## Section 6.2: Volumes by Shells - Worksheet

#17. Find the volume of the solid of revolution obtained by revolving the given region about the given axis using (i) the method of cylindrical shells and (ii) the method of disks/washers.

(a) The region bounded by the curve $y = 2x^3$ and the x-axis for $0 \leqslant x \leqslant 1$ revolved about the x-axis.

(b) The region bounded by the y-axis, the curve $y = 5\sqrt{x}$ and the line $y = 10$ revolved about the line $x = -2$.

(c) The region in the first quadrant bounded by the curves $y = 9 - x^2$ and $y = 1 - \frac{1}{3}x$ revolved about the line $y = 9$.

(d) The region below the graph of $y = \dfrac{2}{\sqrt{x+1}}$ for $0 \leqslant x \leqslant 3$ revolved about the y-axis.

(e) The region below the graph of $y = \dfrac{2}{\sqrt{x+1}}$ for $0 \leqslant x \leqslant 3$ revolved about the x-axis.

#18. Find the volume of the solid of revolution obtained by revolving the given region about the given axis using the method of cylindrical shells.

(a) The region bounded by the curve $y = 3\sqrt{\ln(x)}$, the line $y = 3$ and the line $x = 1$ revolved about the x-axis.

(b) The region below the graph of $y = \dfrac{1}{16 + x^4}$ for $0 \leqslant x \leqslant 2$ revolved about the y-axis.

(c) The region bounded by the curve $x = 4 - (y+1)^2$, the line $x = 4$ and the line $y = x + 7$ revolved about the line $y = -5$.

#19. Consider the region $\mathcal{R}$ bounded by the curve $y = e^{2-x}$, the line $x = 5$ and the line $y = 3$. The region $\mathcal{R}$ is sketched to the right. Set-up integrals computing the volume of the solid obtained by revolving $\mathcal{R}$ about each axis given below using (i) the disk/washer method, and (ii) the shell method.
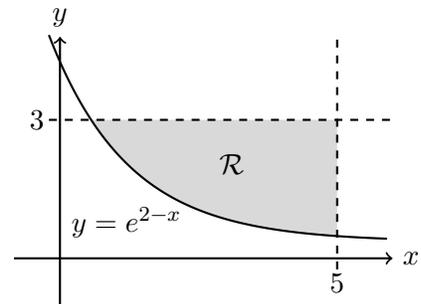
(a) x-axis   (b) y-axis   (c) $y = -1$ (d) $x = 7$ (e) $y = 3$ (f) $x = -4$

#20. Let $\mathcal{R}$ be the region bounded by the curve $y = \dfrac{\cot(x)}{x}$, the x-axis, the line $x = \dfrac{\pi}{6}$ and the line $x = \dfrac{\pi}{4}$. Find the volume of the solid obtained by rotating $\mathcal{R}$ about the y-axis.

**#21.** Let $\mathcal{R}$ be the region bounded by the lines $y = x$, $y = 2x$ and $y = -2x + 12$.

    (a) Sketch the region $\mathcal{R}$. Label the curves, their intersection points and lightly shade the region.

    (b) Calculate the area of the region using (i) an x-integral and (ii) a y-integral.

    (c) The base of a solid is $\mathcal{R}$, and the cross-sections perpendicular to the x-axis are isosceles right triangles with height on the base. Set-up an expression with integrals that calculates the volume of the solid.

    (d) The base of a solid is $\mathcal{R}$, and the cross-sections perpendicular to the y-axis are circles with diameter in the base. Set-up an expression with integrals that calculates the volume of the solid.

    (e) We create a solid of revolution by revolving $\mathcal{R}$ about the line $x = -2$. Set up an expression with integrals that calculates the volume of the solid using (i) the method of disks/washers and (ii) the method of cylindrical shells.

    (f) We create a solid of revolution by revolving $\mathcal{R}$ about the line $y = 8$. Set up an expression with integrals that calculates the volume of the solid using (i) the method of disks/washers and (ii) the method of cylindrical shells.

#22. Consider the disk of equation $(y - 2)^2 + x^2 \leqslant 1$ centered at $(0, 2)$ of radius 1. A torus (or informally, a donut) is created by revolving the disk about the x-axis.

    (a) Find the volume of the torus using the washer method.

    (b) Find the volume of the torus using the shell method.

#23. Let $\mathcal{R}$ be the region under the graph of $y = \dfrac{1}{\sqrt{16 - x^2}}$ for $0 \leqslant x \leqslant 2$. Find the volume of the solid obtained by revolving $\mathcal{R}$ about the line $x = -3$.

#24. A circular cone of height h and radius r is created by revolving a right triangle with base r, height h and right angle at the origin about the y-axis, see figure to the right.
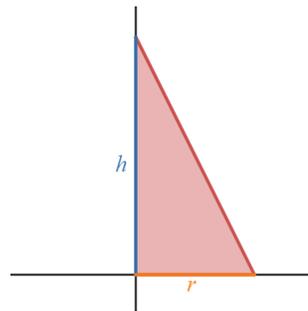
    (a) The triangle is bounded by the coordinate axes and a line. Find an equation of the line bounding the triangle in terms of the constants r and h.

    (b) Calculate the volume of the cone using the disk/washer method.

    (c) Calculate the volume of the cone using the shell method.