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Competing numerical magnitude codes in decimal comparison: Whole number and rational number distance both impact performance

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ABSTRACT

A critical difference between decimal and whole numbers is that among whole numbers the number of digits provides reliable information about the size of the number, e.g., double-digit numbers are larger than single-digit numbers. However, for decimals, fewer digits can sometimes denote a larger number (i.e., 0.8 > 0.27). Accordingly, children and adults perform worse when comparing such *Inconsistent* decimal pairs relative to *Consistent* pairs, where the larger number also has more digits (i.e., 0.87 > 0.2). Two explanations have been posited for this effect. The string length congruity account proposes that participants compare each position in the place value system, and they additionally compare the number of digits. The semantic interference account suggests that participants additionally activate the whole number referents of numbers – the numbers unadorned with decimal points (e.g., 8 < 27) – and compare these. The semantic interference account uniquely predicts that for Inconsistent problems with the same actual rational distance, those with larger whole number distances should be harder, e.g., 0.9 vs. 0.81 should be harder than 0.3 vs. 0.21 because 9 < < 81 whereas 3 < 21. Here we test this prediction in two experiments with college students (Study 1: $n = 58$ participants, Study 2: $n = 78$). Across both, we find a main effect of consistency, demonstrating string length effects, and also that whole number distance interferes with processing conflicting decimals, demonstrating semantic interference effects. Evidence for both effects supports the semantic interference account, highlighting that decimal comparison difficulties arise from multiple competing numerical codes. Finally, for accuracy we found no relationship between whole number distance sensitivity and math achievement, indicating that whole number magnitude interference affects participants similarly across the spectrum of math achievement.

1. Introduction

Rational numbers are a critical bottleneck in the elementary school mathematics curriculum (Bailey, Hoard, Nugent, & Geary, 2012; Fazio, Bailey, Thompson, & Siegler, 2014; Siegler et al., 2012). One source of difficulties with rationals is that many properties of whole numbers (i.e., natural numbers) do not apply to this number class. For example, whole numbers have unique symbols and successors while rational numbers do not. However, some properties of whole numbers do apply to rationals, but only some of the time, making them especially challenging properties to master (Rosenberg-Lee, 2021). For example, among whole numbers, more digits always indicates a larger number (8 < 27), but this is not always the case for decimal numbers (0.8 > 0.27). Unsurprisingly, both children and young adults perform worse on these incongruent problems relative to those congruent with whole number knowledge (0.87 > 0.2) (Avgerinou & Tolmie, 2019; Coulanges et al., 2021; Huber, Klein, Willmes, Nuerk, & Moeller, 2014; Ren & Gunderson, 2021; Varma & Karl, 2013). Two explanations have been offered for this behavioral pattern. The string length congruity account proposes that participants independently compare each position in the place value system, and they compare the number of digits (Huber et al., 2014). In contrast, the semantic interference account proposes that participants additionally activate the whole number referents of numbers – the numbers unadorned with decimal points – and compare these numbers, further influencing comparison processing (Varma & Karl, 2013). Across two studies, we examine the source of interference effects in decimal comparison to determine if just string length drives behavior, or if the whole number magnitudes of decimals also impact performance.

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Distance effects – worse performance on near (8 versus 9) than far (2 versus 9) comparisons – are a hallmark of numerical magnitude processing (Moyer & Landauer, 1967). Studies of distance effects in rational numbers confirm that decimals, like fractions, display distance effects based on their actual, rational magnitudes (DeWolf, Grounds, Bassok, & Holyoak, 2014; Hurst & Cordes, 2016; Hurst & Cordes, 2018). Rational numbers, however, afford more than one distance dimension, which can be leveraged to identify whether participants are processing rational magnitudes holistically or componentially, that is with respect to the distances between their whole number components (i.e., numerators and denominators) (Bonato, Fabbri, Umiltà, & Zorzi, 2007; Ischebeck, Schocke, & Delazer, 2009; Schneider & Siegler, 2010). Unfortunately, for fractions, rational and componential distances are correlated, and confounded with fraction type (i.e. congruent with whole number knowledge, 3/4 > 1/2 vs. incongruent with whole number knowledge 4/7 < 3/5). For example, for congruent comparisons, larger numerator distance results in larger rational distance, but for incongruent comparisons larger numerator distance results in smaller rational distance (Rosenberg-Lee, 2021). These inherent relations in the stimulus space make it challenging to disentangle respective contributions of whole and rational distance in fraction comparison.

Decimals from zero to one, by contrast, are an ideal domain to investigate rational versus whole number distance effects because they have only one whole number dimension. In fact, it is possible to select stimuli that orthogonalize this dimension from rational distance (Rosenberg-Lee, 2021). To do so, we introduce the concept of "whole number distance" in decimal comparison, by which we mean the distance between decimal number pairs when ignoring the decimal point and processing the digits as whole numbers. Fig. 1a depicts several example stimuli for comparisons that are *Consistent* with whole number knowledge where the larger number has more digits (e.g., 0.2 versus 0.74), and *Inconsistent* comparisons with whole number knowledge where the larger number has fewer digits (e.g., 0.8 versus 0.26). In both cases, the rational distance between these numbers is 0.54. However, if we ignore the decimal place we find that comparing 2 vs. 74 has a difference of 72, whereas comparing 8 versus 26 has a difference of 18.

Critically, whole number distance may have different effects for these two problem types. For Consistent problems, larger whole distance should make the problem easier, because ignoring the decimal point still leads to the correct response. By contrast, for Inconsistent problems, larger whole distance should make the problem harder, because ignoring the decimal point provides evidence for the incorrect response. To illustrate that whole number interference can vary while holding rational distance constant, consider the following examples (Fig. 1a). The comparisons 0.3 vs. 0.21 and 0.9 vs. 0.81 are both inconsistent with whole number knowledge, and both have the same rational distance of 0.09. If the semantic interference account is correct, and hence whole number interpretations are also being processed (i.e., 3 vs. 21 and 9 vs. 81), then the first example is a much closer comparison (i.e., a distance of 18) than the second (a distance of 72), making it easier to correctly ignore the whole number comparison and process the decimal information. Among Consistent problems, comparisons like 0.1 vs. 0.19 and 0.7 vs. 0.79 should be equally difficult based on their rational distance of 0.09, but based on whole distance, the first is a much closer comparison (18) than the second (72). The semantic interference account predicts that even when rational distance is held constant, whole number information is processed and affects performance, and thus the latter comparison should be easier, because both the decimal and whole number interpretations lead to the same judgment (Varma & Karl, 2013). In contrast, the string length congruity account proposes that only the magnitude of the individual digits (and the number of digits) matters for decimal comparison and therefore would not predict an effect of whole distance on comparison performance (Huber et al., 2014).

No studies to date have considered rational distance, string length, and whole number distance simultaneously in a decimal comparison task, as we do here. However, one prior study has investigated these accounts in the context of number line estimation (Schiller, Abreu-Mendoza, & Rosenberg-Lee, 2023). The authors reported that decimals are estimated as smaller than the equivalent whole number with the same number of digits; that is, placing 0.20 on a 0–1 number line to the left of the position of 20.0 on a 0–100 number line. Moreover, the same quantity presented as single digit-decimals (e.g., 0.2) was estimated as further to the left still, consistent with the string length congruity account of Huber et al. (2014) that numbers with fewer digits are considered "smaller". In the realm of number line estimation, the semantic interference hypothesis makes an additional prediction beyond the string length congruity effect, namely that larger single-digit decimals (closer to 1) should be underestimated more than smaller decimals (closer to 0) because of the larger mismatch between their actual rational magnitude and that of the corresponding whole number. For example, 0.2 should activate 20 but additionally activates 2, a difference of 18, whereas 0.8 should activate 80, but additionally activates 8, a difference of 72. Consistent with the semantic inference account (Varma & Karl, 2013), overall, participants showed greater underestimation for larger decimals relative to smaller ones. Interestingly, disaggregating the data revealed that this semantic interference effect was only present among participants who estimated decimals after estimating whole numbers, while string length effects were present for all participants regardless of presentation order. Together, these results suggest that string length and semantic interference effects might both be present in the decimal comparison task, although string length effects may be more robust.

To disentangle these two accounts for decimal difficulties, we manipulated string length congruity, rational and whole distance. As can be seen in Fig. 1b, these dimensions are – as in fractions – inherently correlated and confounded, making it necessary to select a subset of the stimuli. Study 1 employed a stimulus set where rational distance was matched between Consistent and Inconsistent trials (Fig. 1c, Table 1). However, in this stimulus set, whole distance was not matched across the two trial types. Study 2 overcomes this shortcoming by employing a stimulus set where both distances are perfectly matched (Fig. 1d, Table 1). These designs enabled us to determine if the string length congruity, semantic interference, or both effects explain decimal comparison performance. In all cases, we expect rational distance to influence performance; it is the interplay between consistency and whole distance that adjudicates between the alternatives. The string length congruity account predicts consistency effects but no impact for whole distance (Huber et al., 2014). The semantic interference account predicts consistency effects, but also that whole distance will impact performance (Varma & Karl, 2013). A final possibility is whole distance impacts performance but there are no effects of consistency. This outcome is not predicted by either account but would indicate that previously reported consistency effects actually reflect interference from the magnitude of whole numbers. Together these studies provide a comprehensive assessment of the role of whole number interference in rational number comparison.

A final question afforded by these studies is whether broad math achievement relates to the proposed distance effects. Rational number distance effects are a hallmark of magnitude-based processing of rational numbers and a primary goal of rational number instruction (Schneider & Siegler, 2010; Siegler & Braithwaite, 2017). Math achievement is associated with rational number outcomes for both decimal (Coulanges et al., 2021) and fraction (Gómez, Jiménez, Bobadilla, Reyes, & Dartnell, 2015) comparison. Yet, no studies have considered math achievement's relationship to numerical distance effects. Given that understanding the magnitude of rational numbers is foundational for other rational number capacities, like arithmetic (Kainulainen, McMullen, & Lehtinen, 2017; Van Hoof, Degrande, Ceulemans, Verschaffel, & Van Dooren, 2018), we would expect greater sensitivity to rational distance to be related to *better* math achievement. Conversely, if sensitivity to whole number magnitude represents interference from the whole number referents when processing decimals, we would expect greater sensitivity to this distance dimension to be related to *worse* math outcomes.

Table 1
Stimulus properties.

	Consistent	Inconsistent	*t*	*P*
Study 1 (Mixed)				
Rational Distance	0.243	0.237	0.124	.902
Whole Distance	69.267	21.333	8.865	<.001
Study 2 (Overlap)				
Rational Distance	0.169	0.169	0.000	1.000
Whole Distance	45.000	45.000	0.000	1.000

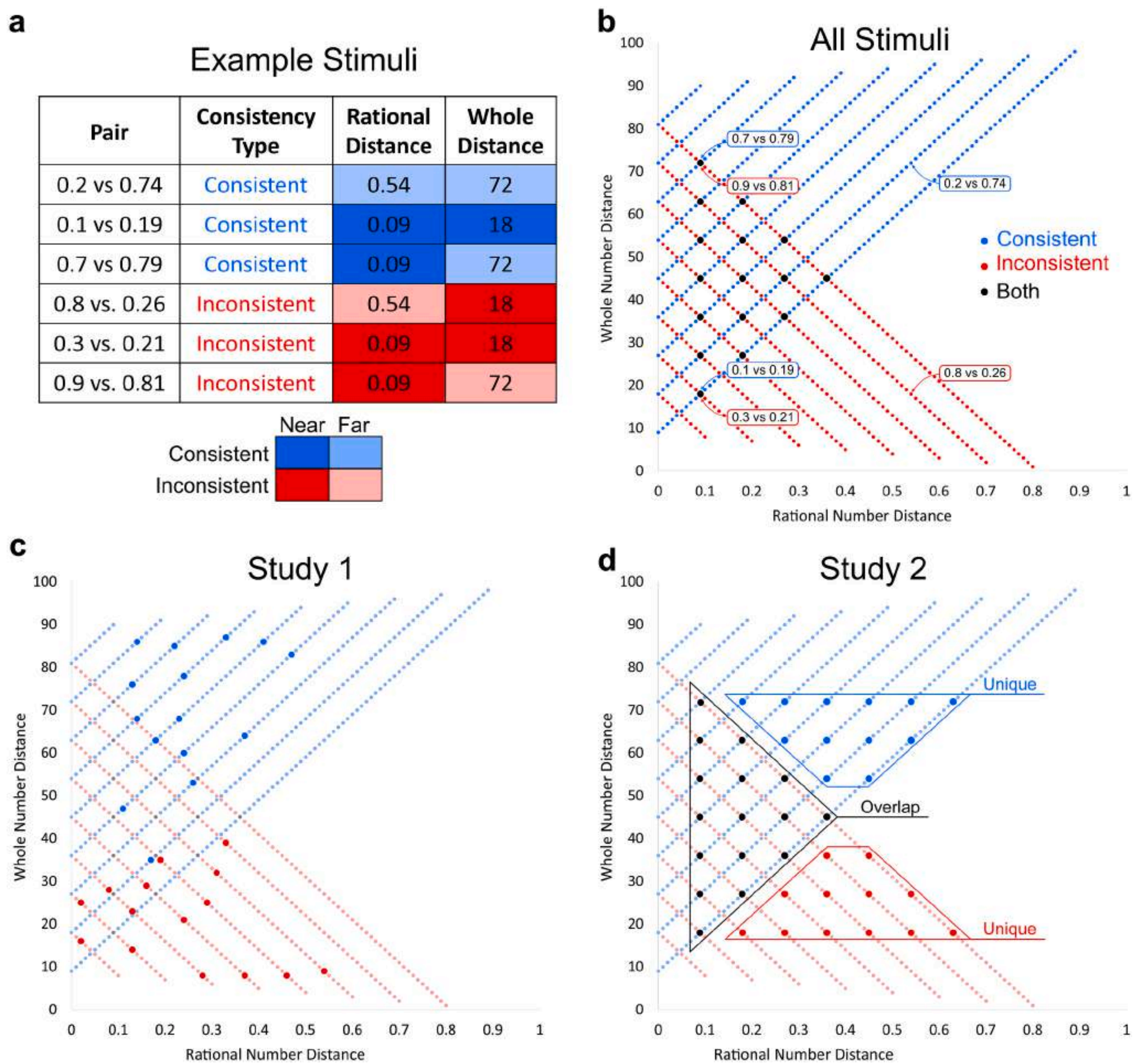

Fig. 1. Decimal comparison stimuli.
Comparison of single-digit vs. double-digit decimals can be mapped in terms of their rational distance and their whole number distance ignoring the decimal point. a) All possible stimuli for comparing [0.1 to 0.9] versus [0.10–0.99]. Consistent stimuli are those where the larger decimal also has more digits (e.g. 0.2 vs 0.74, blue points), while for Inconsistent stimuli, the larger decimal has fewer digits (e.g. 0.8 vs. 0.26). b) Example stimuli highlighted in panel 1a demonstrate that problems of the same rational distance can have different whole distance (and vice versa). Near and far comparison highlighted for illustration purposes only. c) Study 1 *Mixed* stimuli (larger saturated dots) among all comparisons (small pale dots). Consistent stimuli have larger whole distance than Inconsistent stimuli but are matched on rational distance. *Zero* condition stimuli are not shown because for them, whole and rational distance are equivalent. d) Study 2 *Overlap* stimuli (larger saturated black dots) and *Unique* stimuli (larger saturated red and blue dots) among all comparisons (small pale dots). Crucially, in the Overlap stimuli, both whole and rational distances are matched between Consistent and Inconsistent stimuli. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

2. Study 1

2.1. Methods

2.1.1. Participants

Undergraduate students taking psychology courses at Rutgers University - Newark received course credit for participation in this study. Data were collected in-person in Fall 2018 and Spring 2019. Informed consent was obtained prior to data collection and the study was approved by the Rutgers University Institutional Review Board. We implemented a two-step cleaning protocol to ensure 1) participants had sufficient data for analysis (Abreu-Mendoza, Coulanges, Ali, Powell, & Rosenberg-Lee, 2020; Schiller et al., 2023) and 2) they were engaged in the task and not using a strategy that resulted in high accuracy in one condition and low accuracy in the other condition, such as, always selecting the side with more digits in the Mixed conditions. Among the 73 individuals who participated in the study, we excluded those who had fewer than 70% usable trials after trial-level data cleaning, in each condition ($n = 3$; see below). Also, we excluded participants who did not perform above 50% in each condition ($n = 12$). Thus, the final sample consisted of 58 participants (see Table 2 for demographics).

Given that no prior studies have examined whole number distance interference effects, we could not conduct an a priori power analysis to determine a sample size to detect this effect. Instead, we opted to ensure that we had sufficient power to detect other known effects in decimal comparison, namely consistency effects and rational distance effects. For *consistency*, we employed effect sizes from Coulanges et al. (2021), and entered them into G*Power with a target power of 80%, an alpha level of 0.05, and 0.0 correlation between measurements. For accuracy, Cohen's $d = 0.549$ leads to a sample size of 29; for reaction time, Cohen's $d = -0.641$ leads to a sample size of 22 participants.

For *rational distance*, we employed values from Mock et al. (2018) which used double-digit decimal comparison (e.g. 0.78 vs. 0.67). As that study employed linear mixed effects (LME) models, we used the recently developed multilevel power calculator of Murayama, Usami, and Sakaki (2022). This method leverages the equivalence of mixed effect modeling and summary statistics, and therefore only requires summary statistics (i.e., t-scores from the second/participant level) and number of participants from the prior research to determine the target sample sizes. We again sought power of 80% and specified an alpha level of 0.05, with the sample size of $n = 24$ from Mock et al. (2018). For reaction times, we used the reported summary statistic of $t = 3.85$, which led to the sample size of 17 participants. The Murayama tool is not set up for generalized linear models of the type used here for accuracy computations (i.e., logistic regression), as these yield z-scores not t-scores. However, we reasoned that because z-scores and t-scores converge as sample sizes approach 30, using the z-scores would prove a rough effect size estimate. Specifically, the summary statistic of $z = 1.96$ from (Mock et al., 2018) leads to a suggested sample size of 50. Thus, in all cases, our sample size of 58 is sufficiently powered to detect these known effects.

Table 2
Sample demographics.

	Study 1	Study 2
Mean Age (SD)	18.78 (0.44)	20.00 (1.44)
Mean WJ Calculation (range)	112 (87–131)	107 (86–133)
Mean WJ Fluency (range)	101 (77–132)	98.5 (67–124)
Gender Identity		
Female	40	46
Male	17	27
Not Reported	1	5
Race		
Asian or Asian American	18	16
Black or African American	8	13
Latino or Hispanic or Chicano or Puerto Rican	14	24
Middle Eastern or North African	5	12
White or European American	6	5
Multiracial	5	8
Not Reported	2	–
Major		
Biology	16	12
Business	–	3
Computer Science	6	4
Criminal Justice	6	5
Mathematics	2	1
Neuroscience	1	11
Nursing/Medicine	0	1
Other	8	4
Psychology	6	21
Undeclared	6	16
Declined to answer	7	–

2.1.2. Procedure

The decimal comparison task was collected as part of a larger study, where participants completed several tasks. First, they completed two paper- and-pencil assessments: the Math Fluency and Calculation subtests of the Woodcock-Johnson III (Woodcock, McGrew, & Mather, 2001). Next, they completed a series of computerized tasks implemented in PsychoPy: the decimal comparison task, a Colour-Word Stroop task (Stroop, 1935), a fraction comparison task, a Backward Spatial Span task, and a number line estimation task. Finally, they completed a demographic questionnaire on the Qualtrics survey platform. The whole session took approximately 60 min.

2.1.3. Math assessments

Participants first completed the Math Fluency subtest of the Woodcock-Johnson III (Woodcock et al., 2001). This timed test requires participants to complete as many single-digit arithmetic (addition, subtraction, and multiplication) problems as possible in 3 min. Next, they completed the Calculation subtest, a measure of math achievement that starts with single digit arithmetic, continues through double digit and rational arithmetic, and culminates with integral calculus. We focused on the Calculation measure as it provides a better assessment of broad math achievement than Math Fluency. Following prior work in college students finding decreasing standardized math scores with age (Coulanges et al., 2021), we did not use standard scores in the analyses. Instead, we computed the sum of number of correct responses for each participant, which we z-normed for use in the linear mixed effect models. We report the mean and range of the standard scores in Table 2 to provide interpretable information on the participants' math achievement.

2.1.4. Decimal comparison task

The visual display and task timing were modeled after the experiment in Coulanges et al. (2021). The task was implemented in PsychoPy 2, version 1.90.1 (Peirce, 2008), on a 14-in. Lenovo ThinkPad laptop. It began with a practice block consisting of four trials of single-digit decimal comparisons. Each trial began with a 500 ms fixation screen, followed by presentation of the decimal pair for 3000 ms, followed by a blank screen for 500 ms. Participants were instructed to indicate which decimal was greater by pressing either the "Z" key if the larger quantity was on the left or the "M" key if it was on the right (corresponding to the spatial locations of the decimals on the screen). Participants' responses during the blank screen were allowed; thus, participants had 3500 ms to respond. All text was rendered in Arial font in white characters with a black background. The decimals stimuli were displayed with letter height of 0.15 (normalized units in PsychoPy), which resulted in numbers of height 1 cm on the laptop.

All decimal stimuli began with "0.", but differed in the number and type of digits following the decimal. 1) *Mixed* stimuli were comprised of one decimal with one digit after the decimal point and the other with two digits. 2) *Zero* stimuli were constructed by adding a "0" after the single-digit decimal in the Mixed condition. These conditions followed Coulanges et al. (2021), but we omitted the Uniform condition of that study, which consisted of pairs where both decimals had two digits after the decimal point. Consistency was also manipulated (*Consistent* and *Inconsistent*). For Mixed stimuli, both the number of digits (i.e., string length) and the numerical quantity (i.e., decimal magnitudes) led either to the same judgment (*Consistent*: 0.2 versus 0.87) or not (*Inconsistent*: 0.27 versus 0.8). For the Zero stimuli, adding the zero removes the inconsistency between number of digits and magnitude; however, we still use the terminology of Consistent (0.20 versus 0.87) and Inconsistent (0.27 versus 0.80) to align with the corresponding Mixed conditions. Including Zero stimuli enabled us to confirm that poorer performance in the Mixed Inconsistent condition was due to interference from the number of digits, not the specific numerical values employed.

There were 15 unique pairs in each of the four conditions, and each pair was presented twice – once with the larger stimulus on the right and once with it on the left, yielding a total of 120 trials. Presentation of decimal pairs were counterbalanced across two blocks, with a self-paced break between them, which also reminded participants of the task instructions. Fig. 1c presents the specific stimuli plotted in rational vs. whole distance space (see Appendix A for a full list of all stimuli). Notably, the Consistent and Inconsistent stimuli were well matched on rational distance ($t(30) = 0.12$, $p = .902$), but not matched on whole distance ($t(30) = 8.87$, $p < .001$); see Table 1.

2.1.5. Trial-level data cleaning and statistical analyses

We excluded anticipatory responses (reaction times shorter than 250 ms) and outlier responses (reaction times >3 SD from an individual's mean). After applying these criteria, we analyzed 6822 out of the total of 6960 trials (98.0%). All statistical analyses were performed using R 4.2.0 (R Core Team, 2019). ANOVAs for accuracy and reaction time were conducted with the *ez* package (Lawrence, 2016), and with the *stats* package for pairwise comparisons. Generalized linear mixed effects and linear mixed-effects models were conducted using the *glmer* and *lmer* functions, respectively, from the *lme4* package (Bates, Machler, Bolker, & Walker, 2015). For generalized linear mixed effect models of accuracy, we report BIC values as computed by the *glmer* function and use the values to test for overfitting of the data (larger values indicate worse fit). Current approaches to linear mixed effect modeling suggest that BIC is not an appropriate measure and it is not included in the *lmer* output. Therefore we did not consider overfitting when interpreting the *lmer* output for reaction times. Post-hoc comparisons and simple slope analyses were performed using the *emmeans* and *emtrends* functions from the *emmeans* package (Lenth, Love, & Hervé, 2018). Interactions that involved continuous variables (e.g., fraction distance) were plotted using either simple linear regressions or using the *ggpredict* function from the *ggeffects* package (Lüdecke, Aust, Crawley, & Ben-Shachar, 2018). We report Cohen's *d* for *t*-test effect sizes. For ANOVA's, we report partial η^2, to facilitate comparison with prior work (Coulanges et al., 2021; Varma & Karl, 2013). We also report generalized η^2, which affords comparisons of between and within participant designs (Bakeman, 2005).

2.2. Results

2.2.1. Decimal task performance

The full design included Mixed (e.g., 0.27 versus 0.8) and Zero (e.g., 0.27 versus 0.80) conditions. As expected, accuracy was lowest on the Mixed Inconsistent condition (91.01%), relative to the Mixed Consistent condition (97.03%) and both Zero conditions (Consistent = 96.23%, Inconsistent = 97.66%, Fig. 2a). A 2-way repeated measures ANOVA confirmed a significant interaction between Format (Mixed, Zero) and Type (Consistent, Inconsistent) ($F(1, 57) = 31.89$, $p < .001$, *partial* $\eta^2 = 0.359$, $\eta_g^2 = 0.113$). There was also a main effect of Format ($F(1, 57) = 29.00$, $p < .001$, *partial* $\eta^2 = 0.337$, $\eta_g^2 = 0.073$) and of consistency Type ($F(1, 57) = 17.05$, $p < .001$, *partial* $\eta^2 = 0.230$, $\eta_g^2 = 0.046$). Post-hoc *t*-tests confirmed that Mixed Inconsistent trials were significantly less accurate than each of the three other conditions (all *ps* < 0.001). The same pattern held for the analysis of reaction times on correct trials (RT, Fig. 2b). A 2-way repeated measures ANOVA confirmed a significant interaction ($F(1, 57) = 56.08$, $p < .001$, *partial* $\eta^2 = 0.495$, $\eta_g^2 = 0.020$) between Format and Type. There was also a main effect of Format ($F(1, 57) = 5.59$, $p = .002$, *partial* $\eta^2 = 0.089$, $\eta_g^2 = .002$) and a main effect of Type ($F(1, 57) = 29.39$, $p < .001$, *partial* $\eta^2 = 0.340$, $\eta_g^2 = .014$). Post-hoc *t*-tests confirmed that Mixed Inconsistent trials were significantly slower than each of the three other conditions (all *ps* < 0.001).

2.2.2. Whole and rational distance effects

To examine the effects of rational and whole distance on performance, we used a series of generalized and linear mixed effect models (Table 3, Table 4, top panels). For accuracy, confirming the ANOVA results, there was a significant Consistency effect with lower accuracy for Inconsistent than Consistent comparisons ($z = -4.90$, $p < .001$). Fig. 3a depicts the relationship between rational distance and accuracy for both Consistent and Inconsistent trials. Generalized linear mixed effects models revealed a positive main effect of rational distance on accuracy ($z = 2.10$, $p = .035$), with no interaction between consistency type and rational distance ($z = -0.62$, $p = .532$). This pattern, higher accuracy for larger distances for both Consistent and Inconsistent pairs, indicates a typical distance effect for rational distances. By contrast, when accuracies were analyzed by whole number distance (Fig. 3b), there was no main effect of whole number distance ($z = 0.37$, $p = .708$). Instead, there was an interaction between consistency type and whole number distance ($z = -3.17$, $p = .002$) such that larger whole number distance led to worse performance on Inconsistent problems (i.e., a *reverse* effect), but not for Consistent problems. Including terms from both distance models, the resulting Combined model maintained the main effect of rational distance ($z = 2.16$, $p = .031$), and the interaction between whole number distance and consistency ($z = -2.08$, $p = .038$). These results align with the hypothesis that difficulties with Inconsistent decimal comparisons stem not just from the number of digits (i.e., string length congruity), but also from the whole number magnitudes that these decimals elicit (i.e., semantic interference).

To examine any potential interactions between rational and whole distance, we also computed the Full interaction model for accuracy (Supplementary Table S1). As in the Combined model, we found a significant main effect of consistency ($z = -4.49$, $p < .001$) and an interaction between consistency and whole number distance ($z = 2.42$, $p = .015$), although the main effect of rational distance was no longer present ($z = 0.44$, $p = .660$). There were no other significant effects or interaction (all *ps* > .400), including no interactions between whole and rational distance directly or with consistency. Further, the BIC of the Full model (1507) was higher than the Combined model (1494) and the other simple models (1490–1493), suggesting that adding the interaction terms did not improve the fit of the data.

In terms of reaction time, the first model again confirmed a main effect of Consistency ($z = 6.99$, $p < .001$), with slower performance for Inconsistent than Consistent comparisons (Table 4). Turning to distance modulation, there was a main effect of rational distance ($z = -3.04$, $p = .002$) with no interaction ($z = -0.02$, $p = .985$; Fig. 3c), demonstrating faster performance for larger distance regardless of consistency. There was also a main effect of whole number distance ($z = -7.10$ $p < .001$), and a significant interaction ($z = 4.40$, $p < .001$; Fig. 3d), indicating that whole number distance strongly modulated RT in the Consistent comparisons but not in the Inconsistent ones (Fig. 3d). However, when both distance metrics and their interactions with consistency were entered into the Combined model, the main effect of rational distance became an interaction ($z = -2.31$, $p = .031$), likely due to the positive correlation between whole and rational distance for Consistent problems. Crucially, the interaction of consistency with whole distance was significant in this model ($z = -2.80$, $p = .005$).

Finally, we examined a Full model which included interaction terms between the distance metrics (Supplementary Table S2). Here, we found a 3-way interaction of consistency, rational distance and whole distance. As illustrated in Supplementary Fig. S1, for Consistent comparisons, there was the expected whole number distance effect of larger distance eliciting faster performance. For Inconsistent comparisons, there was an interaction with smaller rational distances showing slower responses at larger whole distance, and the opposite pattern for larger rational distance. This pattern suggests that when rational distance is close, whole number information dominates performance. However, these results should be interpreted with caution as whole and rational distance are correlated in these stimuli (Appendix A).

2.2.3. *Distance effects and mathematics achievement*

To examine whether the rational and whole number distance effects were related to math achievement, we again employed a series of generalized and linear mixed effect models. Crucially, here we focused on the Inconsistent comparisons as these stimuli display the predicted effect of semantic interference from whole referents (Varma & Karl, 2013). We first considered whether individual differences in math achievement predicted overall performance on Inconsistent comparisons, then for each of the distance metrics we examined whether math achievement interacted with distance modulation. Specifically, this analysis addresses whether students with higher (or lower) math achievement show stronger (or weaker) distance effects. Finally, we computed Combined and Full models, to determine the strength of any math achievement by distance interaction effects in the context of the other terms. For visualization purposes, we displayed performance 1.5 standard deviations below the mean (*lower math achievement*) and 1.5 standard deviations above the mean *(higher math achievement*). We also used *emtrends* to compute slopes at each of these levels.

Consistent with prior studies relating math achievement and rational number comparison (Coulanges et al., 2021; Gómez et al., 2015), we found that Calculation scores significantly predict accuracy on Inconsistent comparisons. In the Combined model this effect was no longer significant ($z = 0.95$, $p = .344$). Instead, we found a main effect of rational distance ($z = -3.09$, $p = .002$) and a marginal interaction of rational distance with math ability ($z = 1.72$, $p = .085$), driven by

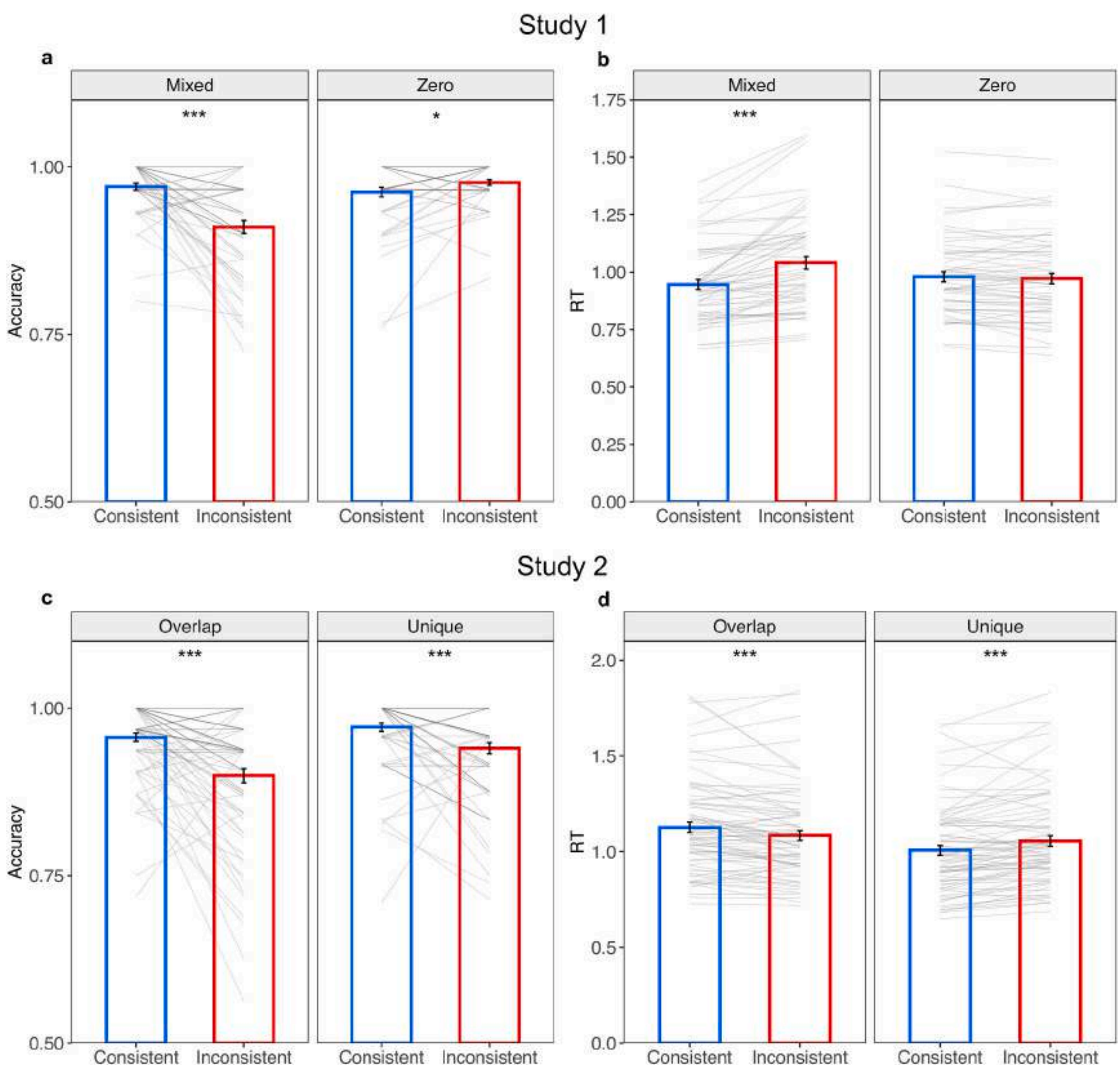

Fig. 2. Consistency effects in all conditions.
Individual responses and sample averages for Consistent and Inconsistent comparisons. a) Participants were less accurate on Mixed Inconsistent than Mixed Consistent pairs, whereas there were no differences on the Zero stimuli. b) Reaction times (RT) were slower on the Mixed Inconsistent than Mixed Consistent pairs. c) Participants were less accurate on the Inconsistent than the Consistent stimuli for both the Overlap and Unique stimuli. d) Performance on the Unique stimuli showed the expected pattern of slower reaction times for Inconsistent versus Consistent pairs, but the opposite pattern for the Overlap stimuli. Error bars represent ±1 Standard Error.

greater sensitivity to rational distance (i.e. a steeper slope) in higher achieving students (*estimate* $= 0.513$, *SE* $= 0.182$, $p = .005$), than those with lower (*estimate* $= 0.041$, *SE* $= 0.143$, $p = .775$) math scores. (Table 5, Fig. 4). There was also a negative main effect of whole distance ($z = -3.70$, $p < .001$), confirming our key finding of worse performance for larger whole number distances. However, there was no interaction between math achievement and whole number distance ($z = -0.78$, $p = .434$). In the Full model, only the main effects of rational ($z = 2.74$, $p = .006$) and whole distance ($z = -4.01$, $p < .001$) were significant. There were no significant interactions with math achievement (all *ps* > 0.14), although there was marginal interaction with between rational and whole distance ($z = 1.93$, $p < .053$), driven by ceiling level performance when rational distance is larger, but lower overall accuracy and modulation by whole distance when rational distance is smaller (see Supplementary Fig. S2). Notably, the BIC value for the Full model (1014) was worse than the other models (BIC from 999 to 1003) suggesting including these interaction terms was overfitting the data.

For reaction time, in the Base model, we found a main effect of Calculation with faster performance among those with better math scores ($z = 2.10$, $p = .041$). Again, in the Combined model, this effect of Calculation was no longer significant ($z = -0.73$, $p = .465$). However, we found a main effect of rational distance ($z = -4.01$, $p < .001$), that is, faster responses for larger rational distances, but no interaction with math achievement ($z = 0.88$, $p = .381$). While there was no main effect of whole distance ($z = -0.90$, $p = .369$), there was an interaction with math achievement ($z = 2.08$, $p = .037$). Specifically, participants with lower math achievement were slower overall, but had a tendency to be faster for larger whole distances (*estimate* $= -0.072$, *SE* $= 0.036$, $p = .032$), while participants with higher math achievement (*estimate* $= 0.044$, *SE* $= 0.032$, $p = .182$) did not show distance modulation. These effects were maintained in the Full model, and there was no 3-way interaction of math achievement, rational, and whole distance ($z = -0.26$, $p = .927$). There was a significant 2-way interaction between rational and whole distance ($z = -2.98$, $p = .003$), reflecting the 3-way interaction reported in the analyses with consistency (Supplementary Table S2 and Fig. S1). All told, these results suggest modest contributions of math achievement on sensitivity to distance metrics, with a possible pattern of higher math-performing participants showing more rational modulation in accuracy and lower math-performing participants being unexpectedly faster for larger whole distances. However, the correlation between whole and rational distance in the stimuli suggests caution in interpreting these results (Appendix A).

2.3. Discussion

In Study 1, we extend prior work showing typical distance effects for rational distance for decimal comparisons of equal string length (Mock et al., 2018) to comparisons of differing lengths. Crucially, we demonstrated for the first time that decimal comparison performance is interfered with by the referents of whole numbers, i.e., the whole number interpretations that result from ignoring the decimal point. Specifically, participants display a reverse distance effect, with worse accuracy on Inconsistent trials when the whole number distances are large and better performance when they are small. While math achievement predicted performance on the challenging Inconsistent comparisons, in terms of accuracy, math achievement did not relate to sensitivity to rational or whole distance. For reaction time, there was some indication that lower math achievement was leading to speeding up on problems with more whole number interference.

The accuracy results of both consistency and whole number magnitude interference effects align with the semantic interference account of Varma and Karl (2013). However, they should be interpreted with caution given some features of the experimental design. Specifically, the stimuli in Study 1 control for the rational distance between Consistent and Inconsistent comparisons. However, they differ on whole number distance (Fig. 1c, Table 1). Moreover, whole and rational distance are positively correlated among the Consistent stimuli ($r(15) = 0.40$) and negatively correlated among the Inconsistent stimuli ($r(15) = -0.35$), suggesting that additional caution is needed when interpreting models that include both terms. Specifically, the main effect of rational distance

Table 3
Accuracy regression results.

Accuracy Study 1	Base		Rational Distance		Whole Number Distance		Combined	
Fixed Factors	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value
Intercept	**3.78 (0.23)**	**16.43**	**3.83 (0.23)**	**16.26**	**3.70 (0.31)**	**11.76**	**3.94 (0.34)**	**11.62**
Consistency	**−1.29 (0.26)**	**−4.90**	**−1.28 (0.27)**	**−4.75**	**−2.04 (0.39)**	**−5.27**	**−2.15 (0.42)**	**−5.16**
Rational			**0.43 (0.20)**	**2.10**			**0.46 (0.22)**	**2.16**
Consistency:Rational			−0.14 (0.22)	−0.62			−0.22 (0.23)	−0.95
Whole					0.09 (0.25)	0.37	−0.13 (0.27)	−0.48
Consistency:Whole					**−1.11 (0.35)**	**−3.17**	**−0.78 (0.38)**	**−2.08**
Marginal / conditional R^2	.098/.223		.123/.247		.105/.231		.132/.255	
BIC	1493		1490		1490		1494	
Participants	58							
Observations	3405							
Accuracy Study 2	Base		Rational Distance		Whole Number Distance		Combined	
Fixed Factors	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value
Intercept	**3.52 (0.19)**	**18.48**	**3.52 (0.19)**	**18.48**	**3.52 (0.19)**	**18.47**	**3.53 (0.19)**	**18.47**
Consistency	**−1.04 (0.21)**	**−5.04**	**−1.00 (0.21)**	**−4.82**	**−1.01 (0.21)**	**−4.86**	**−0.98 (0.21)**	**−4.68**
Rational			0.02 (0.10)	0.17			0.01 (0.10)	0.12
Consistency:Rational			**0.29 (0.13)**	**2.30**			**0.28 (0.13)**	**2.22**
Whole					0.15 (0.10)	1.45	0.15 (0.10)	1.44
Consistency:Whole					**−0.45 (0.12)**	**−3.60**	**−0.42 (0.12)**	**−3.42**
Marginal / conditional R^2	.061/.256		.067/.262		.070/.264		.074/.269	
BIC	2413		2412		2410		2411	
Participants	78							
Observations	4897							

Bolded cells indicate significant effects ($p < .05$).

Table 4
RT regression results.

RT Study 1	Base		Rational Distance		Whole Number Distance		Combined	
Fixed Factors	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value
Intercept	**0.95 (0.02)**	**44.02**	**0.95 (0.02)**	**44.06**	**1.01 (0.02)**	**43.40**	**1.00 (0.02)**	**42.83**
Consistency	**0.09 (0.01)**	**6.99**	**0.09 (0.01)**	**6.96**	0.04 (0.02)	1.85	0.02 (0.03)	0.79
Rational			**−0.02 (0.00)**	**−3.04**			−0.001 (0.008)	−0.18
Consistency:Rational			−0.00 (0.07)	−0.02			**−0.02 (0.01)**	**−2.31**
Whole					**−0.07 (0.01)**	**−7.10**	**−0.07 (0.01)**	**−6.43**
Consistency:Whole					**0.08 (0.02)**	**4.40**	**0.06 (0.02)**	**2.80**
Marginal / conditional R^2	.024/.387		.029/.392		.034/.396		.037/.399	
Participants	58							
Observations	3203							
RT Study 2	**Base**		**Rational Distance**		**Whole Number Distance**		**Combined**	
Fixed Factors	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value
Intercept	**1.13 (0.03)**	**40.88**	**1.13 (0.03)**	**40.85**	**1.13 (0.03)**	**40.88**	**1.13 (0.03)**	**40.85**
Consistency	**−0.04 (0.01)**	**−3.81**	**−0.04 (0.01)**	**−3.73**	**−0.04 (0.01)**	**−3.92**	**−0.04 (0.01)**	**−3.84**
Rational			**−0.03 (0.01)**	**−4.51**			**−0.03 (0.01)**	**−4.60**
Consistency:Rational			0.002 (0.01)	0.23			−0.002 (0.01)	0.33
Whole					**−0.05 (0.01)**	**−7.20**	**−0.05 (0.01)**	**−7.26**
Consistency:Whole					**0.02 (0.01)**	**2.42**	**0.02 (0.01)**	**2.52**
Marginal / conditional R^2	.003/.367		.008/.373		.012/.377		.017/.383	
Participants	78							
Observations	4543							

Bolded cells indicate significant effects ($p < .05$).

on RT disappears when both whole and rational distance (and their interactions with consistency) are included in the model. Further, the RT results suggested that whole number distance does not influence Inconsistent performance. To address these shortcomings and potential confounds, we developed a stimulus set that fully orthogonalizes both rational and whole number distance and investigated whether these distance effects are acting independently during decimal comparison.

3. Study 2

3.1. Methods

3.1.1. Participants

The participants were Rutgers University - Newark undergraduate students taking psychology courses who received course credit. Data were collected in person in Fall 2019 and Spring 2020 (prior to the COVID-19 shutdown). Informed consent was obtained prior to data collection according to a study protocol approved by the Rutgers University Institutional Review Board. Following Study 1, among the 89 individuals who participated in the study, we excluded participants who had <70% usable trials in each condition ($n = 3$) and did not have accuracy above 50% in each condition ($n = 8$). The final sample consisted of 78 participants (see Table 2 for demographics).

Study 1 demonstrated that whole distance influences decimal comparison, and we used that effect size to estimate the sample size for the replication in Study 2. We employed the multilevel power calculator of Murayama et al. (2022), with 80% power, an alpha level of 0.05, and the sample size from Study 1 of $n = 58$. For accuracy, we used the interaction term from the consistency and whole distance model ($z = -3.17$, Table 3); this led to a sample size of 49, while for reaction time this same interaction ($t = 4.40$, Table 4) led to a sample size of 28. The final sample of 78 participants exceeded these values. Another approach would be to use the interaction terms from the Combined models as the summary statistics. For both accuracy and reaction time, the effects are weaker ($z = -2.08$; $t = 2.80$), driven by the co-linearity introduced by the correlation between whole and rational distance in the stimuli set (Appendix A). Using these values results in target sample sizes of 107 for accuracy and 61 for reaction time. Based on these more conservative effect sizes, the sample of 78 participants may be under powered to detect accuracy effects. Due to COVID-19 restrictions, we were unable to collect more data on this protocol.

3.1.2. Decimal comparison task

The visual display of the stimuli, response mode, and task timing were unchanged from Study 1. The only difference was the experimental stimuli. We focused on the *Mixed* condition, where stimuli differ in the number of digits after the decimal place (e.g., 0.27 versus 0.8). Fig. 1b displays the full space of stimuli. For the *Overlap* set, we identify points where Consistent and Inconsistent stimuli had the exact same rational and whole number distance (Fig. 1d, Table 1). Notably, these stimuli occurred at regular 0.09 intervals in the rational distance dimension but only for distances <0.45. To fully sample the stimuli space and provide less challenging stimuli to combat participant fatigue, we also included problems at the same interval (Fig. 1d). Thus, the *Unique* set comprises stimuli that are matched between Consistent and Inconsistent condition in terms of rational distance, but not whole number distance. Given these confounds, the Unique trials are not considered in the mixed effects analyses presented below.

For the Consistent stimuli, there were 16 distinct pairs in the Overlap condition and 12 distinct pairs in the Unique condition; the same was true for the Inconsistent stimuli (see Appendix B). Each pair was presented twice – once with the larger stimulus on the right and once with it on the left – for a total of 112 trials. Presentation order of decimal pairs was counterbalanced across two blocks, with a self-paced break between them, which also reminded participants of the task instructions.

3.1.3. Procedure

The decimal comparison task was collected as part of a larger study, where participants completed two sets of paper- and-pencil assessments and computerized tasks. In the first set, they completed the Addition and Subtraction subtests of the Wechsler Individual Achievement Test – Third Edition (Wechsler, 2009), the Spinners task (Jeong, Levine, & Huttenlocher, 2007) implemented for computer (Abreu-Mendoza et al., 2020), the Hearts and Flower task (Davidson, Amso, Anderson, &

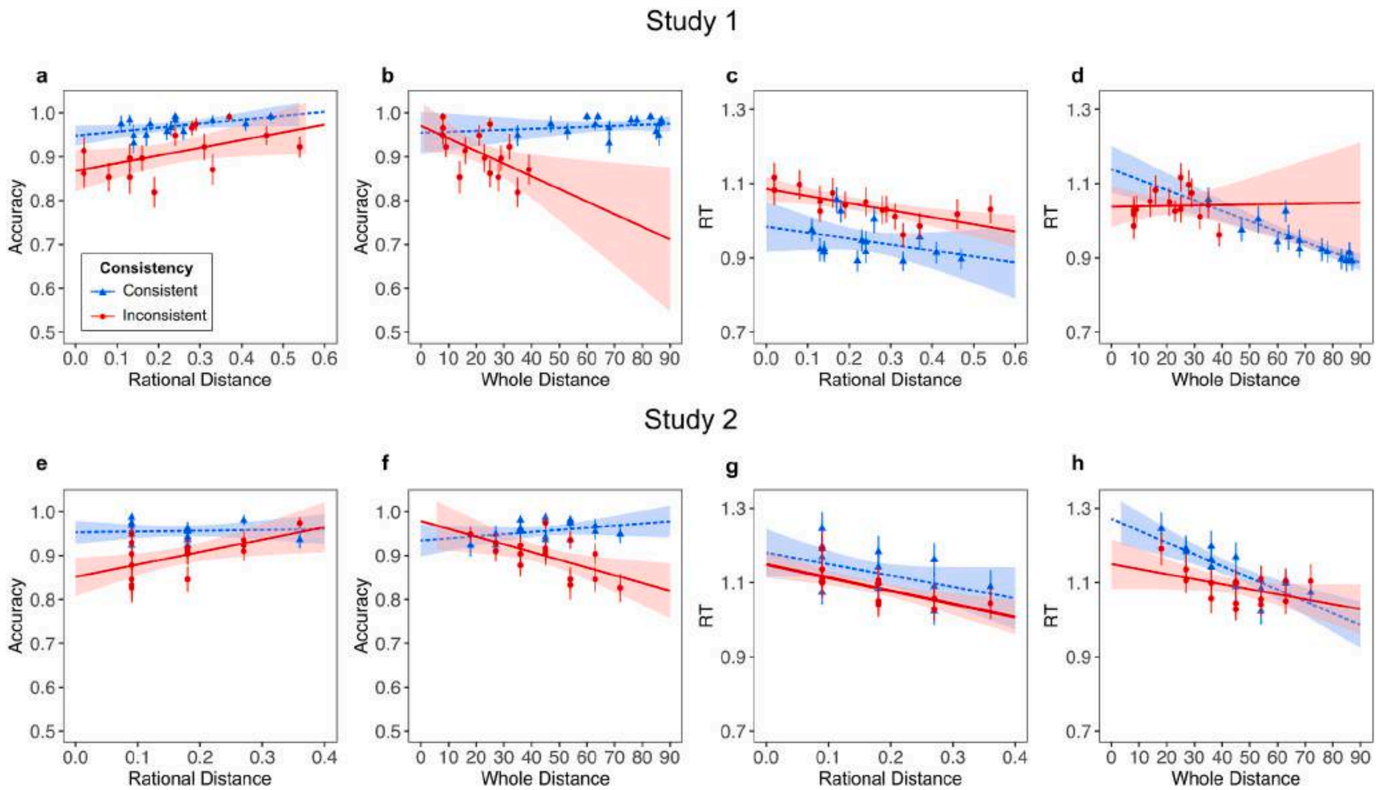

Fig. 3. Distance effects.
Rational and whole distance effects for Studies 1 and 2. a) In Study 1, participants' accuracy was modulated by rational distance for both Consistent (blue) and Inconsistent (red) comparisons, with better performance for large distances. b) Inconsistent comparisons showed a reverse effect of whole distance, that is worse performance for large distances. There was no influence on Consistent accuracy. c) For reaction times (RT), both Consistent and Inconsistent problems displayed the expected distance effect of faster performance for large distances. d) Whole distance only impacted Consistent reaction times, with faster performance for large distances. e) In Study 2, rational distance only impacted Inconsistent accuracy, with higher accuracy for large distances. f) Again, whole distance negatively modulated Inconsistent accuracy with worse performance for large distances. g) For RT, both Consistent and Inconsistent problems displayed the expected distance effect of faster performance for large distances. h) Finally, whole distance impacted both Consistent and Inconsistent comparisons, with faster performance for larger distance, although the effect was stronger for Consistent than Inconsistent comparisons.
Note: Plotted fit lines represent simple regression lines, not the fits from the corresponding models. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

Diamond, 2006), and a symbolic version of the Spinners task. In the second set, they completed a fraction comparison task, the Math Fluency and Calculation subtests of the Woodcock-Johnson III (Woodcock et al., 2001), the decimal comparison task, a Colour-Word Stroop task (Stroop, 1935), a backward spatial span task, and a number line estimation task reported elsewhere (Schiller et al., 2023). Finally, they completed a demographic questionnaire on the Qualtrics survey platform. The full session took approximately 90 min.

3.1.4. *Trial-Level -data cleaning and statistical analyses*

Data cleaning and statistical analyses followed the same procedures as Study 1. After applying these criteria, we analyzed 8575 out of a total of 8736 trials (98.16%).

3.2. *Results*

3.2.1. *Decimal task performance*

In Study 2, the full design comprised Overlap (matched on rational and whole distance) and Unique (matched only on rational distance) conditions. As expected, accuracy was lower for Inconsistent relative to Consistent trials in both the Overlap and Unique conditions (Fig. 2c). A 2-way repeated measures ANOVA confirmed a significant main effect of Type (Consistent vs. Inconsistent; $F(1, 77) = 29.18$, $p < .001$, *partial* $\eta^2 = 0.275$, $\eta_g^2 = 0.089$). There was also a main effect of Set ($F(1, 77) = 25.66$, $p < .001$, *partial* $\eta^2 = 0.250$, $\eta_g^2 = 0.037$) reflecting lower accuracy in the Overlap condition than the Unique condition. Finally, there was an interaction between the two factors ($F(1, 77) = 6.64$ $p = .001$, *partial* $\eta^2 = 0.079$, $\eta_g^2 = 0.008$), suggesting that the difference between Consistent and Inconsistent trials was larger in the Overlap condition than the Unique condition. Post-hoc *t*-tests confirmed that the simple effect on Type was larger in the Overlap condition ($t(77) = 5.20$, $p < .001$, Cohen's $d = 0.590$) than the Unique condition ($t(77) = 4.00$, $p < .001$, Cohen's $d = 0.448$).

For correct trial reaction times, surprisingly, there was no overall difference in latency between Inconsistent and Consistent trials, instead the direction of the effects differed between the Overlap and Unique conditions (Fig. 2d). A 2-way repeated measures ANOVA confirmed no significant main effect of Type (Consistent vs. Inconsistent; $F(1, 77) = 0.17$, $p = .683$, *partial* $\eta^2 = 0.002$, $\eta_g^2 < 0.001$). There was a main effect of Set ($F(1, 77) = 140.56$, $p < .001$, *partial* $\eta^2 = 0.646$, $\eta_g^2 = 0.0245$), driven by slower responses in the Overlap condition than the Unique condition. Finally, the interaction was significant ($F(1, 77) = 38.39$ $p < .001$, *partial* $\eta^2 = 0.333$, $\eta_g^2 = 0.009$). Post-hoc *t*-tests reveal significant effects of Type, but in opposite directions for each condition: for Overlap, Consistent was slower than Inconsistent ($t(77) = -3.70$, $p < .001$, Cohen's $d = 0.424$), and for Unique, Consistent was faster than Inconsistent ($t(77) = 4.40$, $p < .001$, Cohen's $d = 0.501$, Fig. 2d). The reversal of RT differences between Inconsistent and Consistent stimuli among the Overlap set is critical: it indicates that when whole and rational distances are equated between problem types, there is not the typical

Table 5
Accuracy and math achievement regression results.

Accuracy Study 1	Base		Rational Distance		Whole Number Distance		Combined	
Fixed Factors	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value
Intercept	**2.49 (0.13)**	**18.90**	**2.56 (0.14)**	**18.55**	**1.62 (0.23)**	**7.05**	**1.75 (0.25)**	**7.05**
Calculation	**0.35 (0.13)**	**2.70**	**0.40 (0.13)**	**2.97**	0.18 (0.24)	0.77	0.24 (0.26)	0.95
Rational			**0.32 (0.08)**	**3.84**			**0.28 (0.09)**	**3.09**
Calculation: Rational			0.14 (0.09)	1.65			0.16 (0.09)	1.72
Whole					**−1.07 (0.25)**	**−4.32**	**−1.01 (0.27)**	**−3.70**
Calculation:Whole					−0.22 (0.26)	−0.86	−0.22 (0.28)	−0.78
Marginal / conditional R^2	.029/.127		.077/.174		.075/.171		.129/.223	
BIC	1003		1002		999		1003	
Participants	57							
Observations	1672							

Accuracy Study 2	Base		Rational Distance		Whole Number Distance		Combined	
Fixed Factors	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value	Estimate (*SE*)	*z*-value
Intercept	**2.43 (0.13)**	**19.35**	**2.47 (0.13)**	**19.30**	**2.48 (0.13)**	**19.27**	**2.51 (0.13)**	**19.22**
Calculation	**0.34 (0.13)**	**2.57**	**0.33 (0.14)**	**2.43**	**0.38 (0.14)**	**2.76**	**0.36 (0.14)**	**2.60**
Rational			**0.31 (0.08)**	**4.06**			**0.29 (0.08)**	**3.79**
Calculation: Rational			−0.08 (0.09)	−0.89			−0.09 (0.09)	−1.07
Whole					**−0.31 (0.07)**	**−4.43**	**−0.29 (0.07)**	**−4.22**
Calculation:Whole					−0.13 (0.08)	−1.61	−0.12 (0.08)	−1.59
Marginal / conditional R^2	.024/.187		.044/.207		.057/.219		.069/.232	
BIC	1550		1547		1545		1544	
Participants	78							
Observations	2457							

Bolded cells indicate significant effects ($p < .05$).

Table 6
RT and math achievement regression results.

RT Study 1	Base		Rational Distance		Whole Number Distance		Combined	
Fixed Factors	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value
Intercept	**1.04 (0.03)**	**38.10**	**1.04 (0.03)**	**38.12**	**1.05 (0.03)**	**38.38**	**1.03 (0.03)**	**32.14**
Calculation	**−0.06 (0.03)**	**−2.10**	**−0.06 (0.03)**	**−2.10**	−0.03 (0.03)	−0.93	−0.02 (0.03)	−0.73
Rational			**−0.02 (0.01)**	**−3.93**			**−0.02 (0.01)**	**−4.01**
Calculation: Rational			0.001 (0.01)	0.13			0.005 (0.01)	0.88
Whole					0.01 (0.02)	0.58	−0.02 (0.02)	−0.90
Calculation:Whole					0.03 (0.02)	1.85	**0.04 (0.02)**	**2.08**
Marginal / conditional R^2	.031/.411		.037/.416		.033/.412		.039/.418	
Participants	57							
Observations	1523							

RT Study 2	Base		Rational Distance		Whole Number Distance		Combined	
Fixed Factors	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value	Estimate (*SE*)	*t*-value
Intercept	**1.09 (0.03)**	**41.34**	**1.09 (0.03)**	**41.36**	**1.09 (0.03)**	**41.23**	**1.09 (0.03)**	**41.26**
Calculation	−0.04 (0.03)	−1.48	−0.04 (0.03)	−1.50	−0.04 (0.03)	−1.46	−0.04 (0.03)	−1.48
Rational			**−0.03 (0.01)**	**−4.60**			**−0.03 (0.01)**	**−4.50**
Calculation: Rational			**0.01 (0.01)**	**2.13**			**0.01 (0.01)**	**2.06**
Whole					**−0.03 (0.01)**	**−4.11**	**−0.03 (0.01)**	**−4.00**
Calculation:Whole					**0.01 (0.01)**	**2.14**	**0.01 (0.01)**	**2.08**
Marginal / conditional R^2	.011/.373		.017/.379		.016/.279		.022/.385	
Participants	78							
Observations	2210							

Bolded cells indicate significant effects ($p < .05$).

consistency effect in RT, with worse performance on Inconsistent problems, a challenge to the string length congruity account.

3.2.2. Whole and rational distance effects

The Overlap stimuli in Study 2 were designed to overcome the confound between whole number distance and consistency among the Study 1 stimuli. The goal was to replicate the finding of whole number referents interfering with processing of decimals in the Inconsistent comparisons. We again used a series of generalized and linear mixed effect models to examine the effects of rational and whole number distance on performance (Table 3, Table 4, bottom panels). Fig. 3e depicts the relationship between rational distance and accuracy for both Consistent and Inconsistent trials. Here, we found a significant interaction between rational distance and consistency type ($z = 2.30, p = .022$),

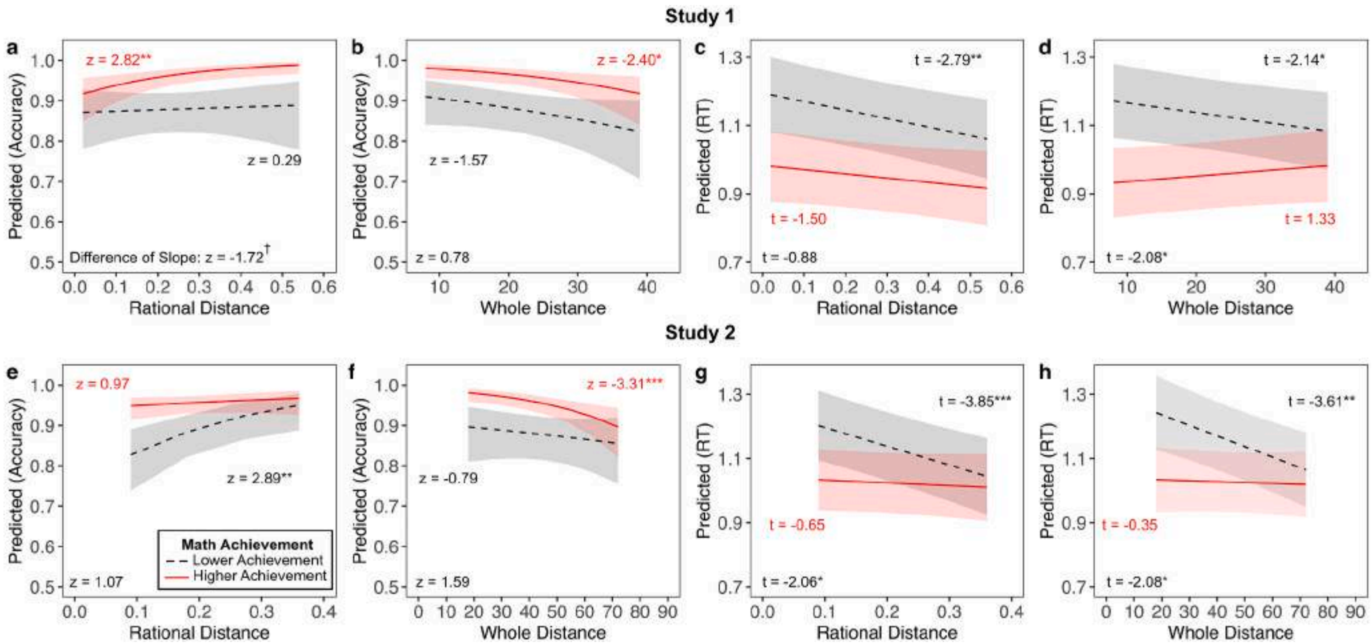

Fig. 4. Interplay of math achievement and distance effects for Inconsistent comparisons.

Rational and whole distance effects modulated by math achievement among Inconsistent trials for Studies 1 and 2. In Study 1, a), accuracy was marginally modulated by an interaction between rational distance and math achievement. This interaction is illustrated by stronger rational distance effects among higher achieving students (1.5 SD above the mean, red solid line) than lower achieving students (1.5 SD below the mean, black dashed line). b) In contrast, there was only a main effect of whole distance and no interactions with math achievement. c) For reaction times (RT), there was a main effect of rational distance but no interaction with math achievement. d) Notably, there was a significant interaction between whole distance and math achievement on reaction times. While higher achieving students did not show any whole distance modulation, lower achieving students showed an unexpected pattern of faster performance for larger whole distance. In Study 2, e and f), there were main effects of math achievement, rational and whole distance, but no interactions between math and distance metrics. g) For RT, rational distance and math achievement showed an interaction, with higher achieving students faster overall, but not sensitive to rational distance and lower achieving students showing the expected effect of faster response for larger distances. h) As in Study 1, there was a significant interaction between whole distance and math achievement. While higher achieving students did not show any whole distance modulation, lower achieving students again showed the unexpected pattern of faster performance for larger whole distance.

Note: Lines represent the fitted lines from the corresponding generalized and linear mixed effect models (Table 5 and Table 6), and shaded areas represent 95% confidence intervals. $^{\dagger}p < .10$, $^{*}p < .05$, $^{**}p < .01$, $^{***}p < .001$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

such that only Inconsistent trials were modulated by rational distance, while performance was flat for Consistent problems (indicating a ceiling effect). By contrast, in Study 1 both Consistent and Inconsistent problems showed an effect of rational distance. With respect to whole number distance (Fig. 3f), there was again no main effect of whole number distance ($z = 1.45$, $p = .145$), and again an interaction between consistency type and whole distance ($z = -3.60$, $p < .001$). As in Study 1, larger whole distance led to worse performance on Inconsistent problems, but not Consistent problems. Including terms from both models into the Combined model maintained all of these effects, most notably the interaction of consistency with whole distance ($z = -3.42$, $p < .001$). When we consider the Full model, we found no interactions between rational and whole distance alone ($z = -0.22$, $p = .828$) or with consistency ($z = 1.13$, $p = .260$). Notably, the interaction of consistency and whole distance was no longer significant in this model ($z = -1.61$, $p = .107$). However, the BIC values for this Full model (2425) relative to other models (2410–2413) again suggested including the interactions terms in the model resulted in overfitting. Together these results confirm that, when we control for the confounds between consistency and whole distance that were present in Study 1, we find evidence for the interfering effects of whole number distance on Inconsistent decimal comparisons. That is, the farther the distance between the incorrect whole number interpretations of the decimals, (i.e. the more the semantic interference), the lower the accuracy.

In terms of reaction time, as in the ANOVA results, there was an unexpected effect of worse performance on Consistent than Inconsistent pairs ($z = -3.81$, $p < .001$), which was maintained in all the subsequent models (Table 4). Turning to the distance metrics, there was a main effect of rational distance ($z = -4.51$, $p < .001$) and no interaction ($z = 0.23$, $p = .820$; Fig. 3g). There was also a negative main effect of whole distance ($z = -7.20$, $p < .001$) and a significant interaction ($z = 2.42$, $p = .016$; Fig. 3h), driven by faster performance for larger distances for Consistent and to a lesser extent for Inconsistent comparisons. These effects were all maintained in the Combined and Full models, with no significant interactions between rational and whole distance alone ($z = -0.60$, $p = .546$) or with consistency ($z = 0.51$, $p = .612$). To further examine the effects of whole distance, we conducted a follow-up analysis which showed that whole distance more strongly modulates performance on Consistent comparisons ($z = -7.63$, $p < .001$) than Inconsistent comparisons ($z = -3.32$, $p = .001$), a significant difference ($z = -2.84$, $p = .005$). For Inconsistent pairs, negative modulation of RT for whole distance is unexpected, as larger whole distance should make these problems more difficult because ignoring the decimal point provides even greater evidence for the incorrect response. In sum, for reaction times, when whole and rational distance are equated, we do not see the expected detriment in performance for the Inconsistent condition, nor interfering effects for whole distance.

3.2.3. *Distance effects and mathematics achievement*

As in Study 1 and prior research (Coulanges et al., 2021; Gómez et al., 2015), Calculation scores significantly predicted accuracy on Inconsistent comparisons ($z = 2.57$, $p = .010$). For accuracy, in the Combined model, we found the expected positive effect of rational distance ($z = 3.79$, $p < .001$) and a negative effect of whole number distance ($z = 2.08$, $p = .037$) but no interactions between these distance metrics and Calculation performance (rational: $z = -1.07$, $p = .284$; whole: $z = -1.59$ $p = .112$). In the Full model, the main effects of Calculation and rational distance were maintained, and there were no significant interactions with math achievement. We also found again a marginal interaction between rational and whole number distance ($z = 1.93$, $p = .053$), driven by strong distance modulation for whole number distance when rational distance is small, but higher performance and no whole number interference for larger rational distances. As in Study 1, the BIC value for the Full model (1557) is worse than the other models (BIC from 1544 to 1550) suggesting this model was overfitting the data.

For reaction time, in the Base model, we did not find a main effect of Calculation ($t = -1.48$, $p = .140$). However, in the Combined model, we found the expected main effect of rational distance ($t = -4.50$, $p < .001$) and an interaction between Calculation and rational distance ($t = 2.06$, $p = .039$), driven by faster responses for larger rational distances for lower achieving students (*estimate* $= -0.049$, SE $= 0.013$, $p < .001$) but faster responses overall with no distance modulation for higher achieving students (*estimate* $= -0.007$, SE $= 0.011$, $p = .512$). The same pattern held for whole number distance, with a main effect ($t = 4.00$, $p < .001$) and interaction with math achievement ($t = 2.08$, $p = .038$). Again, lower achieving students had faster responses for larger whole number distances (*estimate* $= -0.047$, SE $= 0.013$, $p < .001$), but higher achieving students were faster overall and and were not modulated by whole number distance (*estimate* $= -0.004$, SE $= 0.011$, $p = .725$). All told, these results suggest that math achievement is primarily influencing reaction time, with higher achievers showing less sensitivity to rational or whole number distance, likely because they are faster overall, while low achievers are faster for large distances, as predicted for rational distance, but unexpectedly for whole number distance.

3.3. Discussion

Study 1 provided initial evidence that the whole number referents of decimal numbers impact their comparison, especially for Inconsistent problems, i.e., when the incorrect whole number interpretations and the correct decimal interpretations pull for different judgments. However, the whole number distances in that study were not well-controlled between the Consistent and Inconsistent problems, making unclear the independent role of whole distance in decimal comparison. Study 2 overcomes this limitation by exactly matching whole distance among the Overlap stimuli. For accuracy, we again find a string length consistency effect and a negative distance effect, that is worse performance for larger whole number distances on Inconsistent comparisons, supporting the semantic interference account of Varma & Karl, 2013. Interestingly, math achievement was not related to sensitivity to either rational or whole distance. With regards to RT, participants did not display a typical consistency effect and were in fact worse for the Consistent than Inconsistent condition. Further, not only did whole number distance not interfere with performance, it seemed to facilitate it, and this effect was strongest in students with low math achievement. Together, these results provide further evidence of the role of whole number interference on decimal performance and hint at a strategy that lower performing students may be employing when working with Inconsistent decimals.

4. General discussion

Across two studies, we found that both rational and whole number distance impacted decimal comparison performance. Specifically, for Inconsistent pairs, where the larger number has fewer digits, we found a typical rational distance effect, with better accuracy for far than near problems. However, performance on these pairs also demonstrated a *reverse* distance effect for whole distance, that is, worse accuracy for far than near pairs. Or put another way, the larger the “signal” for an incorrect judgment from the corresponding whole number comparison, the larger the interference on the decimal comparison, and the worse the accuracy. Notably, we also found an independent effect of consistency, with lower accuracy for Inconsistent pairs than Consistent pairs, even after including rational and whole distance in the models. This pattern of results – both consistency effects and whole number magnitude effects – aligns with the predictions of the semantic interference account (Varma & Karl, 2013) over the string length congruity account (Huber et al., 2014), which posits that only the number of digits is the source of interference effects in decimal comparison. Taken together, these results demonstrate that decimal comparison performance is impacted by the number of digits, their rational magnitude, and the magnitude of the whole number referents.

Several studies have demonstrated that rational numbers display distance effects (Binzak & Hubbard, 2020; Bonato et al., 2007; DeWolf et al., 2014; Hurst & Cordes, 2016; Hurst & Cordes, 2018; Kalra, Binzak, Matthews, & Hubbard, 2020). Other efforts have attempted to disentangle the effects of rational vs. whole distance (Ischebeck et al., 2009; Obersteiner, Van Dooren, Van Hoof, & Verschaffel, 2013; Toledo, Abreu-Mendoza, & Rosenberg-Lee, 2023). These studies have focused on fractions, where the actual rational distance between fraction pairs has been contrasted with the distance between the components (numerators and denominators). Unfortunately, for rational numbers expressed as fractions, these distances are inherently correlated (Rosenberg-Lee, 2021). Furthermore, the correlations differ between different problem types, such as, congruent problems where larger numerals indicate larger fractions ($7/8 > 1/2$) and incongruent problems where smaller numerals indicate larger fractions ($3/5 > 4/9$) (Rosenberg-Lee, 2021). In an imaging paper, Ischebeck et al. (2009) examined fraction pairs where the components could be near (i.e., 1) or far distance (i.e., 3), while continuously varying rational distance. Interestingly, while component distance drove accuracy, the intraparietal sulcus, a key region for whole number magnitude processing (Sokolowski, Fias, Mousa, & Ansari, 2017), was sensitive to rational distance regardless of condition. Conversely, no brain region's activity was related to whole distance. While this result suggests that only rational distances are processed neurally by skilled adults, the confounds in the stimuli warrant caution (Rosenberg-Lee, 2021).

Decimals, like fractions, also show correlations between rational number and whole number distance in the full stimulus space (Fig. 1b). Fortunately, the Overlap stimuli introduced in Study 2 overcome these limitations by selecting decimal stimuli where rational distance and whole distance are perfectly orthogonal. This orthogonalization was successful, as indicated by the numerical similarity of the estimates of models which look at one distance metric vs. both. This stimulus set enabled us to definitely demonstrate, in Study 2, that both rational distance and whole distance affect rational number comparison, setting the stage for more conclusive neuroimaging investigations of whole vs. rational distance processing.

The two explanations posited for performance decrements on Inconsistent decimal comparison – the string length congruity account (Huber et al., 2014) and semantic interference account (Varma and Karl (2013) – are not mutually exclusive. Since both accounts predict worse performance on Inconsistent comparisons than Consistent comparisons, the semantic interference account can be seen as a superset of the string length congruity account which includes the effects of interference from the magnitude of whole number referents. The accuracy results bore out the semantic interference account by identifying significant independent effects of consistency and whole number distance. In the current studies, string length was only manipulated in terms of single-digit versus double-digit numbers, rather than the continuously varying whole number distance. Directly manipulating the number of digits (e.g., including triple-digit decimals) along with whole distance would be one approach to capturing the limits of each effect.

A somewhat different pattern emerges for the reaction time data. Specifically, both studies found the expected main effect of rational distance for both Inconsistent and Consistent comparisons, with no interactions. However, for whole distance, we found the expected effect for Consistent problems (better performance for far distances) but flatter effects for Inconsistent problems. For Study 1, we interpreted the lack of whole distance effect for Inconsistent comparisons to mean that if participants are able to reply correctly, they must be doing so by completely ignoring whole distance. In Study 2, both Consistent and Inconsistent problems show the typical, negative distance effect of better performance for far than near whole distances. Although the effect was less steep for Inconsistent comparisons, it still contradicts the prediction that larger whole number distances should lead to worse performance. A challenge to both accounts is the result of slower reaction times for the Consistent than Inconsistent stimuli in Study 2. Neuroimaging studies could provide insights into how individuals are able to overcome whole number interference on accuracy to successfully and quickly compare these decimals pairs.

Many studies report relations between rational number comparison and math achievement (Coulanges et al., 2021; Gómez et al., 2015; Gómez & Dartnell, 2018). However, none have looked explicitly at sensitivity to numerical magnitude and math achievement. We predicted that greater sensitivity to rational distance would be related to better math skills, while greater whole number magnitude interference would be associated with lower math achievement. With respect to accuracy, these results were not borne out as there were no significant interactions between math achievement and either distance metric. Interestingly, however, we found relationships between math achievement and numerical distance for reaction times. For both whole and rational distance, individuals with higher math scores were faster and had less modulation by distance relative to those with lower scores, likely indicating a floor effect. Examining math scores also provided insights into the unexpected reaction time finding of a negative distance effect for whole distance, that is, faster performance for larger distances. This aggregate pattern was driven by lower math achieving students. One possible explanation for the pattern of performance in these students is that they may have been using some form of "reverse strategy". In fraction comparison, a common approach after students realize that larger numerals do not always indicate larger fractions (e.g. $3/5 > 4/9$) is for them to instead decide that larger fractions have smaller numerals (Leib et al., 2023; Miller Singley, Crawford, & Bunge, 2020; Rinne, Ye, & Jordan, 2017). Use of these reverse strategies in fraction comparison is associated with worse math achievement (Gómez & Dartnell, 2018). In decimal comparison, a small subset of students report always selecting the number with fewer digits, regardless of magnitude (Ren & Gunderson, 2019; Resnick et al., 1989). In the current context, it is possible that lower achieving students may be considering the larger mismatch between whole and rational outcomes as further reason to select the decimal with fewer digits. This interpretation would also explain the unexpected finding of faster reaction times for Inconsistent than Consistent comparisons in Study 2. Strategy self-reports may be a method to determine the origin of this unexpected effect.

An alternative explanation for the whole number distance effects reported here is that they reflect the ratio between the decimal values, rather than the distance. Indeed, 0.3 vs. 0.21 and 0.9 vs. 0.81, while matched in rational distance (0.09), do differ in ratio (1.43 vs. 1.11), making the first one easier than the second, as is also predicted by semantic interference account. While debate surrounds whether ratio or distance effects best explain whole number comparison, these constructs are intrinsically linked (Dehaene, Izard, Spelke, & Pica, 2008). In the current studies, within the Inconsistent stimuli sets, the two metrics (rational distance and ratio) are strongly correlated (Study 1: $r(13) = 0.87$; Study 2: $r(14) = 0.82$). Decimal ratio also correlates with whole distance, but less strongly and in the negative direction (Study 1: $r(13) = -0.74$; Study 2: $r(14) = -0.53$). Given the stronger co-linearity with rational distance, we reasoned that in Study 2, where rational distance and whole distance are orthogonal (i.e. $r(14) = 0.00$), rational ratio could not explain the reported effects. Another possibility is that participants are comparing the ratio of whole number referents, rather than their whole distance (3 vs. 21, ratio = 7; 9 vs. 81, ratio = 9). Such an outcome would still be consistent with semantic interference account, just refining the metric used for comparing these automatically activated whole magnitudes. Future work, with appropriately tuned stimuli, is needed to more fully explore the impact of the ratio between the rational and whole magnitudes in decimal comparison.

The question of whether the magnitude of whole number referents affects decimal processing is an important one that will require a multi-paradigm approach. It can also be addressed using the number line estimation (NLE) task. Schiller et al. (2023) found that decimals are consistently estimated as smaller than equivalent whole numbers. For example, 0.20 on a 0–1 number line is estimated as smaller than (e.g., farther to the left of) 20.0 on a 0–100 number line, and this effect is exacerbated for single-digits decimals (e.g., 0.2 vs. 0.20 on a 0–1 number line). Crucially, the semantic interference account also predicts that larger decimals should show even greater underestimation because 0.2 should activate 20 but instead activates 2, a difference of 18 from the correct value, whereas 0.8 should activate 80, but only activates 8, a difference of 72. Interestingly, the predicted effect was found, but only for participants who first completed a whole number NLE task. This finding suggests that working with whole numbers negatively primes participants to (incorrectly) activate the whole number referents of decimals (Roell, Viarouge, Houde, & Borst, 2019). In the current study, we did not manipulate task order, and all participants completed a set of rational number activities. Potentially, the presence of two decimals differing in string length is sufficient to induce whole number magnitude effects. An important question for future work is whether exposure to whole numbers exacerbates the string length and whole distance interference effects observed here for comparison tasks. The converse question is also interesting: Does exposure to decimal magnitudes, through first completing an NLE task, reduce whole number interference?

A growing literature has linked the executive function capacity of inhibitory control with proficiency with rational numbers (Abreu-Mendoza et al., 2020; Avgerinou & Tolmie, 2019; Coulanges et al., 2021; Gómez et al., 2015; Leib et al., 2023). Moreover, Inconsistent decimal performance mediates the relationship between inhibition and math achievement (Coulanges et al., 2021). This finding suggests that interventions focused on bolstering decimal comparison in participants with poor inhibitory control could counteract some of the difficulties this group has with mathematics. The current study provides insights to further refine this educational implication. Specifically, having identified two contradictory numerical codes as sources of interference in decimal comparison (the number of digits and the whole number referents), we can ask which source is most affected by poor inhibitory control? Targeting this source in education interventions should yield larger gains in learning. Alternatively, inhibition itself can be trained and it would be interesting to see if these interventions act equally on each numerical code (Brookman-Byrne, Mareschal, Tolmie, & Dumontheil, 2018; Wilkinson et al., 2019). Finally, coupled with the finding of underestimation of decimals in a numberline task (Schiller et al., 2023), this work highlights the importance of using a consistent number of significant digits when communicating numerical information.

5. Conclusion

This research has investigated the processes by which people compare decimals. Across two studies, we find a robust effect of problem consistency, as predicted by the string length congruity account of Huber et al. (2014). Crucially, we also find that performance is worse on conflicting decimal comparisons when the whole number distance is larger, pushing participants further towards the wrong answer. The presence of whole number magnitude-based interference aligns with the unique prediction of the semantic interference account of Varma and Karl (2013). Thus, we find evidence for two independent effects leading to decimal comparison difficulties (as well as the expected effect of rational distance). Interestingly, the interference effects were largely independent of math achievement, suggesting susceptibility to whole number interference is present across the ability spectrum. Establishing the existence and robustness of multiple competing numerical codes within a single rational number task sets the stage for future work examining the development of these effects and their neural basis.

CRediT authorship contribution statement

Miriam Rosenberg-Lee: Conceptualization, Visualization, Supervision, Formal analysis, Writing – original draft, Writing – review & editing. **Sashank Varma:** Conceptualization, Writing – review & editing. **Michael W. Cole:** Writing – review & editing. **Roberto A. Abreu-Mendoza:** Data curation, Formal analysis, Visualization, Writing – review & editing.

Data availability

The authors do not have permission to share data.

Appendix A. Stimuli for Study 1

Consistent				Inconsistent			
Mixed Item	Zero Item	Rational Distance	Whole Number Distance	Mixed Item	Zero Item	Rational Distance	Whole Number Distance
0.4 vs. 0.51	0.40 vs. 0.51	0.11	47	0.18 vs. 0.2	0.18 vs. 0.20	0.02	16
0.6 vs. 0.74	0.60 vs. 0.74	0.14	68	0.28 vs. 0.3	0.28 vs. 0.30	0.02	25
0.8 vs. 0.94	0.80 vs. 0.94	0.14	86	0.32 vs. 0.4	0.32 vs. 0.40	0.08	28
0.2 vs. 0.37	0.20 vs. 0.37	0.17	35	0.17 vs. 0.3	0.17 vs. 0.30	0.13	14
0.5 vs. 0.68	0.50 vs. 0.68	0.18	63	0.27 vs. 0.4	0.27 vs. 0.40	0.13	23
0.7 vs. 0.92	0.70 vs. 0.92	0.22	85	0.34 vs. 0.5	0.34 vs. 0.50	0.16	29
0.5 vs. 0.73	0.50 vs. 0.73	0.23	68	0.41 vs. 0.6	0.41 vs. 0.60	0.19	35
0.7 vs. 0.83	0.70 vs. 0.83	0.23	76	0.26 vs. 0.5	0.26 vs. 0.50	0.24	21
0.4 vs. 0.64	0.40 vs. 0.64	0.24	60	0.12 vs. 0.4	0.12 vs. 0.40	0.28	8
0.6 vs. 0.84	0.60 vs. 0.84	0.24	78	0.31 vs. 0.6	0.31 vs. 0.60	0.29	25
0.3 vs. 0.56	0.30 vs. 0.56	0.26	53	0.39 vs. 0.7	0.39 vs. 0.70	0.31	32
0.6 vs. 0.93	0.60 vs. 0.93	0.33	87	0.47 vs. 0.8	0.47 vs. 0.80	0.33	39
0.3 vs. 0.67	0.30 vs. 0.67	0.37	64	0.13 vs. 0.5	0.13 vs. 0.50	0.37	8
0.5 vs. 0.91	0.50 vs. 0.91	0.41	86	0.14 vs. 0.6	0.14 vs. 0.60	0.46	8
0.4 vs. 0.87	0.40 vs. 0.87	0.47	83	0.16 vs. 0.7	0.16 vs. 0.70	0.54	9

Note. In the Consistent trials, the correlation strength between Rational and Whole Number distance was $r = 0.45$. In the Inconsistent trials, the correlation strength between Rational and Whole Number distance was $r = -0.35$.

Appendix B. Stimuli for Study 2

Consistent				Inconsistent			
Item	Type	Rational Distance	Whole Number Distance	Item	Type	Rational Distance	Whole Number Distance
0.1 vs. 0.19	Overlap	0.09	18	0.21 vs. 0.3	Overlap	0.09	18
0.2 vs. 0.29	Overlap	0.09	27	0.31 vs. 0.4	Overlap	0.09	27
0.3 vs. 0.39	Overlap	0.09	36	0.41 vs. 0.5	Overlap	0.09	36
0.4 vs. 0.49	Overlap	0.09	45	0.51 vs. 0.6	Overlap	0.09	45
0.5 vs. 0.59	Overlap	0.09	54	0.61 vs. 0.7	Overlap	0.09	54
0.6 vs. 0.69	Overlap	0.09	63	0.71 vs. 0.8	Overlap	0.09	63
0.7 vs. 0.79	Overlap	0.09	72	0.81 vs. 0.9	Overlap	0.09	72
0.1 vs. 0.28	Overlap	0.18	27	0.32 vs. 0.5	Overlap	0.18	27
0.2 vs. 0.38	Overlap	0.18	36	0.42 vs. 0.6	Overlap	0.18	36
0.3 vs. 0.48	Overlap	0.18	45	0.52 vs. 0.7	Overlap	0.18	45
0.4 vs. 0.58	Overlap	0.18	54	0.62 vs. 0.8	Overlap	0.18	54
0.5 vs. 0.68	Overlap	0.18	63	0.72 vs. 0.9	Overlap	0.18	63
0.1 vs. 0.37	Overlap	0.27	36	0.43 vs. 0.7	Overlap	0.27	36
0.2 vs. 0.47	Overlap	0.27	45	0.53 vs. 0.8	Overlap	0.27	45
0.3 vs. 0.57	Overlap	0.27	54	0.63 vs. 0.9	Overlap	0.27	54
0.1 vs. 0.46	Overlap	0.36	45	0.54 vs. 0.9	Overlap	0.36	45
0.6 vs. 0.78	Unique	0.18	72	0.22 vs. 0.4	Unique	0.18	18
0.4 vs. 0.67	Unique	0.27	63	0.23 vs. 0.5	Unique	0.27	18
0.5 vs. 0.77	Unique	0.27	72	0.33 vs. 0.6	Unique	0.27	27
0.2 vs. 0.56	Unique	0.36	54	0.24 vs. 0.6	Unique	0.36	18
0.3 vs. 0.66	Unique	0.36	63	0.34 vs. 0.7	Unique	0.36	27
0.4 vs. 0.76	Unique	0.36	72	0.44 vs. 0.8	Unique	0.36	36
0.1 vs. 0.55	Unique	0.45	54	0.25 vs. 0.7	Unique	0.45	18
0.2 vs. 0.65	Unique	0.45	63	0.35 vs. 0.8	Unique	0.45	27
0.3 vs. 0.75	Unique	0.45	72	0.45 vs. 0.9	Unique	0.45	36
0.1 vs. 0.64	Unique	0.54	63	0.26 vs. 0.8	Unique	0.54	18
0.2 vs. 0.74	Unique	0.54	72	0.36 vs. 0.9	Unique	0.54	27
0.1 vs. 0.73	Unique	0.63	72	0.27 vs. 0.9	Unique	0.63	18

Note. Within each of the four conditions, the correlation strength between Rational and Whole Number distance was $r = 0.00$.

Appendix C. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.cognition.2023.105608.

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