Formation and Patterns of Air Bubbles when Freezing Water

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Introduction

One can appreciate the attractive patterns seen in wood grain or marble, but the irregular lines, warped regions, recurring bands, and needle-like air bubbles produced in ice cubes are equally mesmerizing. While easily reproducible from freezing water in a household freezer, this complex array of patterns is largely unreported and unexplained, posing an essential question to how exactly they arise. To understand the processes, finite difference models in MATLAB are coded to reproduce the patterns seen in experiments. Simulations show the combination of diffusion and interaction between temperature and the air concentration, revealing a connection between freezing ice and Turing patterns, such as Liesegang ring formation in metamorphic rocks.

Background

Freezing water in a standard freezer tray rarely produces clear transparent crystals of ice as one may expect. Instead, you may notice there is usually a foggy white region towards the center, composed of numerous tiny air bubbles that diffused out of the liquid as the water was cooled from the surface [1]. However, there are more intriguing patterns, including:

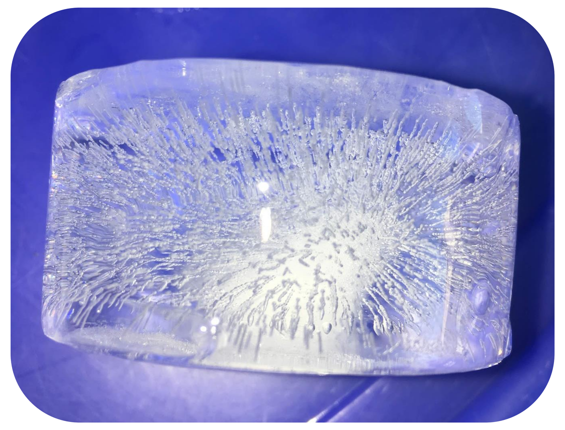

Figure 1: Image of an ice cube frozen in a standard freezer with distilled water.

- warping in the central white foggy region
- needle-like bubbles oriented radially towards the surface of the ice
- periodic high to low air concentration bands
- train-like bubbles that are connected together

Objectives with MATLAB simulation:

- reproduce the observed behaviors virtually
- investigate other phenomena generated by different initial conditions
- assess what parameters produce more accurate behaviors

Understanding these phenomena is key to connecting the ice patterns with other Turing instabilities, which are similarly constructed patterns usually found in biological or geological systems [2].

Methods

Reproducing the patterns in ice cubes requires a system of reaction-diffusion equations, as first proposed by Alan Turing for the formation of unstable and irregular patterns [3]. Using finite difference (FD) modeling in MATLAB, a system of partial differential equations (PDEs) was coded to describe the evolution of air concentration and temperature at each location and time.

Parameters: 2000 time steps, 128 × 64 × 64 voxel resolution.

Equations:

$$\frac{\partial}{\partial t}T(x,y,z,t) = D_T(A) \cdot \nabla^2 T \qquad (1)$$

$$\frac{\partial}{\partial t}A(x,y,z,t) = D_A(T) \cdot \nabla^2 A + f(A,T) \qquad (2)$$

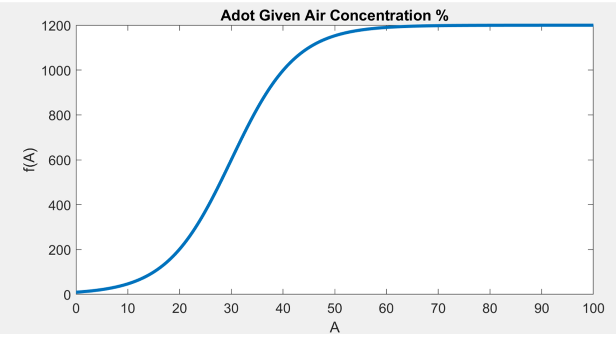

Figure 2.1: f(A,T) with T held constant

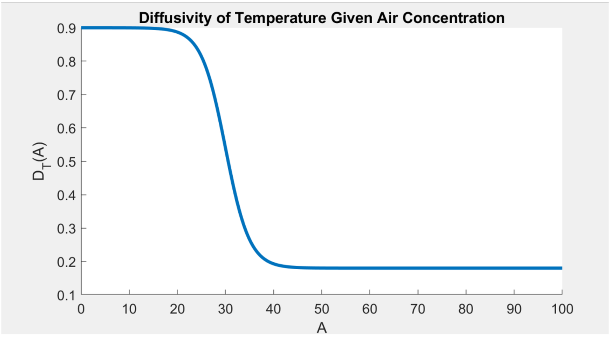

Figure 2.2: $D_T(A)$ using hyperbolic tangent

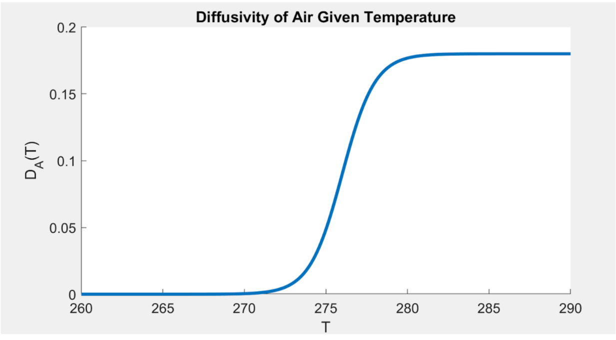

Figure 2.3: $D_A(T)$ using hyperbolic tangent

- Figures 2.2 and 2.3 show how air concentration and temperature interact with each other by varying the rate of diffusion.
- *f(A,T)* from Equation 2 or `Adot` is an additional term responsible for keeping existing bubbles from dissipating away. Areas of high *A* will increase *A* faster (with a maximum at 100%).
- For a sufficiently low value of *T*, say $T < 273$, $f(A,T) = 0$.

Results

- Figures 3-5 compare phenomena with simulated patterns.
- Air concentration from 0 to 100% shown with a color gradient from blue to red, respectively.

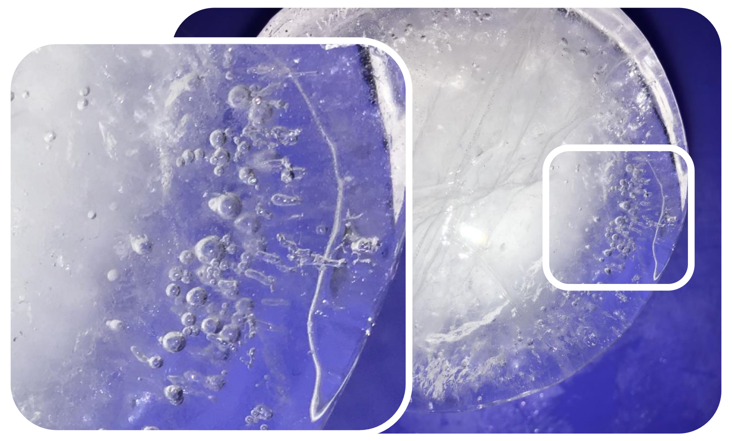

Figure 3.1: Image of round bubbles

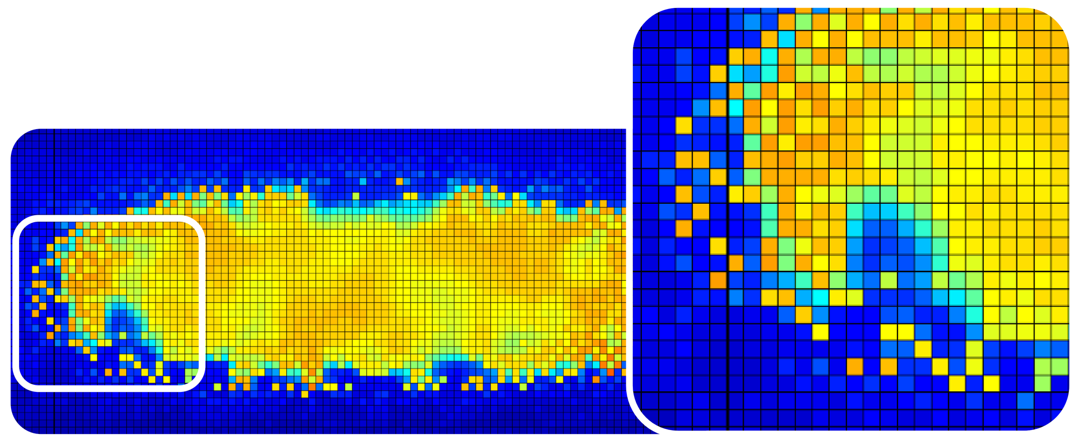

Figure 3.2: Simulation showing round bubbles, slice at $z = 32$

Phenomenon 1: Round bubbles dotted near edge of foggy region

- Figures 3.1 and 3.2 show dense grouping of round bubbles along edge of foggy region
- Figure 3.2 shows cross section $z = 32$. Parameters: Noise 0-6%, $D_T = 0.9$, $D_A = 0.18$.

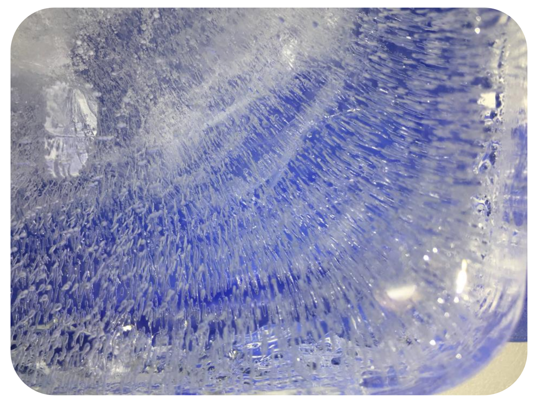

Figure 4.1: Image of Liesegang bands

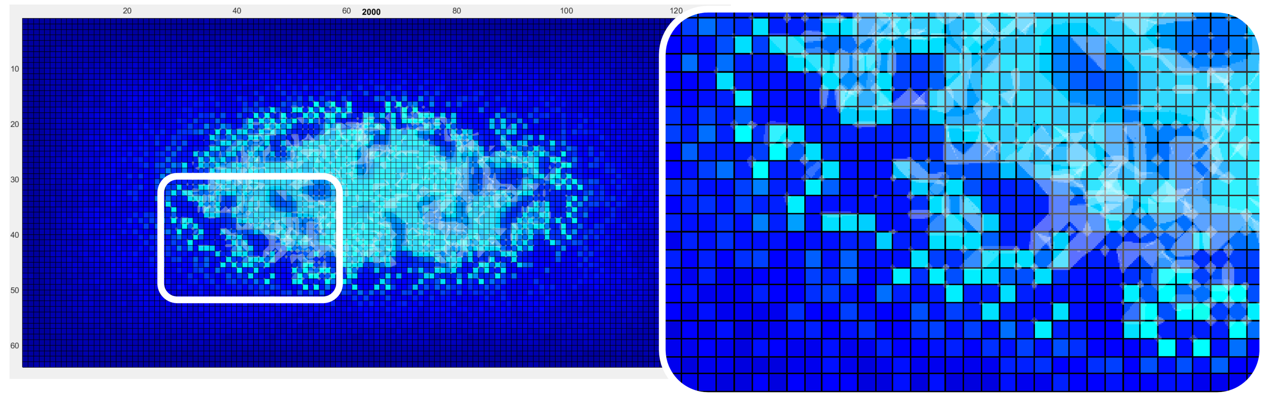

Figure 4.2: Simulation showing Liesegang bands, slice at $z = 32$

Phenomenon 2: Bands of needle-like bubbles, periodic high to low concentration

- Multiple separated bands of bubbles pointed radially from the center.
- Figure 4.2 Parameters: Noise 0-5%, $D_T = 0.9$, $D_A = 0.25$.

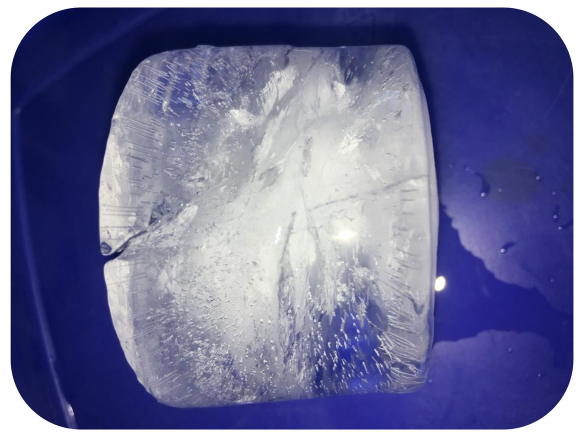

Figure 5.1: Image of finger-like patterns,

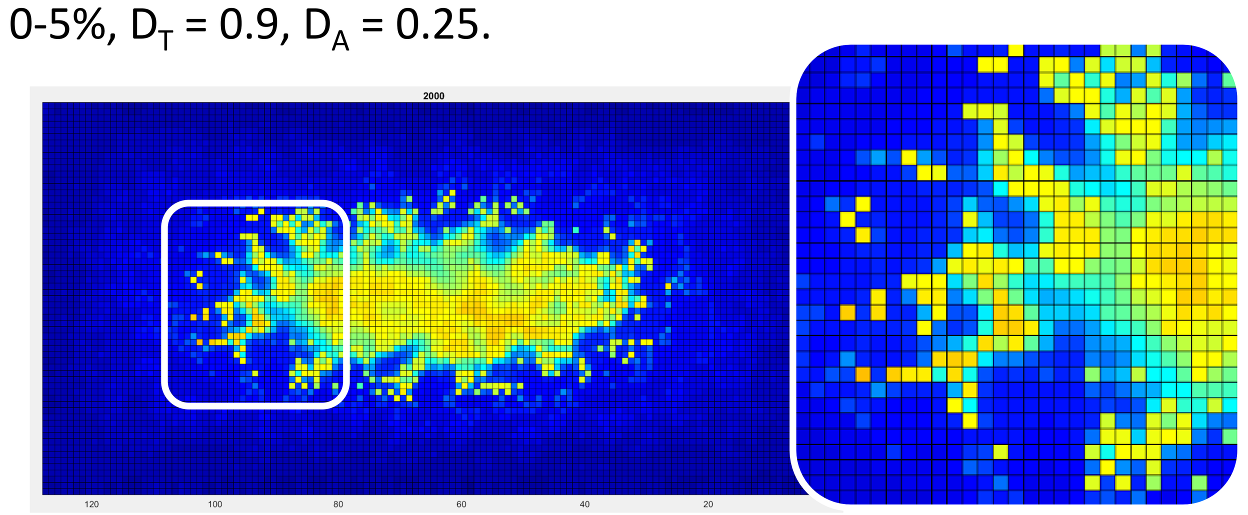

Figure 5.2: Simulation showing finger-like patterns, slice at $z = 32$

Phenomenon 3: Finger-like patterns and warped central region

- Pockets and streaks of liquid regions create paths into the central foggy region
- Figure 5.2 Parameters: Noise 0-6%, $D_T = 0.9$, $D_A = 0.1$.

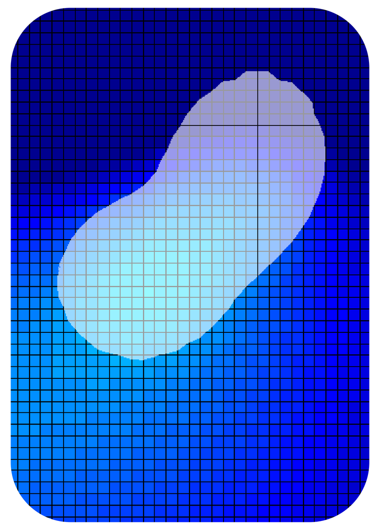

Figure 6: Simulation starting with two colinear bubbles with origin that merge into one elongated bubble.

Observations:

- Higher initial noise conditions produced larger central regions
- Lower initial noise conditions produced more warped and chaotic air regions
- Higher diffusivity of air resulted in higher final air concentration

Bubble in Figure 6 acts as a test scenario for MATLAB to see how two bubbles diffuse inward given a directional cooling front. Here, temperatures are shown with color, and the bubble is the white surface.

Conclusions

Accomplishments:

- Able to produce 3D simulations that resembled some of the phenomena seen in real life, mostly warping of the central white foggy region.
- All patterns are variations of nonhomogeneous regions of high and low air concentration, supporting that these patterns are Turing Patterns.

Shortcomings and Future Additions:

- Unable to produce isolated needle-shaped bubbles, only slight bubble elongation.
- High concentration bands only occurred close to foggy region, never closer to outer surface.
- Surface tension in the bubble is not simulated, Can be accomplished with $(\nabla A)^2$ term from KPZ equation.
- Conservation of air is not obeyed.

These simulations reveal that patterns generated from freezing ice are the result of diffusion-reaction equations of two quantities. This connects ice patterns with Turing patterns, seen in many other scientific fields.

Acknowledgements

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References

[1] The Secret of Clear Ice Cubes (p. 141–). (2020). Indiana University Press.

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