{n=1}^{\infty} \frac{3^n}{n5^n}$$

$$(f) \sum_{n=0}^{\infty} \frac{2^{2n}}{3^n + 11n^2}$$

$$(g) \sum_{n=3}^{\infty} \frac{\ln(n)^2}{\sqrt{n}}$$

$$(h) \sum_{n=2}^{\infty} \frac{1}{\sqrt{n} \ln(n)^2}$$

$$(i) \sum_{n=2}^{\infty} \frac{1}{n \ln(n)^2}$$

$$(j) \sum_{n=0}^{\infty} \left( \frac{n}{n+3} \right)^n$$

$$(k) \sum_{n=1}^{\infty} \frac{7 - 3 \cos(n^2)}{n^5 + 3}$$

$$(l) \sum_{n=2}^{\infty} n \sin\left(\frac{5}{n^3}\right)$$

## Section 10.5: Absolute Convergence, Ratio & Root Tests - Worksheet

#71. Determine if the series below converge or diverge. Make sure to clearly label and justify the use of any convergence test used. **Note:** some of these problems require convergence tests from previous sections.

$$(a) \sum_{n=1}^{\infty} \frac{(-1)^n}{n^e}$$

$$(e) \sum_{n=1}^{\infty} \frac{\cos(8n) + 6}{4^n}$$

$$(i) \sum_{n=1}^{\infty} (-1)^n \frac{((2n)!)^2}{(4n)!}$$

$$(b) \sum_{n=1}^{\infty} (-1)^n \frac{n^n}{3^{2n}}$$

$$(f) \sum_{n=1}^{\infty} \frac{8 \cos(n) + 6}{4^n}$$

$$(j) \sum_{n=1}^{\infty} \frac{n^n}{3^n(n+2)!}$$

$$(c) \sum_{n=1}^{\infty} \frac{\sqrt[3]{8n^6 + 7n + 11}}{3n^7 + 8n^5 - 1}$$

$$(g) \sum_{n=1}^{\infty} \frac{\ln(n)}{\ln(\ln(n))}$$

$$(k) \sum_{n=1}^{\infty} \left( \frac{2n + 5 \sin(n)}{3n} \right)^n$$

$$(d) \sum_{n=1}^{\infty} \frac{(2n+1)!}{e^n n! (n+1)!}$$

$$(h) \sum_{n=1}^{\infty} 4^n \left( \frac{n-2}{n} \right)^{n^2}$$

$$(l) \sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln(4n)^5}$$

#72. Let  $a_n$  be the sequence defined recursively by

$$a_1 = 7, \quad a_{n+1} = a_n \left( \frac{n}{n+3} \right)^n \quad \text{for } n \geq 1.$$

Determine whether the series  $\sum_{n=1}^{\infty} a_n$  converges or diverges. Make sure to clearly label and justify the use of any convergence test used.

## Section 10.6: Alternating Series and Conditional Convergence - Worksheet

**#73.** Determine if the series below converge absolutely, converge conditionally or diverge. Make sure to clearly label and justify the use of any convergence test used. **Note:** this problem is a comprehensive review of series - many of these problems require convergence tests from previous sections.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[7]{n^3 + 1}}$	(h) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2n^3 + 5\sqrt{n}}$	(o) $\sum_{n=3}^{\infty} \left(\frac{n+3}{n}\right)^{n^2}$
(b) $\sum_{n=1}^{\infty} \frac{\sqrt{9n^2 + 2}}{2n^4}$	(i) $\sum_{n=0}^{\infty} \frac{1}{3^n + \cos(n)}$	(p) $\sum_{n=1}^{\infty} \frac{(n+3)!}{(-5)^n}$
(c) $\sum_{n=1}^{\infty} \frac{(-5)^n + 1}{3^{2n+1}}$	(j) $\sum_{n=2}^{\infty} \frac{\sec(\pi n)}{\sqrt{n}}$	(q) $\sum_{n=3}^{\infty} \cos\left(\frac{\pi}{n}\right)^{n^2}$
(d) $\sum_{n=3}^{\infty} \frac{(-1)^n}{n \log_2(n)}$	(k) $\sum_{n=1}^{\infty} \frac{5 \cos(n) - 2}{n^2}$	(r) $\sum_{n=0}^{\infty} \frac{3^n + 2}{7\sqrt{25^n + 1}}$
(e) $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{n!}$	(l) $\sum_{n=2}^{\infty} (-1)^n \ln\left(\frac{n+1}{n}\right)$	(s) $\sum_{n=1}^{\infty} \frac{(n!)^2 e^n}{(2n)!}$
(f) $\sum_{n=1}^{\infty} \left(5 + \frac{3}{n}\right)^2$	(m) $\sum_{n=0}^{\infty} \frac{1}{e\sqrt{n}}$	(t) $\sum_{n=1}^{\infty} \frac{2n^2 + 3 \cos(n)}{n^4 + 1}$
(g) $\sum_{n=0}^{\infty} \frac{n \arctan(n)}{\sqrt[3]{8n^6 + 1}}$	(n) $\sum_{n=0}^{\infty} (-1)^n \frac{n}{2n+1}$	(u) $\sum_{n=1}^{\infty} \frac{\ln(n)^2}{n}$

**#74.** Consider the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt[3]{7n+4}}$ .

(a) Show that this series meets the conditions of the Alternating Series Estimation Theorem.

(b) Find the smallest integer  $N$  for which the partial sum  $S_N = \sum_{n=1}^N \frac{(-1)^n}{\sqrt[3]{7n+4}}$  approximates the sum of the series with an error of at most 0.1.

**#75.** Consider the series  $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{2^{3n-7} + 9}$ .

(a) Show that this series meets the conditions of the Alternating Series Estimation Theorem.

(b) Find the smallest integer  $N$  for which the partial sum  $S_N = \sum_{n=0}^N \frac{(-1)^{n+1}}{2^{3n-7} + 9}$  approximates the sum of the series with an error of at most  $10^{-3}$ .

**#76.** Consider the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+2}}$ .

(a) Show that this series meets the conditions of the Alternating Series Estimation Theorem.

(b) Find the smallest integer  $N$  for which the partial sum  $S_N = \sum_{n=1}^N \frac{(-1)^n}{\sqrt{n+2}}$  approximates the sum of the series with an error of at most  $\frac{3}{20}$ .

**#77.** Consider the series  $S = \sum_{n=1}^{\infty} \frac{(-1)^n}{(3n+11)^2}$ .

(a) Show that this series meets the conditions of the Alternating Series Estimation Theorem.

(b) Find the smallest integer  $N$  for which the partial sum  $S_N = \sum_{n=1}^N \frac{(-1)^n}{(3n+11)^2}$  approximates the sum of the series  $S$  with an error of at most 0.0001.

**#78.** Determine whether the series  $\sum_{n=3}^{\infty} \frac{(-1)^n}{n\sqrt{\ln(n)^2+1}}$  converges absolutely, converges conditionally or diverges.

## Section 10.7: Power Series - Worksheet

**#79.** Find the radius and interval of convergence of the power series below. Specify for which values of  $x$  in the interval of convergence the series converges absolutely and for which it converges conditionally.

(a)  $\sum_{n=1}^{\infty} \frac{(x-3)^n}{\sqrt[3]{n}5^n}$

(d)  $\sum_{n=0}^{\infty} \frac{n^2 x^n}{5n^2}$

(g)  $\sum_{n=0}^{\infty} \frac{n^n(x+2)^n}{6^n}$

(b)  $\sum_{n=1}^{\infty} \frac{(x-4)^n}{3^n n^2}$

(e)  $\sum_{n=0}^{\infty} \frac{(-1)^n(x-9)^{3n}}{8^n(n+1)}$

(h)  $\sum_{n=1}^{\infty} \frac{(-1)^n(x-4)^{2n}}{36^n \sqrt{n}}$

(c)  $\sum_{n=1}^{\infty} \frac{2^n(x+3)^n}{n}$

(f)  $\sum_{n=0}^{\infty} n3^n(2x+1)^n$

(i)  $\sum_{n=0}^{\infty} \frac{(3x+2)^n}{n^2+4}$

**#80.** Find the radius of convergence of the following power series.

(a)  $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^{2n}$

(b)  $\sum_{n=1}^{\infty} \left(1 - \frac{3}{n}\right)^{n^2} (x+5)^n$

(c)  $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$

**#81.** Suppose that a power series converges absolutely at  $x = 5$ , converges conditionally at  $x = -3$  and diverges at  $x = 11$ . What can you say, if anything, about the convergence or divergence of the power series at the following values of  $x$ ?

(a)  $x = -4$

(b)  $x = 2$

(c)  $x = 15$

(d)  $x = 7$

**#82.** Let  $f(x) = \frac{3}{2+7x}$ . Use the power series representation of  $\frac{1}{1-x}$  and power series operations to find a power series representation of  $f(x)$  centered at  $a = 0$ . What are the radius and interval of convergence of the resulting power series?

**#83.** Consider the power series  $f(x) = \sum_{n=0}^{\infty} \frac{(x+1)^n}{3^n(n+1)}$ .

(a) Find the radius and interval of convergence of  $f$ .

(b) Find a power series representation of  $f'(x)$  centered at  $a = -1$ . What are its radius and interval of convergence?

(c) Let  $g(x)$  be the antiderivative of  $f(x)$  such that  $g(-1) = -8$ . Find a power series representation of  $g(x)$  centered at  $a = -1$ . What are its radius and interval of convergence?

**#84.** (a) Use term-by-term differentiation to find a power series representation of  $\frac{1}{(1-x)^2}$ . What is its radius of convergence?

(b) Find the sum of the series  $\sum_{n=1}^{\infty} \frac{(-1)^n n}{5^n}$ .

**#85.** Find the radius and interval of convergence of  $\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x+5)^{3n}}{\sqrt{64^n n + 1}}$

## Section 10.8: Taylor and Maclaurin Series - Worksheet

**#86.** Find the Taylor polynomials for the following functions at the order and center indicated.

- |                                                                            |                                                               |
|----------------------------------------------------------------------------|---------------------------------------------------------------|
| (a) $f(x) = 2 \cos\left(\frac{\pi}{3} - 5x\right)$ , $T_4(x)$ at $a = 0$ . | (d) $f(x) = \ln(\cos(x))$ , $T_3(x)$ at $a = \frac{\pi}{4}$ . |
| (b) $f(x) = \sqrt[3]{4 + 2x}$ , $T_3(x)$ at $a = 2$ .                      | (e) $f(x) = \frac{6}{5-3x}$ , $T_4(x)$ at $a = 1$ .           |
| (c) $f(x) = 2^{3-x}$ , $T_4(x)$ at $a = 1$ .                               | (f) $f(x) = \ln(5 + x)$ , $T_3(x)$ at $a = -4$ .              |

**#87.** In the previous problem, you found the third degree Taylor polynomial of  $f(x) = \sqrt[3]{4 + 2x}$  centered at  $a = 2$ . Use this Taylor polynomial to estimate  $\sqrt[3]{8.6}$ .

- #88.** (a) For  $f(x) = \sum_{n=0}^{\infty} \frac{x^n}{(n+1)!3^n}$ , find  $f^{(8)}(0)$  and  $f^{(9)}(0)$ .
- (b) For  $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n}}{(n+1)!3^n}$ , find  $f^{(8)}(0)$  and  $f^{(9)}(0)$ .

**#89.** Consider the function  $f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{9^n(n+1)}(x-4)^{2n+1}$ .

- (a) Find the radius and interval of convergence of  $f$ .
- (b) Find  $f^{(7)}(4)$ ,  $f^{(8)}(4)$  and  $f^{(9)}(4)$ .

**#90.** Use the reference Maclaurin series to calculate the Maclaurin series of the following functions.

- |                                 |                                |                                        |
|---------------------------------|--------------------------------|----------------------------------------|
| (a) $f(x) = \frac{5}{3+7x}$     | (d) $f(x) = x \sin(3x) - 3x^2$ | (g) $f(x) = \sin(2x) - 2 \tan^{-1}(x)$ |
| (b) $f(x) = x^7 \cos(4x^5)$     | (e) $f(x) = \frac{x}{1-4x^3}$  | (h) $f(x) = \frac{4}{5+x^6}$           |
| (c) $f(x) = e^{-x^3} - 1 + x^3$ | (f) $f(x) = \arctan(5x)$       | (i) $f(x) = \ln(1-6x^2)$               |

**#91.** Find the first three non-zero terms of the Maclaurin series of the following functions.

- |                               |                             |                            |
|-------------------------------|-----------------------------|----------------------------|
| (a) $f(x) = (1+x^2) \cos(5x)$ | (b) $f(x) = \sin(2x)e^{3x}$ | (c) $f(x) = \cos(\sin(x))$ |
|-------------------------------|-----------------------------|----------------------------|

## Section 10.9: Convergence of Taylor Series - Worksheet

**#92.** Use the Remainder Estimation Theorem to estimate the error made when approximating  $f(x) = \sqrt{1+3x}$  by its 2<sup>nd</sup> degree Maclaurin polynomial  $T_2(x)$  on the interval  $[0, 0.1]$ .

**#93.** Consider the function  $f(x) = \frac{1}{1+2x}$ .

- Find the Maclaurin series of  $f$  using geometric series. What are its radius and interval of convergence?
- Find the Maclaurin series of  $f$  using its definition. (Hint: compute the first few derivatives of  $f$  and identify a pattern to find a formula for  $f^{(n)}(x)$ .)
- Find is the smallest integer  $N$  for which the Maclaurin polynomial  $T_N(x)$  of  $f(x)$  approximates  $f(x)$  with an error of at most  $10^{-4}$  on the interval  $[-0.1, 0.1]$ .

**#94.** Find the Maclaurin series of the following functions.

- (a)  $f(x) = x^3(\cos(3x^5) - 1)$       (b)  $f(x) = x^4 \ln(1 - 5x^3)$       (c)  $f(x) = e^{-x^2/2} - \cos(x)$

## Section 10.10: Applications of Taylor Series - Worksheet

**#95.** Use Maclaurin series to compute the following limits.

$$(a) \lim_{x \rightarrow 0} \frac{1 - \cos(5x^2)}{x \sin(4x) - 4x^2}$$

$$(e) \lim_{x \rightarrow 0} \frac{e^{-2x^2} - \cos(2x)}{x^2 \ln(1 + 5x) - 5x^3}$$

$$(b) \lim_{x \rightarrow 0} \frac{\ln(1 + 2x^2) - x \arctan(2x)}{7x^4}$$

$$(f) \lim_{x \rightarrow \infty} x^3 \left( \tan^{-1} \left( \frac{4}{x} \right) - 2 \sin \left( \frac{2}{x} \right) \right)$$

$$(c) \lim_{x \rightarrow \infty} x^2 \left( 1 - e^{-3/x^2} \right)$$

$$(g) \lim_{x \rightarrow 0} \frac{\sin(x^6)}{\cos(x^3) - 1}$$

$$(d) \lim_{x \rightarrow 0^+} \frac{\cos(2\sqrt{x}) - 1 + 2x}{x^2}$$

$$(h) \lim_{x \rightarrow \infty} x^2 \left( 5 \ln \left( 1 + \frac{3}{x} \right) - 3 \ln \left( 1 + \frac{5}{x} \right) \right).$$

**#96.** Use Maclaurin series to write each integral below as the sum of an infinite series of numbers (your final answer should not contain  $x$ ).

$$(a) \int_0^{1/2} \cos(5x^2) dx$$

$$(c) \int_0^{1/2} \frac{3x^2}{1 + 7x^5} dx$$

$$(e) \int_0^{1/3} x^7 \sin(2x^5) dx$$

$$(b) \int_0^1 x^3 e^{-4x^3} dx$$

$$(d) \int_0^1 \frac{5}{128 + x^7} dx$$

$$(f) \int_0^1 \frac{\sin(x)}{x} dx$$

**#97.** Suppose that  $f$  and its derivatives have the following values at  $x = 0$ :

$$f(0) = -4, \quad f'(0) = 2, \quad f''(0) = -5, \quad f^{(3)}(0) = 24.$$

Use this function in parts (a)-(b) below.

(a) Find the 3<sup>rd</sup> degree Maclaurin polynomial of  $f(x)$ .

(b) Use your answer from part (a) to evaluate the limit

$$\lim_{x \rightarrow 0} \frac{f(x) + 4 - 2x}{7x^2}.$$

You must use the Maclaurin polynomial in this question – no credit will be given to solutions using L'Hôpital's Rule.

**#98.** (a) Use Maclaurin series to write the integral  $I = \int_0^1 e^{-x^2} dx$  as the sum of an infinite series of numbers.

- (b) Use the Alternating Series Estimation Theorem to find how many terms of the series found in (a) need to be summed in order to obtain an approximation of  $I$  with an error of less than  $10^{-5}$ .

**#99.** (a) Find the first three non-zero terms of the Maclaurin series of  $e^{3x} \ln(1+x)$ .

(b) Use your answer to part (a) to approximate  $\int_0^{1/2} e^{3x} \ln(1+x) dx$ .

**#100.** (a) Find the first three non-zero terms of the Maclaurin series of  $e^{6 \sin(x)}$ .

(b) Use your answer to part (a) to evaluate  $\lim_{x \rightarrow 0} \frac{e^{6 \sin(x)} - 1 - 6x}{x^2}$ .

**#101.** (a) Find the first three non-zero terms of the Maclaurin series of  $\sqrt{1+x}$ .

(b) Use your answer to part (a) to find the first three non-zero terms of the Maclaurin series of  $\sqrt{1+x^4}$ .

(c) Use your answer to part (b) to approximate the length of the curve  $y = \frac{1}{3}x^3$ ,  $0 \leq x \leq 1$ .

**#102.** Find the sums of the following series. **Note:** some of these problems use methods from earlier sections.

(a) 
$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}$$

(c) 
$$\sum_{n=0}^{\infty} \frac{1}{3^n n!}$$

(b) 
$$\sum_{n=0}^{\infty} \frac{5}{2^n}$$

(d) 
$$\sum_{n=1}^{\infty} \left( e^{1/n} - e^{1/(n+1)} \right)$$

## Sections 11.1-2: Parametric Curves - Worksheet

**#103.** Find an equation of the tangent line to the given parametric curve at the point defined by the given value of  $t$ .

(a)  $\begin{cases} x = 5t^2 - 7 \\ y = t^4 - 3t \end{cases}, t = -1$

(c)  $\begin{cases} x = \sec(3t) \\ y = \cot(2t - \pi) \end{cases}, t = \frac{\pi}{12}$ .

(b)  $\begin{cases} x = e^{4t} - e^t + 2 \\ y = t - 3e^{2t} \end{cases}, t = 0.$

(d)  $\begin{cases} x = \arcsin(2t) \\ y = \arctan(4t) \end{cases}, t = \frac{1}{4}.$

**#104.** Find all points on the following parametric curves where the tangent line is (i) horizontal, and (ii) vertical.

(a)  $\begin{cases} x = t^4 - 8t^2 \\ y = t^2 - 6t \end{cases}$

(c)  $\begin{cases} x = 3t - t^3 \\ y = t^2 + 4t + 3 \end{cases}$

(b)  $\begin{cases} x = \sin(2t) + 1 \\ y = \cos(t) \end{cases}, 0 \leq t < 2\pi.$

(d)  $\begin{cases} x = 4t - e^{2t} \\ y = t^2 - 18 \ln|t| \end{cases}$

**#105.** Consider the ellipse of equation  $x^2 + 4y^2 = 4$ .

- (a) Find a parametrization of the ellipse.
- (b) Find the area enclosed by the ellipse.
- (c) Find the area of the surface obtained by revolving the top-half of the ellipse about the  $x$ -axis.

**#106.** For each of the following parametric curves: (i) find the arc length, (ii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the  $x$ -axis and (iii) set-up (but do not evaluate) an integral that computes the area of the surface obtained by revolving the curve about the  $y$ -axis.

(a)  $\begin{cases} x = e^{4t} \\ y = e^{5t} \end{cases}, 0 \leq t \leq 1.$

(d)  $\begin{cases} x = \ln(t) \\ y = t + \frac{1}{4t} \end{cases}, 1 \leq t \leq 2.$

(b)  $\begin{cases} x = \ln(t) \\ y = \sin^{-1}(t) \end{cases}, \frac{1}{2} \leq t \leq \frac{1}{\sqrt{2}}.$

(e)  $\begin{cases} x = \tan^{-1}(t) \\ y = \frac{1}{2} \ln(t^2 + 1) \end{cases}, 0 \leq t \leq 1.$

(c)  $\begin{cases} x = t^3 - t \\ y = \sqrt{3}t^2 \end{cases}, 0 \leq t \leq 1.$

(f)  $\begin{cases} x = t + \sin(t) \\ y = 1 - \cos(t) \end{cases}, \pi \leq t \leq 2\pi.$

**#107.** Consider the parametric curve  $\begin{cases} x = \sqrt{4-t} \\ y = \sqrt{4+t} \end{cases}, 0 \leq t \leq 2.$

- (a) Find the length of the curve.
- (b) Find the area of the surface obtained by revolving the curve about the  $y$ -axis.

**#108.** Consider the parametric curve  $x = \sin^3(4t), y = \cos^3(4t), 0 \leq t \leq \frac{\pi}{8}.$

- (a) Find an equation of the tangent line to the curve at the point corresponding to  $t = \frac{\pi}{16}.$
- (b) Find the length of the curve.
- (c) Find the area of the surface obtained by revolving the curve about the  $x$ -axis.
- (d) Find the area of the surface obtained by revolving the curve about the line  $x = -2.$

**#109.** Consider the parametric curve  $x = \ln(\sec(t) + \tan(t)), y = \sec(t), 0 \leq t \leq \frac{\pi}{4}.$

- (a) Find the length of the curve.
- (b) Find the area of the surface obtained by revolving the curve about the  $y$ -axis.

**#110.** Consider the curve with parametric equations  $x = 2t, y = 3 \ln(t) + 2, \frac{3}{2} \leq t \leq 3.$

- (a) Calculate the length of the curve.
- (b) Calculate the area of the surface of revolution obtained by revolving the curve about the  $y$ -axis.
- (c) Set-up (but do not evaluate) an integral that computes the area of the surface of revolution obtained by revolving the curve about the  $x$ -axis.

## Sections 11.3-4: Polar Coordinates - Worksheet

#111. Convert the following Cartesian equations to polar.

(a)  $y = 11$

(c)  $(x - 3)^2 + y^2 = 9$

(e)  $x^2 + y^2 + xy = 2$

(b)  $x + y = 0$

(d)  $y = 7 + 2x$

(f)  $y^2 = 3x^2$

#112. Convert the following polar equations to Cartesian. Then describe the graph.

(a)  $r = -7 \sec(\theta)$

(c)  $\theta = \frac{\pi}{6}$

(e)  $r = 5 \sin(\theta)$

(b)  $r = \frac{5}{3 \sin(\theta) - 4 \cos(\theta)}$

(d)  $r = 3 \cot(\theta) \csc(\theta)$

(f)  $r = 2 \cos(\theta) + 6 \sin(\theta)$

#113. For each the following polar curves, identify the symmetries and sketch the graph.

(a)  $r = 9$

(c)  $r = 3 \sin(2\theta)$

(e)  $r = 1 - \cos(\theta)$

(b)  $r = 8 \cos(\theta)$

(d)  $r = 5 \cos(5\theta)$

(f)  $r = \sqrt{3} + 2 \sin(\theta)$ .

#114. Find an equation of the tangent line to the following polar curves at the given value of  $\theta$ .

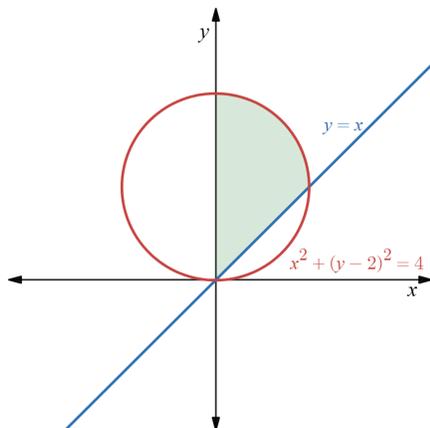
(a)  $r = \cos(3\theta)$ ,  $\theta = \frac{\pi}{4}$

(b)  $r = 1 + 2 \sin(\theta)$ ,  $\theta = \frac{\pi}{6}$

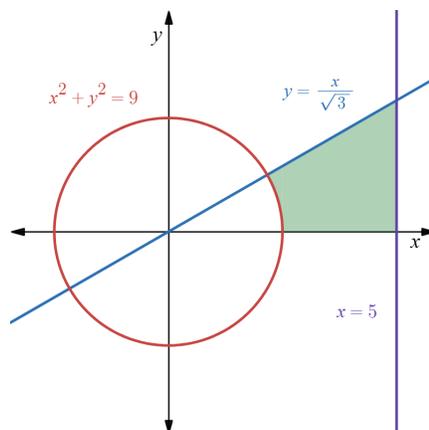
## Section 11.5: Areas and Lengths in Polar - Worksheet

#115. Find the areas of the regions sketched below using polar coordinates.

- (a) The region in the first quadrant contained in the circle of equation  $x^2 + (y - 2)^2 = 4$  and above the line  $y = x$ .



- (b) The region in the first quadrant bounded by the circle  $x^2 + y^2 = 9$  and the lines  $x = 5$ ,  $y = \frac{x}{\sqrt{3}}$ .



#116. Find the areas of the given regions.

- The region shared by the circles  $r = 2 \sin(\theta)$  and  $r = 2 \cos(\theta)$ .
- The region contained inside the leaves of the rose  $r = 6 \sin(2\theta)$  and outside the circle  $r = 3$ .
- The region inside the cardioid  $r = 1 + \sin(\theta)$  and below the line  $x = \sqrt{3}y$ .
- The region inside the circle  $r = \cos(\theta)$  and outside the cardioid  $r = 1 - \cos(\theta)$ .
- The region shared by one leaf of the rose  $r = 2 \cos(3\theta)$  and the circle  $r = 1$ .

#117. Consider the region  $\mathcal{R}$  contained in the circle  $r = 4 \cos(\theta)$  to the right of the line  $x = 3$ .

- Find the area of the region  $\mathcal{R}$  using integration with respect to  $x$ .
- Find the area of the region  $\mathcal{R}$  using integration with respect to  $y$ .
- Find the area of the region  $\mathcal{R}$  using integration with respect to  $\theta$ .

#118. Find the lengths of the given polar curves.

- $r = \theta^2$ ,  $0 \leq \theta \leq 2$
- $r = \sqrt{1 + \cos(2\theta)}$ ,  $0 \leq \theta \leq \frac{\pi}{2}$ .
- $r = \frac{2}{1 - \cos(\theta)}$ ,  $\frac{\pi}{2} \leq \theta \leq \pi$ .
- $r = e^{3\theta}$ ,  $0 \leq \theta \leq \pi$ .

## Section 18.1: Complex Numbers - Worksheet

#119. If  $z_1 = -1 + i$  and  $z_2 = 4 + 6i$ , find the rectangular forms of the following complex numbers

(a)  $z_1 z_2$

(b)  $\frac{z_1}{z_2}$

(c)  $z_1^{18}$

#120. Find all complex solutions to the following equations. Write your answers in rectangular form.

(a)  $z^3 = -125i$

(c)  $z^6 = -64$

(e)  $z^3 = 8i$

(b)  $z^2 = -1 + i\sqrt{3}$

(d)  $z^4 = 81$

(f)  $z^3 = 216$

#121. Find all complex solutions to the following equations. Write your answers in rectangular form

(a)  $z^2 - 6z + 25 = 0$

(b)  $z^4 - 8z^2 + 64 = 0$