Section 6.3: Arc Length - Worksheet

#25. Calculate the arc length of the given curves.

    (a) $y = 11 - 2(x-5)^{3/2}$, $5 \leqslant x \leqslant 6$.

    (b) $y = \dfrac{1}{3}\ln\left(\cos(3x)\right) + 4$, $\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{3}$.

    (c) $x = \dfrac{1}{4}\sqrt[3]{y} - \dfrac{9}{5}\sqrt[3]{y^5}$, $1 \leqslant y \leqslant 2$.

    (d) $y = \sqrt{36 - x^2}$, $0 \leqslant x \leqslant 3$.

    (e) $y = \dfrac{1}{3}(2x+1)^{3/2}$, $0 \leqslant x \leqslant 1$.

    (f) $x = \sqrt{16y - y^2}$, $4 \leqslant y \leqslant 12$.

    (g) $y = \dfrac{1}{6}\ln\left(\sin(3x)\cos(3x)\right)$, $\dfrac{\pi}{18} \leqslant x \leqslant \dfrac{\pi}{9}$.

    (h) $y = \dfrac{e^{5x} + e^{-5x}}{10}$, $0 \leqslant x \leqslant \dfrac{1}{5}$.

    (i) $x = \dfrac{4}{5}y^{5/4}$, $0 \leqslant y \leqslant 9$.

    (j) $x = \dfrac{6}{7}y^{7/6}$, $0 \leqslant y \leqslant 1$.

    (k) $y = \cos\left(\dfrac{x}{2}\right) - \ln\left(\csc\left(\dfrac{x}{2}\right) + \cot\left(\dfrac{x}{2}\right)\right)$, $\dfrac{\pi}{2} \leqslant x \leqslant \pi$.

#26. Find a curve of the form $y = f(x)$ passing through $(4, 13)$, having negative derivative, and whose length integral on $1 \leqslant x \leqslant 7$ is given by

$$L = \int_1^7 \sqrt{1 + \frac{25}{x^3}}\,dx.$$

#27. Consider the ellipse of equation $9x^2 + 4y^2 = 36$. Set-up an integral that calculates the perimeter of the ellipse.