Multiple Input Structures with Robust Estimator MISRE

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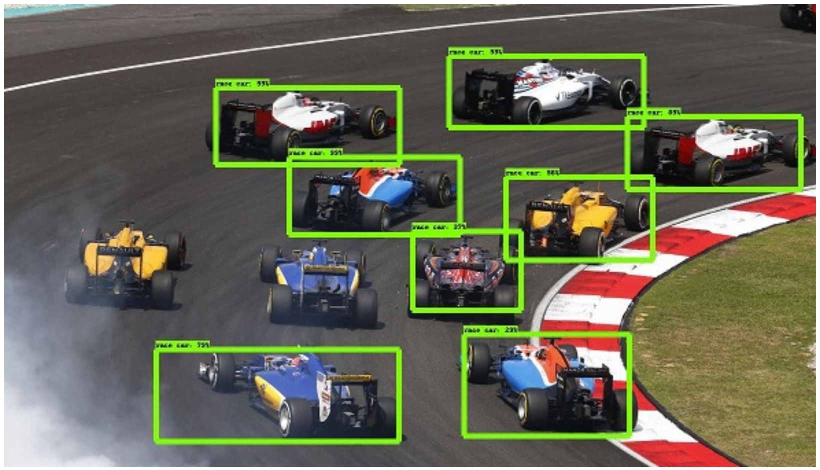


convolutional neural network

parametric estimation

Robust parametric estimation estimates a mathematical relation

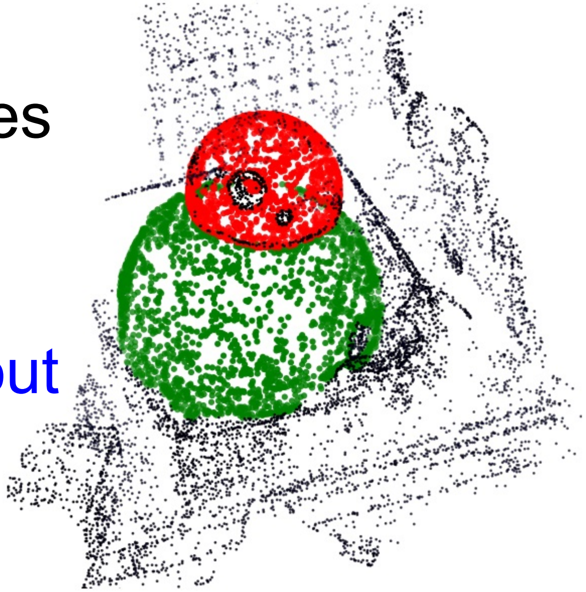
* for the inliers objective function
* using a given algorithm
* with no training necessary.

But

* the user has to give threshold(s) before the estimation
* and the structures are not processes independently.

Should be this way:

a **set** of 2D images viewed in 3D and segmented with 3D spheres without task depended thresholds.



Input measurements $\mathbf{y}_i = [y_{i,1} \;\; y_{i,2} \ldots y_{i,l}]^\top$. Inliers objective function

$f(\mathbf{y}_i)$ is solved by linearization and elemental subsets.

The objective function $f(\mathbf{y}_i)$ is interpreted as the linear relation

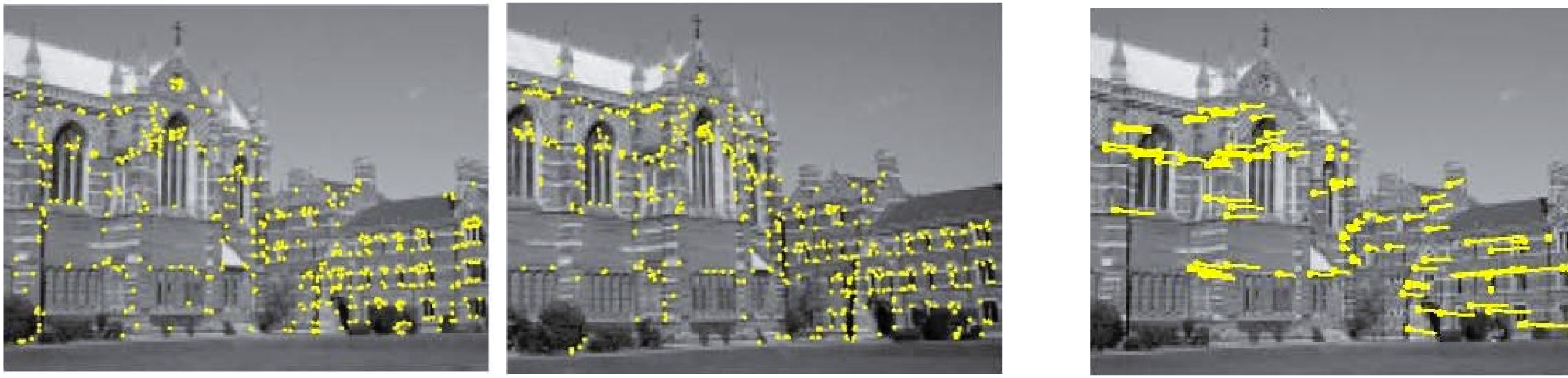
$$f(\mathbf{y}_i) \rightarrow \mathbf{x}_i^\top \boldsymbol{\theta} - \alpha \quad i = 1, \ldots, n$$

The carrier vector $\mathbf{x}_i$ contains both the $\mathbf{y}_i$ and $y_{i,j} y_{i,k}$ (computer vision).
The l unknown in $f(\mathbf{y}_i)$ give rise to m variables in $\boldsymbol{\theta}$.

An elemental subset is the miminum number of points needed for the solution of the m variables. For scalar $f(\mathbf{y}_i)$ number of points equal m.

$$\mathbf{x}_i^\top \boldsymbol{\theta} - \alpha = 0 \quad i = 1, \ldots, m.$$

elemental subset $\longrightarrow$ $\boldsymbol{\theta}$, α Ambiguity is reduced if $||\boldsymbol{\theta}|| = 1$.



inliers projected on first image

example: Fundamental matrix between two 2D images uses object point correspondences to solve for the 3×3 matrix F

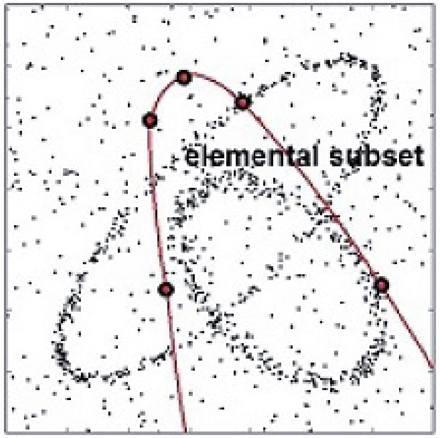
$$f(\mathbf{y}) = [x' \;\; y' \;\; 1] \, \mathrm{F} \, [x \;\; y \;\; 1]^\top \qquad \mathbf{y} = [x \;\; y \;\; x' \;\; y']^\top$$

gives eight carriers $\quad \mathbf{x} = [x \;\; y \;\; x' \;\; y' \;\; xx' \;\; xy' \;\; x'y \;\; yy']^\top$

matrix F $\rightarrow$ vector $\boldsymbol{\theta}$ and scalar α

$\mathbf{x}_i^\top \boldsymbol{\theta} - \alpha = 0 \quad i = 1 \ldots 8 \quad ||\boldsymbol{\theta}|| = 1$ an elemental subset

example: 2D ellipse



m = 5

original variables $\mathbf{y} = [x \;\; y]^\top$ are on the ellipse

$$f(\mathbf{y}) = (\mathbf{y} - \mathbf{y}_c)^\top \mathbf{Q}(\mathbf{y} - \mathbf{y}_c) - 1$$

if $\mathbf{Q}$ is 2×2 positive definite symmetric matrix

$\mathbf{y}_c$ is the ellipse center. $\mathbf{y}_c \rightarrow \theta_1, \theta_2$. $\mathbf{Q} \rightarrow \theta_3, \theta_4, \theta_5$.

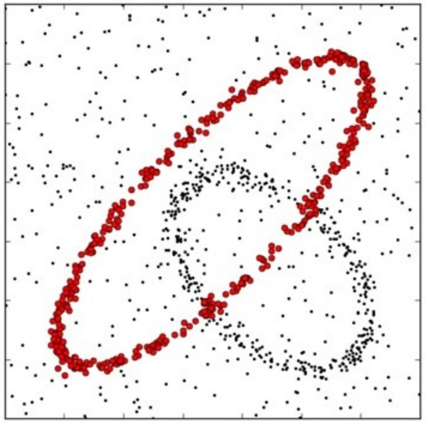
The carrier vector $\mathbf{x} = [x \;\; y \;\; x^2 \;\; xy \;\; y^2]^\top$ gives

$$\theta_1 x_i + \theta_2 y_i + \theta_3 x_i^2 + \theta_4 x_i y_i + \theta_5 y_i^2 - \alpha = 0 \quad i = 1 \ldots 5$$

and a valid elemental subset must satisfy

$$4\theta_3\theta_5 - \theta_4^2 > 0$$

If the inlier scale is given, both ellipses will be entirely recovered only if all the inlier points have similar noise. (Here not.)

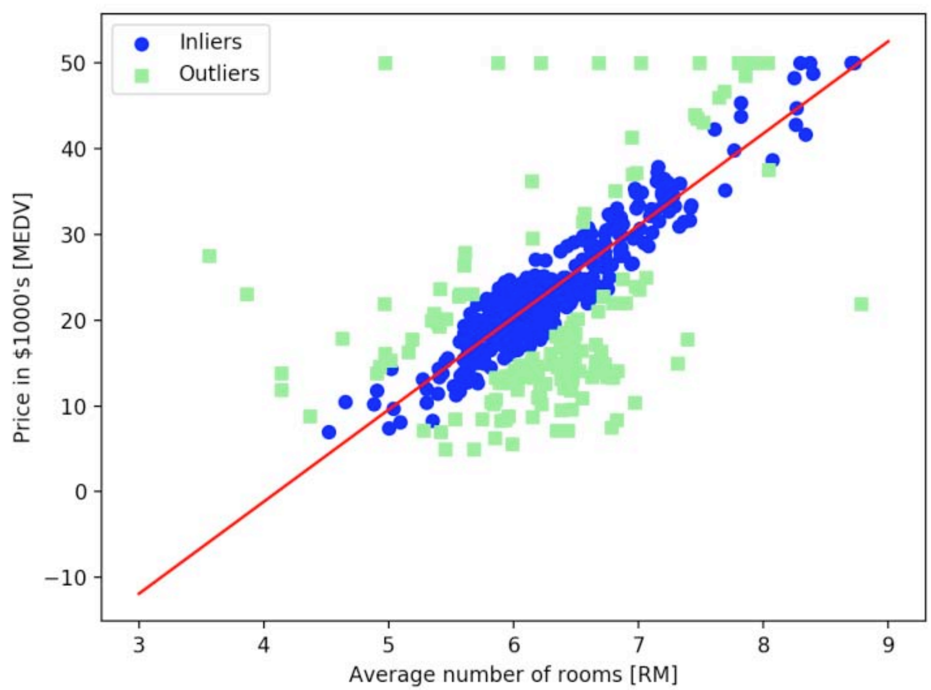


RANdom SAmple Consensus   RANSAC

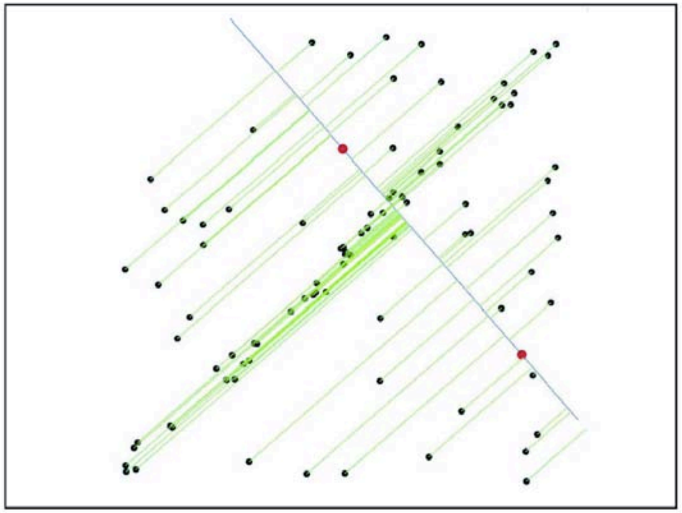
Fischler, Bolles *Communications of **A**ssociation for **C**omputing **M**achinery* 1981

The user has to give before the estimation

M, the number of elemental subsets...



...and the inlier scale.

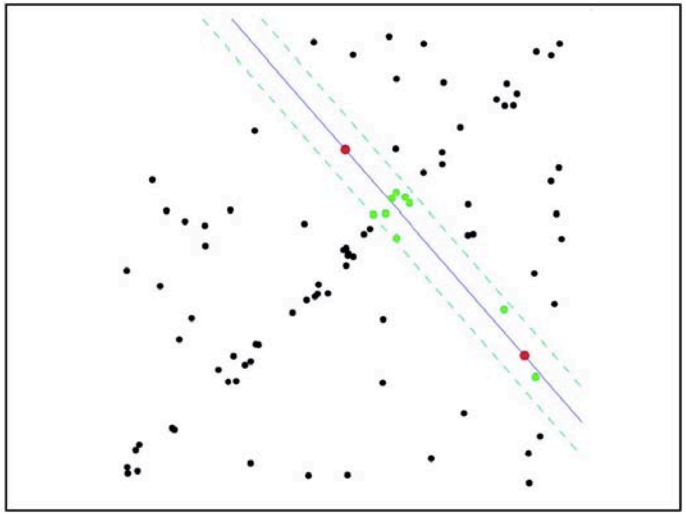


example:

2D line estimation

elemental subset has two points

$$\theta_1 x + \theta_2 y - \alpha = 0$$



RANSAC

given: inlier scale; *M* number of trials (sufficiently large)

Repeat *M* times:

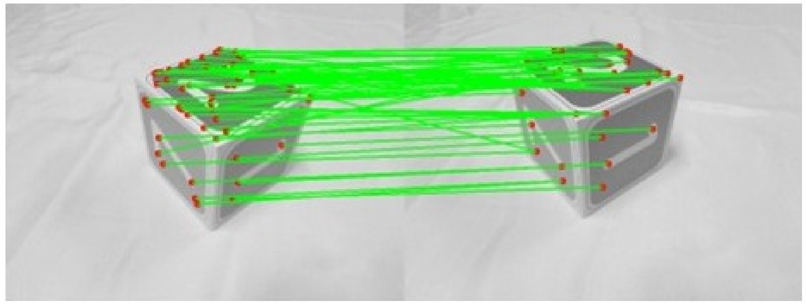
* choose an elemental subset
* find the linear model estimate
* assume the estimate valid for all *n* points
* distances less than the scale are inliers.

Largest consensus set gives the RANSAC estimate.

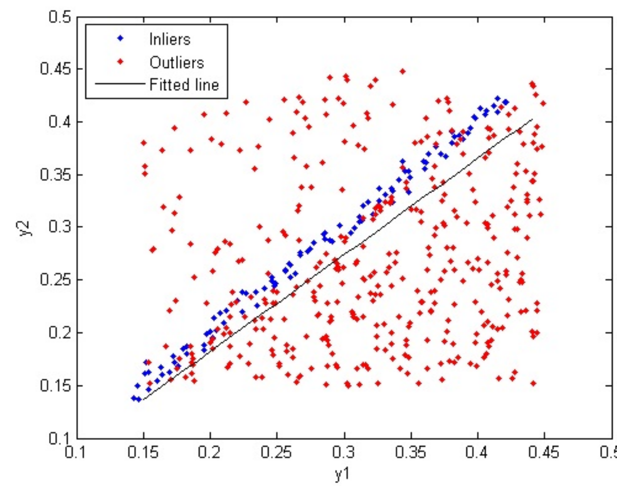
Total Least Squares (TLS) with the inliers: $\hat{\boldsymbol{\theta}}^{tls}, \hat{\alpha}^{tls}$. If needed, project back to the input space and find the original estimates.

RANSAC may fail

* if the scale is incorrectly guessed by the user



* if an image sequence have big changes of the scale
* if the outliers are asymmetric



* if there are multiple inlier structures

A single scale is not enough in many cases.

Multiple Input Structures with Robust Estimator
MISRE

Each structure (inlier or outlier) estimated independently.

Each structure has three steps:
* scale estimation
* refinement with mean shift
* compute the structure's density

When the remaining data is not enough for a structure: STOP

Sort the structures based on the decreasing densities.

The user decides on the number of detected inlier structures.

Building the linear relation for the estimation

The l unknowns in the function $f(\mathbf{y})$ have covariance $\sigma^2\mathbf{I}_{l\times l}$
where σ is unknown and different for each structure (iteration).

If $\mathbf{f}(\mathbf{y}_i)$ is a vector, $\mathbf{y}_i$ have several carrier vectors $\mathbf{x}_i^{[c]}$ $\quad c = 1\ldots\zeta$
Number of points required $m_e = \lceil m/\zeta \rceil$ for $\boldsymbol{\theta}$ and α.

Each column of the $m \times l$ Jacobian matrix $\mathbf{J}^{[c]}_{\mathbf{x}_i|\mathbf{y}_i}$ have the derivatives of the carrier vector in one original variable. $\qquad \mathbf{x}_i^{[c]} \simeq \mathbf{J}^{[c]}_{\mathbf{x}_i|\mathbf{y}_i}\mathbf{y}_i$

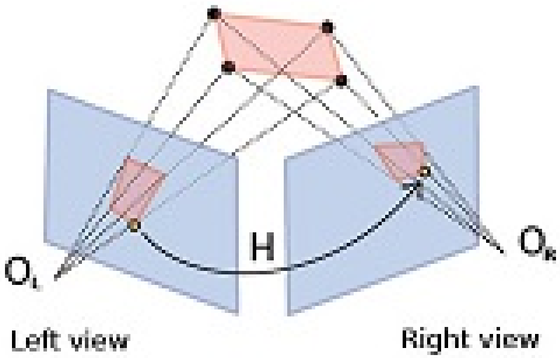
$m \times m$ covariance of $\mathbf{x}_i^{[c]}$ is $\sigma^2\mathbf{C}_i^{[c]} = \sigma^2\mathbf{J}_{\mathbf{x}_i^{[c]}|\mathbf{y}_i}\mathbf{J}^\top_{\mathbf{x}_i^{[c]}|\mathbf{y}_i}$ $\quad c = 1\ldots\zeta$

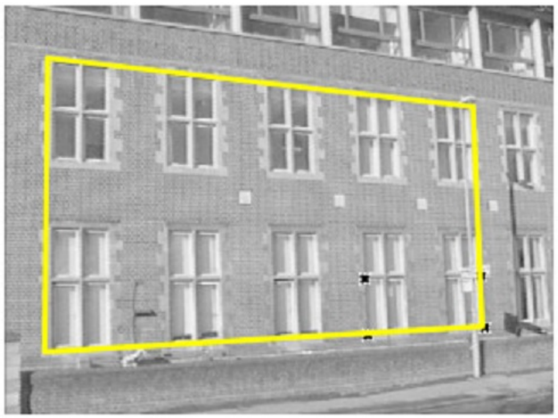
example: $\zeta = 1$ fundamental matrix

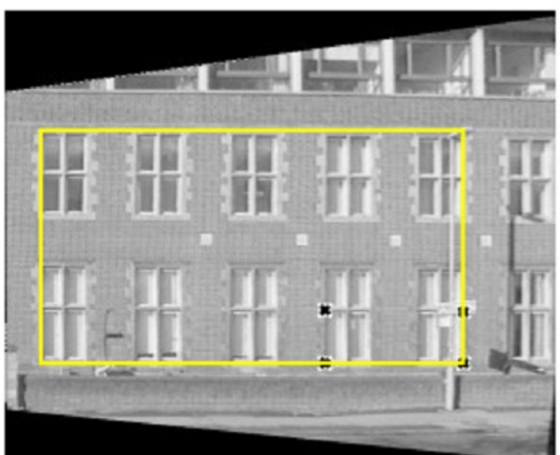
$$f(\mathbf{y}) = [x' \;\; y' \;\; 1]\,\mathbf{F}\,[x \;\; y \;\; 1]^\top$$

$$\mathbf{x} = [x \; y \; x' \; y' \; xx' \; xy' \; x'y \; yy']^\top$$

$$\mathbf{J}^\top_{\mathbf{x}_i|\mathbf{y}_i} = \begin{bmatrix} 1 & 0 & 0 & 0 & x_i' & y_i' & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & x_i' & y_i' \\ 0 & 0 & 1 & 0 & x_i & 0 & y_i & 0 \\ 0 & 0 & 0 & 1 & 0 & x_i & 0 & y_i \end{bmatrix}$$







example: $\zeta = 2$

2D homography between two 2D images is a plane correspondence found through a 3×3 matrix

$$\mathbf{H} = [\mathbf{h}_1 \;\; \mathbf{h}_2 \;\; \mathbf{h}_3]^\top \qquad \mathbf{y} = [x \;\; y \;\; x' \;\; y']^\top$$

$$\begin{bmatrix} x'_h \\ y'_h \\ w'_h \end{bmatrix} - \begin{bmatrix} \mathbf{h}_1^T \\ \mathbf{h}_2^T \\ \mathbf{h}_3^T \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \mathbf{0} \quad x' = \frac{[x \;\; y \;\; 1]\mathbf{h}_1}{[x \;\; y \;\; 1]\mathbf{h}_3} \quad y' = \frac{[x \;\; y \;\; 1]\mathbf{h}_2}{[x \;\; y \;\; 1]\mathbf{h}_3}$$

written with the unknown vec H^T and the carriers $\mathbf{x}^{[1]}, \ \mathbf{x}^{[2]}$

$$\begin{bmatrix} -x & -y & -1 & 0 & 0 & 0 & x'x & x'y & x' \\ 0 & 0 & 0 & -x & -y & -1 & y'x & y'y & y' \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} = \mathbf{0}$$

Jacobian matrices $\mathbf{J}^{[1]}_{\mathbf{x}_i|\mathbf{y}_i}, \mathbf{J}^{[2]}_{\mathbf{x}_i|\mathbf{y}_i}$ are 9×4 and $\alpha = 0$.

An elemental subset $\begin{bmatrix} \mathbf{x}_1^{[1]\top} \\ \mathbf{x}_1^{[2]\top} \\ \dots \\ \mathbf{x}_4^{[1]\top} \\ \mathbf{x}_4^{[2]\top} \end{bmatrix} \begin{bmatrix} \mathbf{h}_1 \\ \mathbf{h}_2 \\ \mathbf{h}_3 \end{bmatrix} = 0$ has 8 equations.

$$\mathbf{x}_i^{[c]\top}\boldsymbol{\theta} - \alpha = 0 \quad c = 1,\ldots,\zeta \quad i = 1,\ldots,m_e \longrightarrow \boldsymbol{\theta} \text{ and } \alpha$$

and projects $\mathbf{x}_i^{[c]}$ into $z_i^{[c]} = \mathbf{x}_i^{[c]\top}\boldsymbol{\theta} \quad i = 1,\ldots,n.$

Mahalanobis distance from α is $\quad d_i^{[c]} = \dfrac{|\mathbf{x}_i^{[c]\top}\boldsymbol{\theta} - \alpha|}{\sqrt{\boldsymbol{\theta}^\top \mathrm{C}_i^{[c]}\boldsymbol{\theta}}} \geq 0$

without the unknown σ.

For each $\mathbf{y}_i$ retain only the largest Mahalanobis distance

$$\widetilde{c}_i = \arg\max_{c=1\ldots\zeta} \; d_i^{[c]} \qquad d_i^{[\widetilde{c}_i]} = \widetilde{d}_i = \frac{|\widetilde{\mathbf{x}}_i^\top\boldsymbol{\theta} - \alpha|}{\sqrt{\boldsymbol{\theta}^\top \widetilde{\mathrm{C}}_i\boldsymbol{\theta}}} \geq 0$$

Input $\mathbf{y}_i$ corresponds to the carrier vector $\widetilde{\mathbf{x}}_i$ in the estimation.

Scale estimation

Refinement with mean shift

Compute the structure's density

All MISRE estimators use the same two constants.

Per iteration M elemental subset trials are given by the user.

An iteration has $n \leq n_T$ datapoints

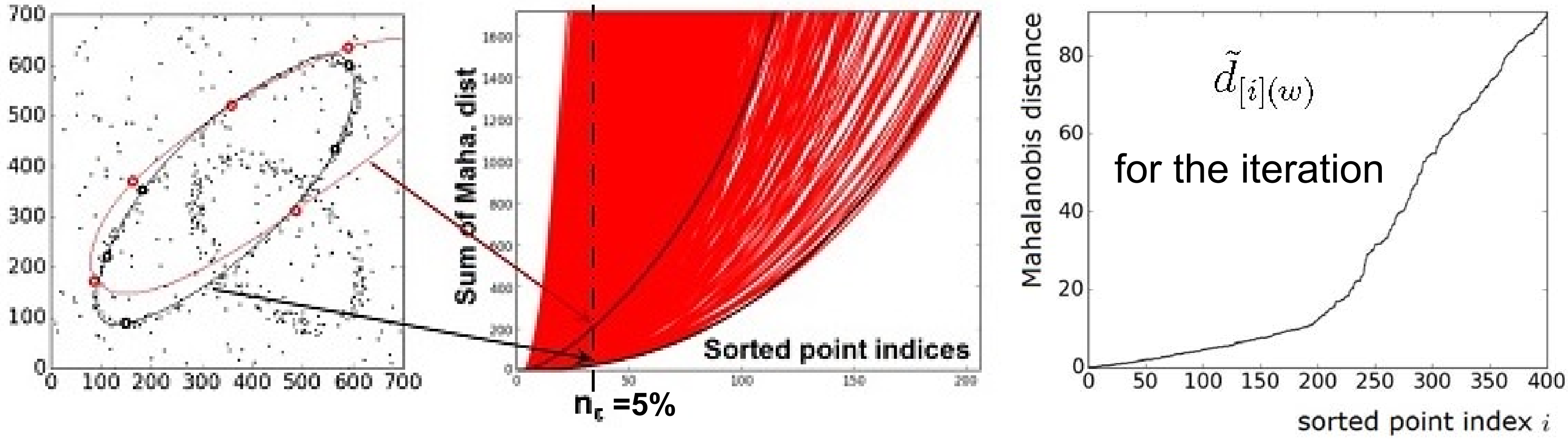
where n_T is the total number of datapoints.

First constant

Each iteration starts from n_ϵ points which is the larger between:

* $0.05 n_T$ (5%) from the total number of datapoints, or
* five times number of unknowns m in an elemental subset.

A second condition quick-in only when the data is relative small and the elemental subset uses a large carrier vector.



In each sequence the Mahalanobis distances are ascendingly ordered

$\tilde{d}_{[i](j)}$ $i = 1 \ldots n$ and are $j = 1 \ldots M$ sequences.

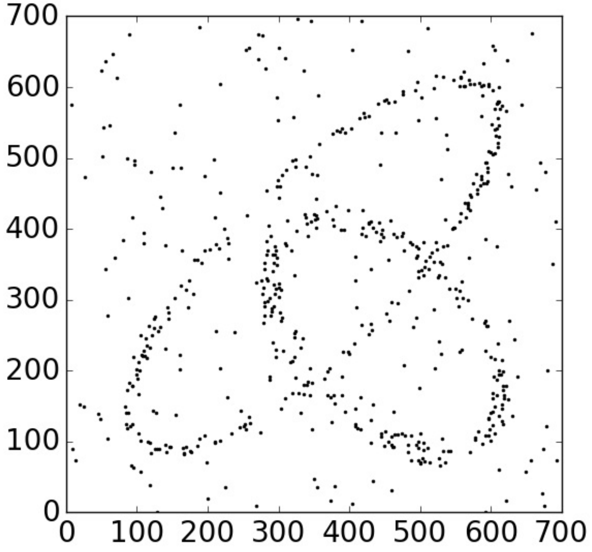
The working sequence $\tilde{d}_{[i](w)}$ is the sequence with minimum sum of the Mahalanobis distances

$$\min_{j \in M} \sum_{i=1}^{n_\epsilon} \tilde{d}_{[i](j)} \quad \text{for the first } n_\epsilon \text{ points.}$$

The corresponding elemental subset returns $\hat{\boldsymbol{\theta}}_w, \hat{\alpha}_w$.

example of how MISRE is built:

two ellipses n_{in} = 200, n_{out} = 200. M = 2000 n_e = 30

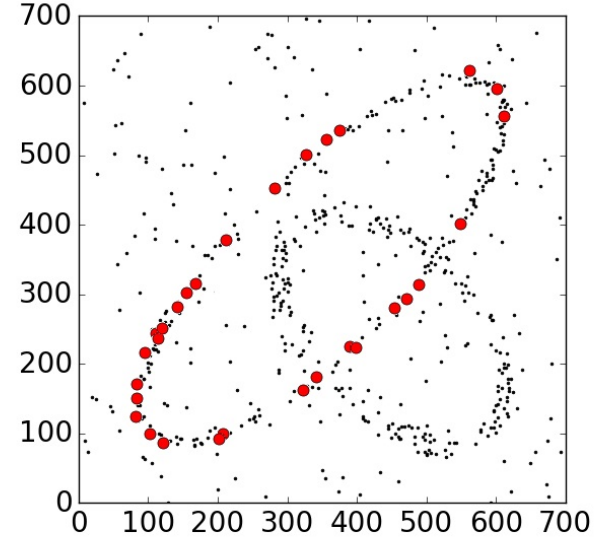


$\sigma_g = 5$ or 10

Gaussian standard deviations

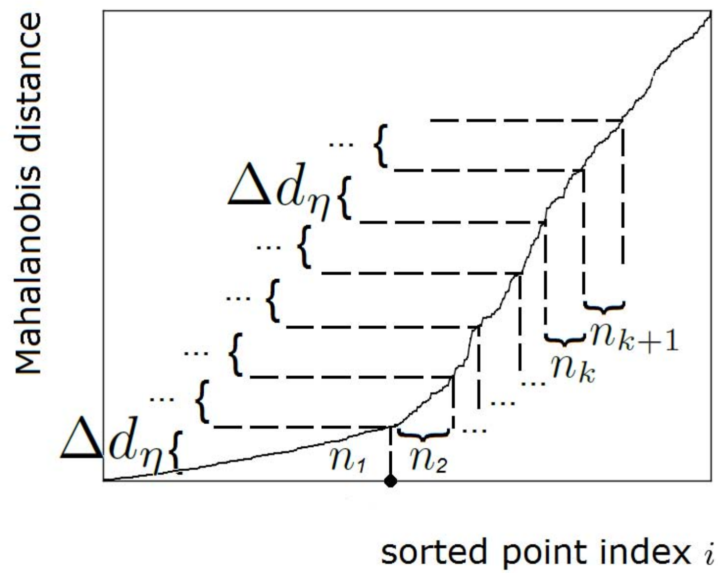
inlier scale $\sim 2.5\sigma_g$

first iteration



number of datapoints n_T = 200+200+200 = 600

starting from n_e = 0.05x600 = 30 points



Starting from point $\hat{\alpha}_w$ the $\eta\%$ $(\geq 0.05 n_T)$ points correspond to the Mahalanobis distance Δd_η and have n_1 points.

Δd_η divides the working sequence $\tilde{d}_{[i](w)}$ $i = 1, \ldots, n$ in equal parts.

Δd_η have n_k points in the k-th segment $k = 1, 2, \ldots$

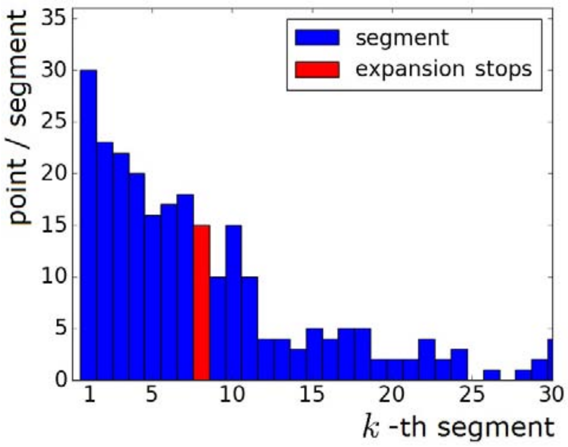
The expansions are independent since each $\eta\%$
increases with $0.01 n_T$ (1%) relative to the previous $\eta\%$.

Second constant

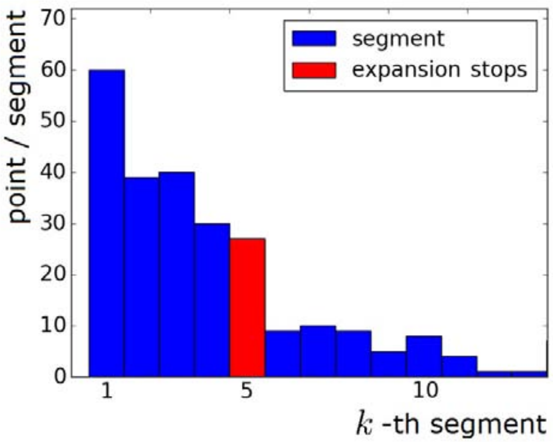
* If the average number of processed points in *k* segments is larger than twice the number of points in the (*k+1*)-th segment, the expansion terminates.

This condition is verified for $k = 1, 2, \ldots$ at each Δd_η

$$\frac{1}{k}\sum_{i=1}^{k} n_i > 2\, n_{k+1}$$

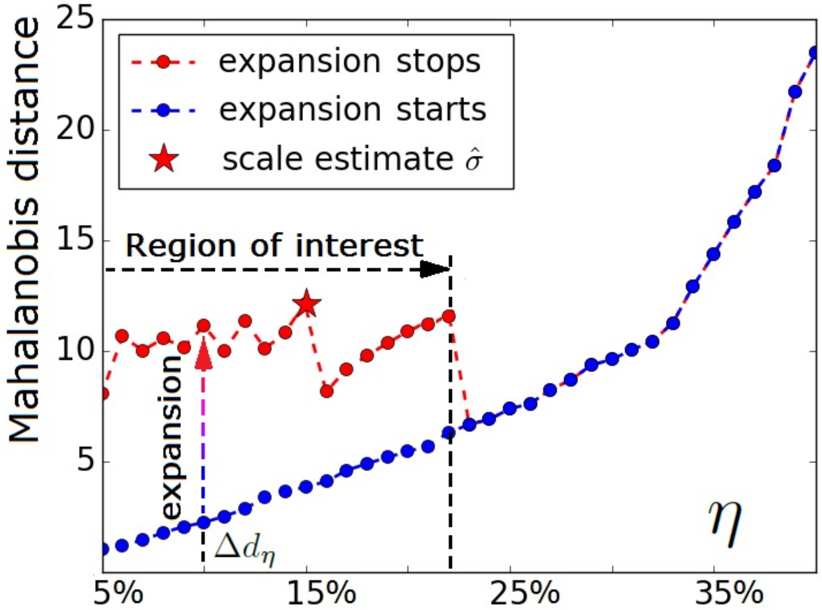


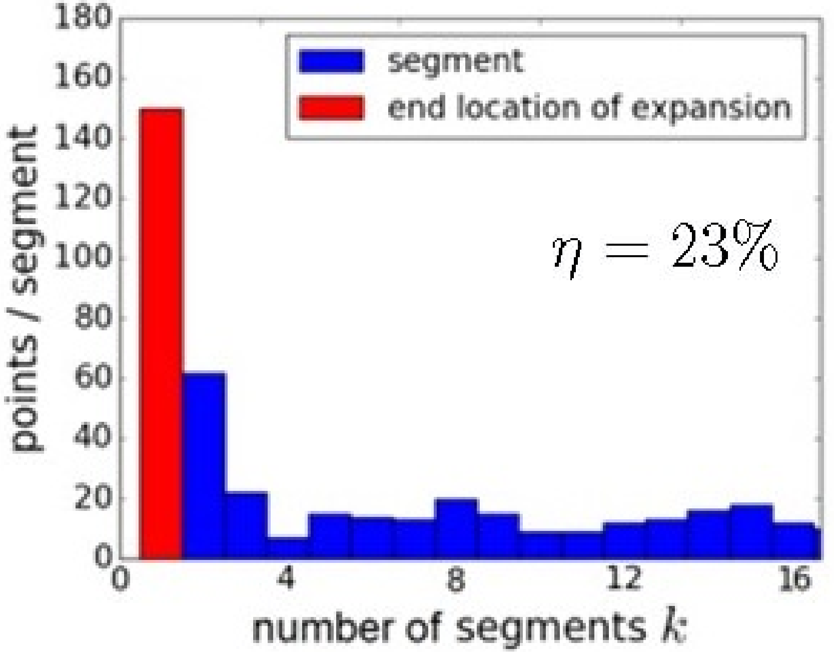
$\Delta d_5,\ k_{t_5} = 8,\ \hat{\sigma} = 8.06$



$\Delta d_{10},\ k_{t_{10}} = 5,\ \hat{\sigma} = 11.10$

scale should be $\sim 2.5 * 5 \simeq 12.5$





Region of interest is defined from n_ϵ, corresponding to Δd_5 until the first $\eta\%$ where the second constant already holds from the beginning of the expansion.

Largest expansion gives the scale estimate

$$\hat{\sigma} = \max_{\eta=5\%,\ldots,\eta_f} k_{t_\eta} \Delta d_\eta$$

$\hat{\sigma} = 12.54$ in the example. $n_{\hat{\sigma}}$ points between $\hat{\alpha}_w \pm \hat{\sigma}$.

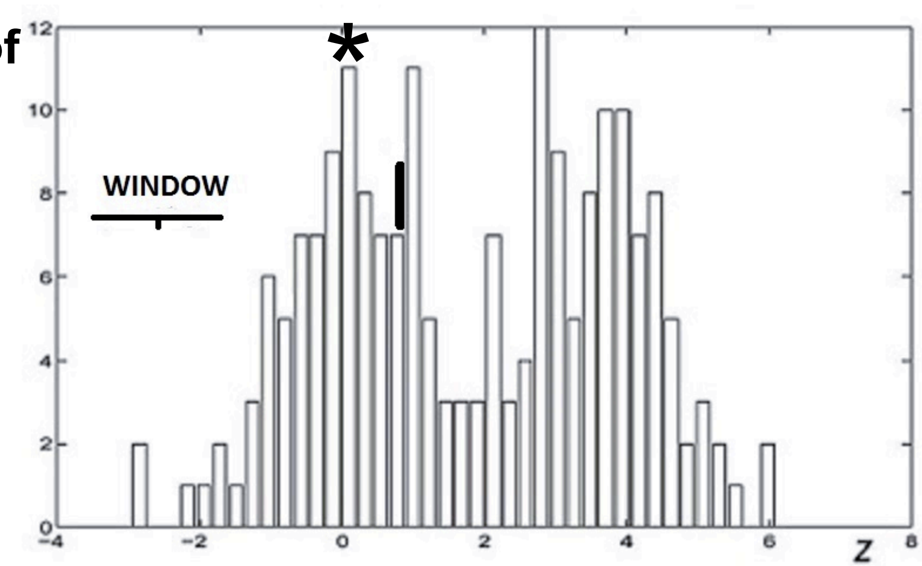
Scale estimation

Refinement with mean shift

Compute the structure's density

Mean shift is an iterative procedure finding the modes of the distribution function of a given window.

number of points

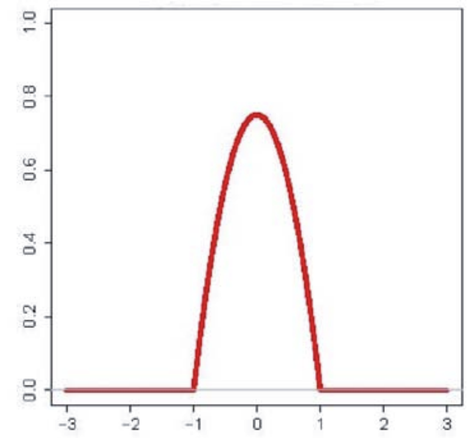


datapoints quantized here

Interested in the mode * closest to the point marked | for $\hat{\alpha}_w$
Convergence achieved after only a few iterations.

Setting up of the mean shift

$$K(u) = \begin{cases} \frac{3}{4}(1-u^2) & |u| \le 1 \\ 0 & otherwise \end{cases}$$



Epanechnikov kernel

$$g(u) = -K'(u^2) = -\kappa'(u) = -\kappa'\left((z - \tilde{z}_i)^\top \widetilde{B}_i^{-1} (z - \tilde{z}_i) \right)$$

$$\kappa(u) = \begin{cases} 1-u & 0 \le u \le 1 & g(u) = 1 \\ 0 & u > 1 & g(u) = 0 \end{cases}$$

$\hat{\sigma}$ defines $\widetilde{B}_i = \hat{\sigma}^2 \boldsymbol{\theta}^\top \mathbf{J}_{\tilde{\mathbf{x}}_i|\mathbf{y}_i} \mathbf{J}_{\tilde{\mathbf{x}}_i|\mathbf{y}_i}^\top \boldsymbol{\theta}$ $\quad \tilde{z}_i = \tilde{\mathbf{x}}_i^\top \boldsymbol{\theta}$

From $n_{\hat{\sigma}}$ points choose another $N = M/10$ elemental subsets.

Mean shift is applied to all n points.

$$\arg\max_{\hat{\alpha}} \sum_{i=1}^{n} \kappa\left((z-\tilde{z}_i)^\top \widetilde{B}_i^{-1}(z-\tilde{z}_i)\right) \qquad z \to \hat{\alpha}$$

Current value is z_{old} and all points $\tilde{z}_i$ contribute equally

$$z_{new} = \frac{\sum_{i=1}^{n} g\left(u_i\right)\tilde{z}_i}{\sum_{i=1}^{n} g\left(u_i\right)}$$

$$\text{if } |z_{old}-\tilde{z}_i| \leq \sqrt{\widetilde{B}_i} \qquad g(u_i) = 1$$

$$\text{if } |z_{old}-\tilde{z}_i| > \sqrt{\widetilde{B}_i} \qquad g(u_i) = 0$$

After N trials, the window having the most points $\tilde{z}_i$ at the convergence with $g(u_i) = 1$ is the mode $\hat{\alpha} = z_{final}$.

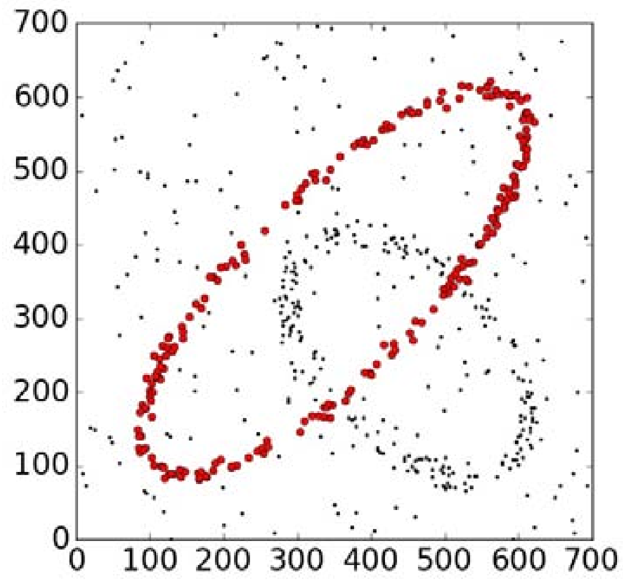
n_{st} points converge to $\hat{\alpha}$.

Nonrobust total least squares (TLS) estimates the structure from all the n_{st} points

$$\tilde{\mathbf{x}}_i^T \boldsymbol{\theta} - \alpha = 0 \quad i = 1, ..., n_{st} \quad \longrightarrow \quad \hat{\boldsymbol{\theta}}^{tls} \; \hat{\alpha}^{tls} \; \hat{\sigma}^{tls}$$

n_{st} points between $\hat{\alpha}^{tls} \pm \hat{\sigma}^{tls}$

$\hat{\sigma}^{tls} = 12.37 \quad n_{st} = 219$ in the example.



Scale estimation

Refinement with mean shift

Compute the structure's density

The density for the structure is the ratio between the number of points and the scale of the structure.

$$\rho = n_{st} / \hat{\sigma}^{tls}$$

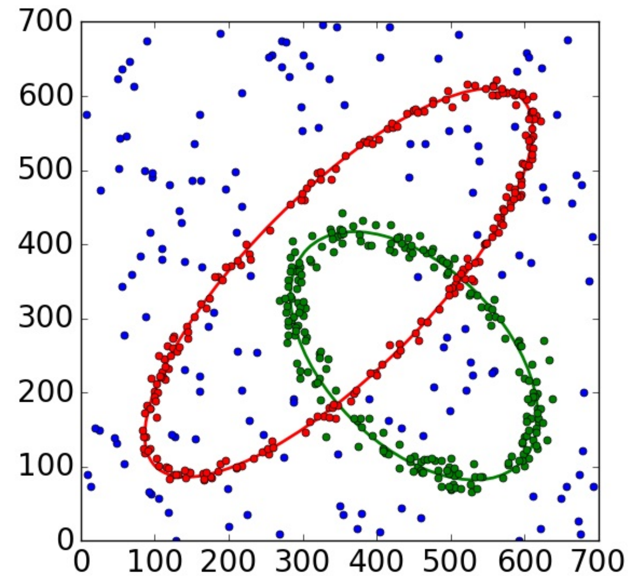
$$\rho = 17.7 \text{ in the example}$$

n_{st} are removed from the input and the processing of the next structure begins ..... until less than n_e of points.

Sorting the structures

The detected structures are sorted in descending order based on the densities.

	red	green	blue
nr. points :	219	210	163
TLS scale :	12.37	28.4	708.7
density :	17.7	7.4	0.23



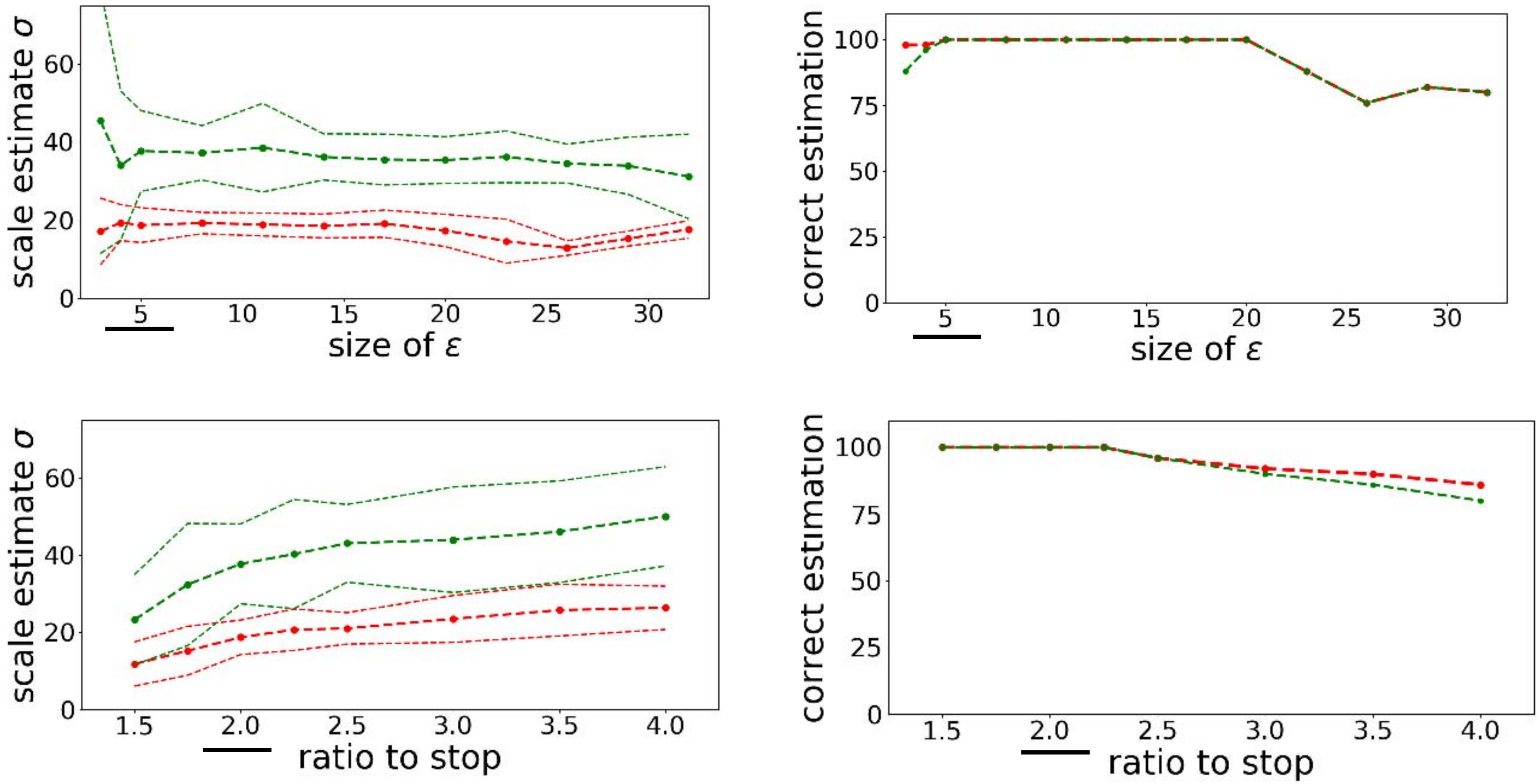
Significant inliers structures have much smaller scales and much larger densities.

The user has only to specify how many inlier structures at the beginning are returned in the estimation.

Here retains the first two structures.

Why these constants?

ellipses: inliers 2x33% (elemental subset x 5 = 4.17%) 100 trials



M increasing till 1000, the output is better. We took 2000.

Higher *M*-s do not help the results in a statistical way.

Pre/post-processing, not in MISRE, could maybe improve only.

Summary of MISRE

For each structure:

Scale is the largest expansion in the region of interest.

Refinement returns n_{st} points between $\hat{\alpha}^{tls} \pm \hat{\sigma}^{tls}$.

Density of the structure is $\rho = n_{st}/\hat{\sigma}^{tls}$.

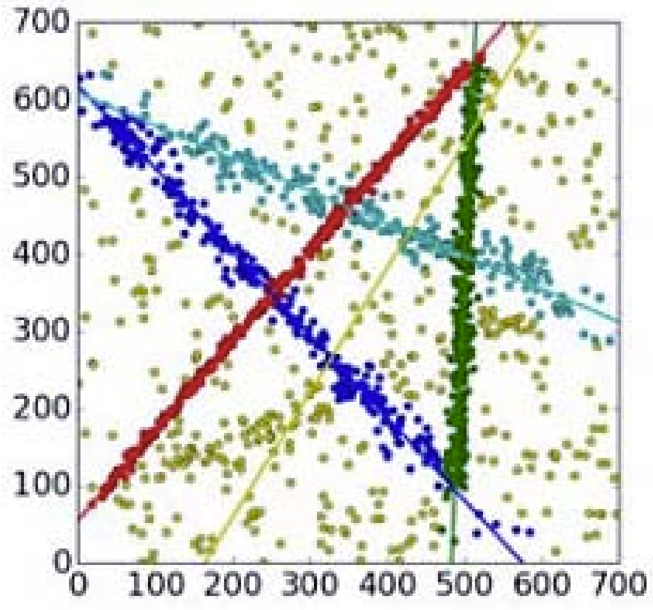
Sorting by decreasing densities.

Separating the inlier structures from outliers requires the user just to decide where the $\hat{\sigma}^{tls}$ increased a lot.

MISRE is as good as the RANSAC-type estimators if similar noise corrupts all inliers and RANSAC is correctly tunes.

Superior when the noise is very different for each inlier structure.

Outliers



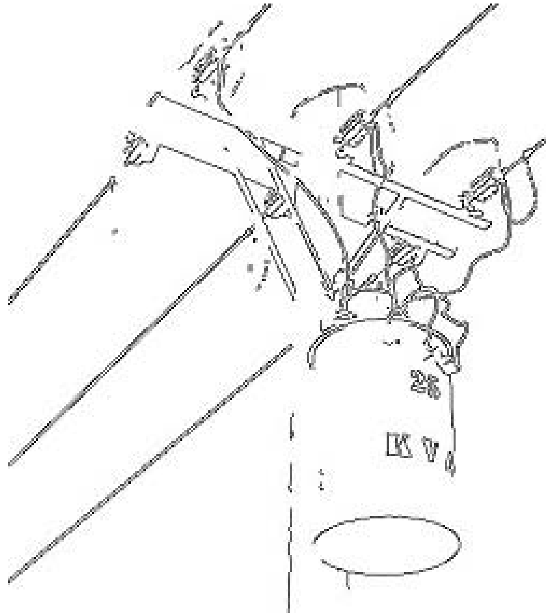
MISRE with 500 outliers
only the weakest disapp.

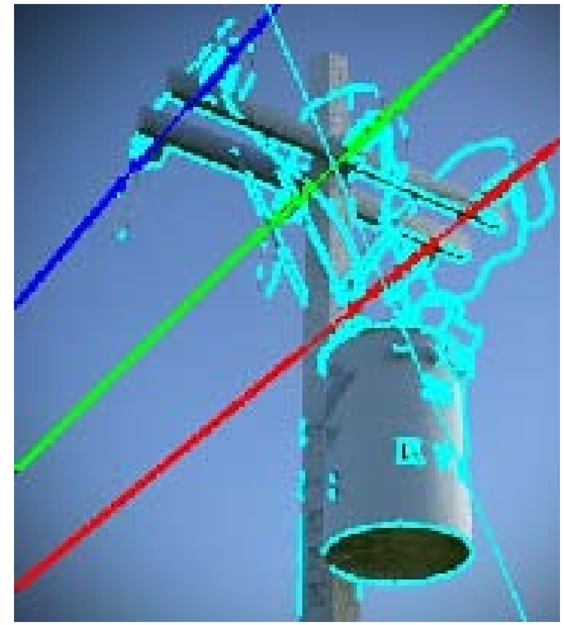
MISRE degrades gradually when number of outliers increases
while the RANSAC-type estimators fail competely.
(Will be discussed at the end of the talk.)

The number outliers can exceed the number of inliers.
Processing times are based on i7-2617M with a 1.5GHz clock.

2D lines

Canny edge detection 8072 points. $M = 1000$





n_e=404 t_p=4.35 seconds First three structures are inliers.

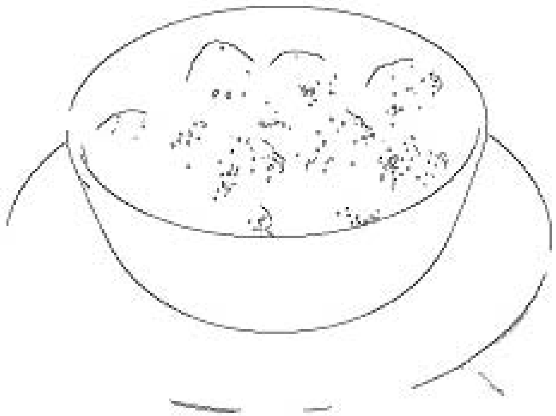
(**2732** points red, green, blue)

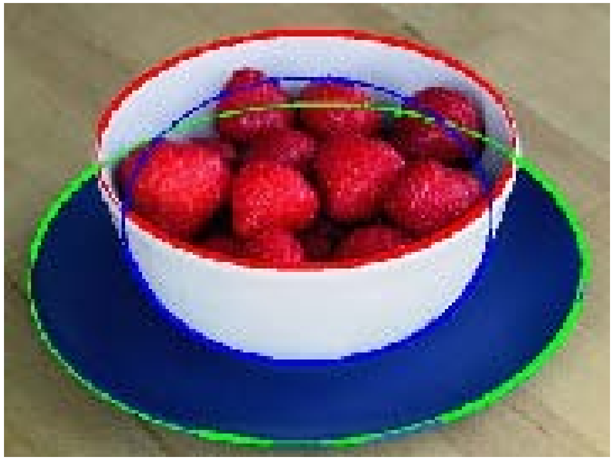
Outliers <30 times larger scale.(cyan)

(Later will see how multiple inlier types can be also detected.)

2D ellipses

Canny edge detection 4343 points. M = 5000





n_e=218 t_p=18.90 seconds

First three structures are inliers.

(**2285** points red, green, blue)

Outliers <100 times larger scale.

(not shown)

Fundamental matrices

Parts (moving) together in 3D give a structure in 2D. $M = 5000$

608 input pairs $n_e=8x5=40$ $t_p=1.75$ seconds

First two structures are inliers (**508** pairs red/green).

Outliers <15 timers larger scale (blue).

2D homographies

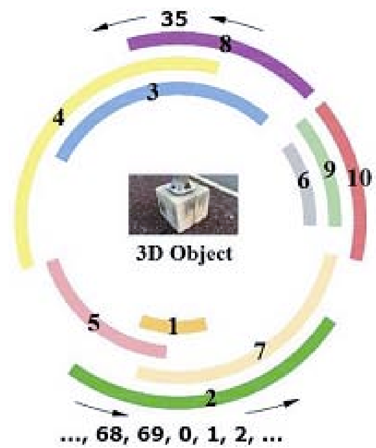
Correspondences in 3D may not correspond to correspondences in 2D for the homographies. $M = 2000$

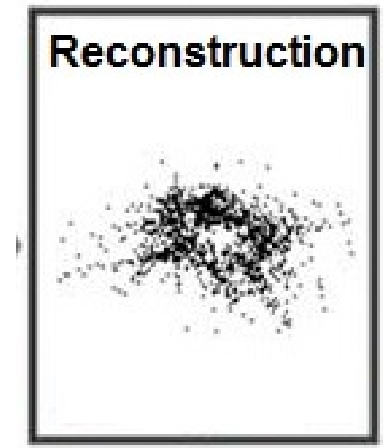
1940 input pairs n_e=97 t_p=3.78 seconds

First four structures are inliers (**1747** pairs red/green/blue/cyan).

Outliers <50 times larger scale (yellow).

Experiments with 3D point clouds





* From a sequence of 2D images
* build 3D tracks covering small parts of the entire 3D scene
* which are fused together into a single 3D point cloud.

Structure from Motion algorithm (SfM)

MISRE used repeatedly during this 3D reconstruction.

Processing time starts after the 3D point cloud was estimated.

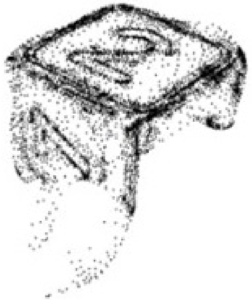
3D point cloud can also be generated with the Autodesk professional program ReMake

input images in 2D

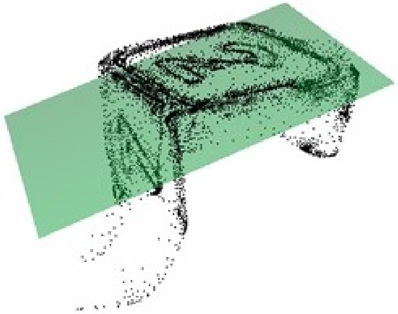
give a 3D mesh model

but in ReMake the different surfaces in the data must be selected before the estimation.

Sample 2D image



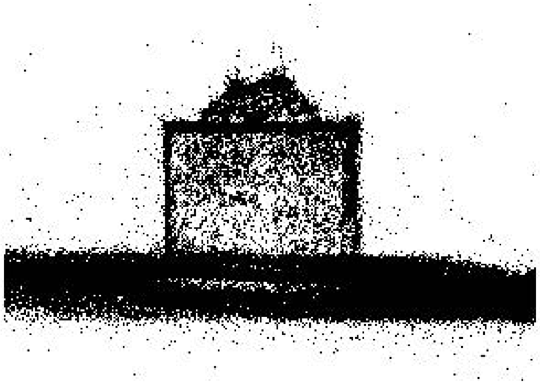
3D point cloud

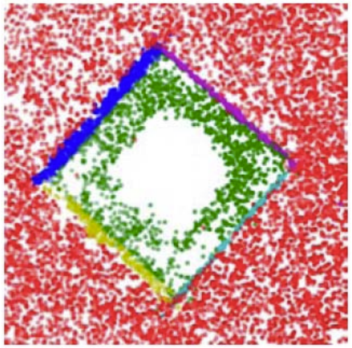


Direct plane fitting in professional software

3D planes

Structure from Motion algorithm. M = 1000





70 - 2D images give 23077 points in the 3D point cloud.

n_e=1154. t_p=7.04 seconds.

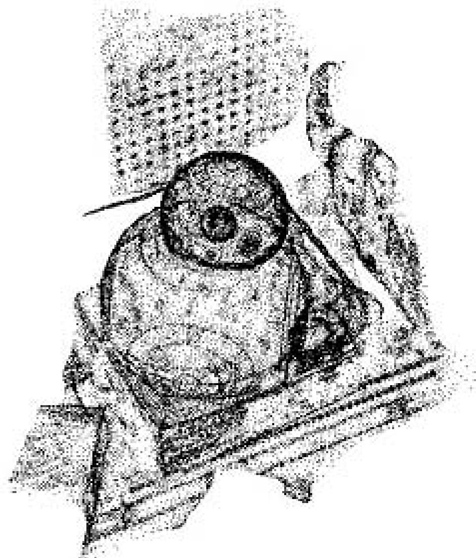
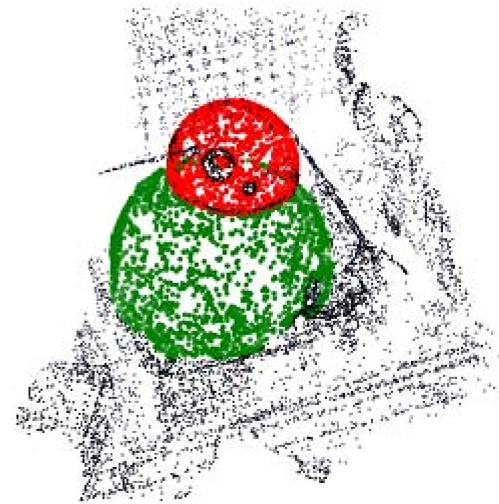
First six structures are inliers (**21758** points all 6 colors).

Outliers much larger scale and smaller density.

(not shown)

3D spheres

$\mathbf{x} = [X \;\; Y \;\; Z \;\; X^2 + Y^2 + Z^2]^\top$ Autodesk ReMake M = 1000

36 - 2D images give 10854 points in the 3D point cloud.

n_e=543. t_p=7.24 seconds.

First two structures are inliers (**3504** points red/green).

Outliers have much larger scale and radius (black).

3D circular cylinders

Nine points solution $\mathbf{x} = [X \;\; Y \;\; Z \;\; X^2 \;\; XY \;\; XZ \;\; Y^2 \;\; YZ \;\; Z^2]^\top$ is valid for a cylinder when the elemental subsets have to satisfy additional relations resulting in only five degrees of freedom.

Autodesk ReMake $M = 2000$

22 - 2D images give 6500 points in the 3D point cloud

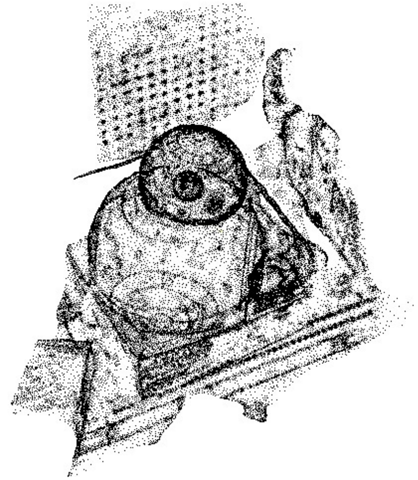
n_e=325. t_p=12.65 seconds.

First two structures are inliers (**2262** points red/green).

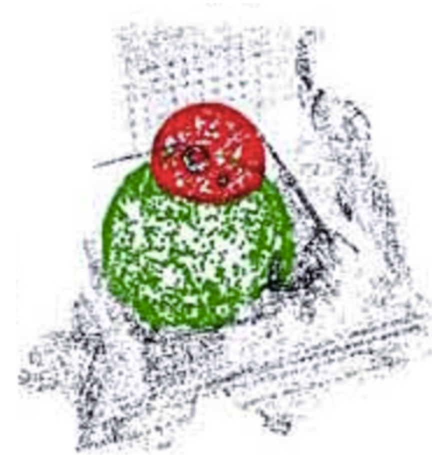
Outliers have much larger scale and no height (black).

3D point cloud gives two spheres or three planes. Post-processing of both outputs together, after some reallocation of points, solves both tasks.

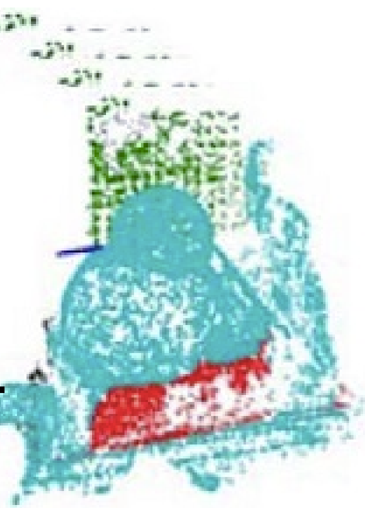
36 - 2D images



3D point cloud



two spheres



three planes

both tasks

Better pre-processing can increase the number of inliers. Better post-processing can recover more inlier struct.

Pre and post processing (needing thresholds) are not in MISRE.

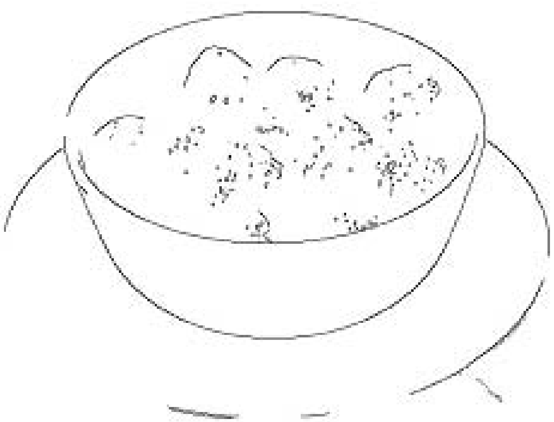
Significant MISRE inliers are returned (based on the two constants) but the inlier structures can vary in multiple runs.

Universality comes with this caveat.

for *example*: two runs for the 2D ellipse.

	red	*green*	*blue*	*cyan*
nr. points :	1068	690	532	2056
TLS scale :	2.18	1.76	1.46	843
density :	488.4	392.2	363.6	2.44

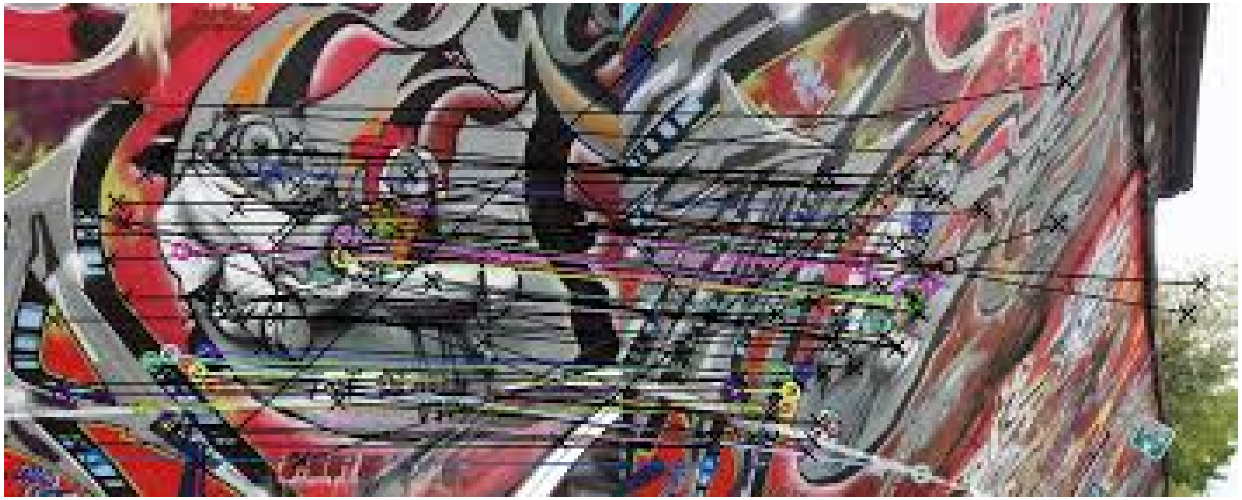
	red	*green*	*blue*	*cyan*
nr. points :	927	662	626	2122
TLS scale :	1.39	1.73	2.10	10805
density :	667	383.6	298	0.20



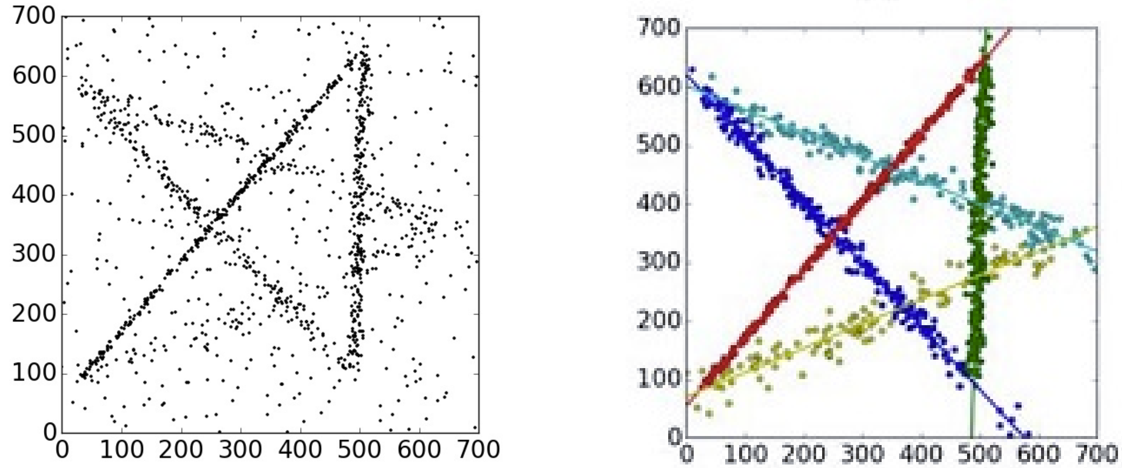
The inlier structures are the same, however the number of points and the TLS scales are somewhat different.

Limitations of MISRE

Every robust estimator fails if too many outliers are present.
RANSAC fails if already a single inlier structure failed.



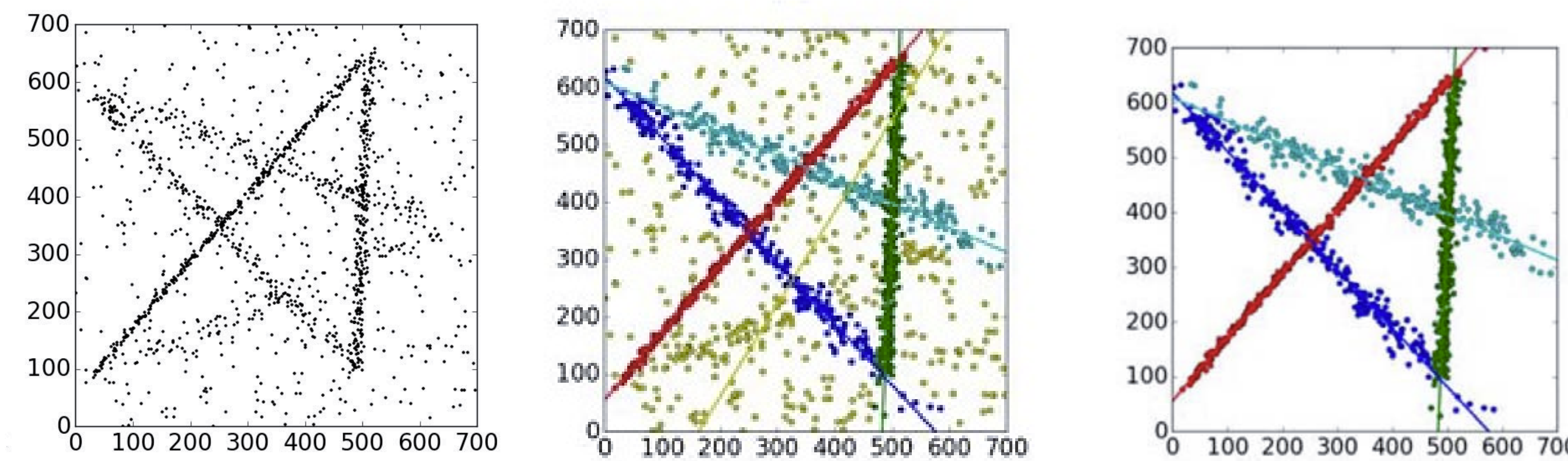
MISRE estimates each structure independently, therefore at the beginning only the "weakest" inlier structure, the lowest inlier density, become outliers.



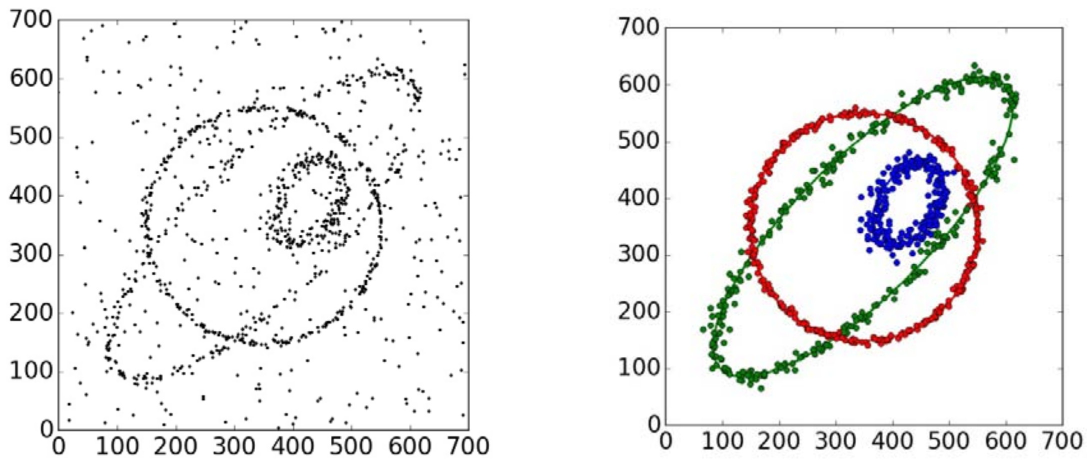
M = 1000

five lines, "weakest" n_{in}=100 gaussian noise=15, n_{out}=350

"weakest" 6/100 are outliers



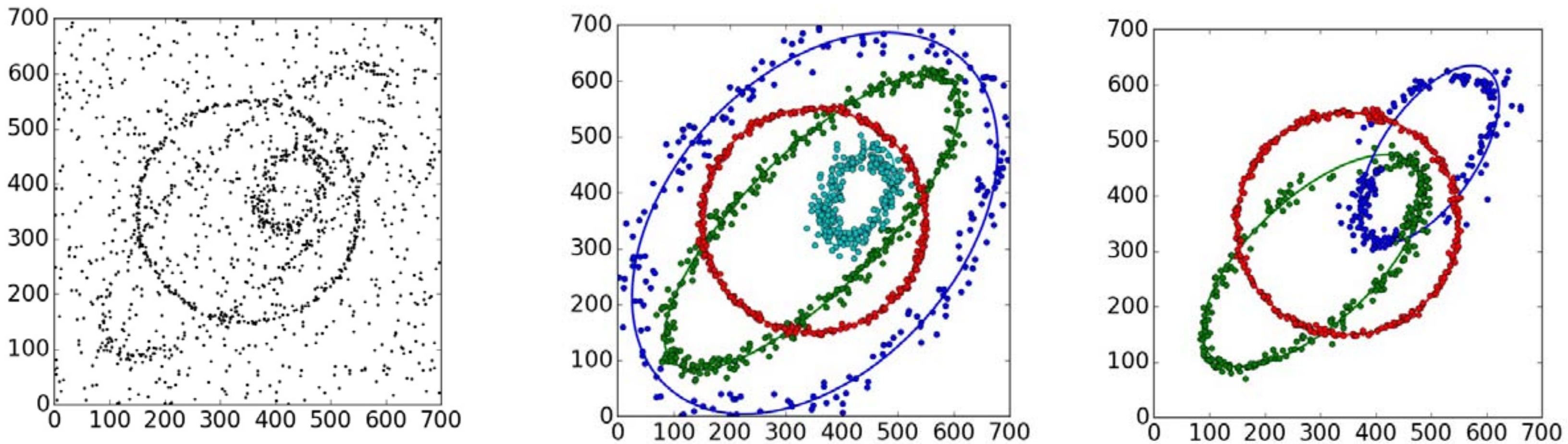
n_{out}=500 "weakest" 34/100, next one 2/100 too are outliers



M = 5000

three ellipses, smallest n_{in}=200 gaussian noise=9, n_{out}=350

smallest 10/100, middle 6/100 too are outliers

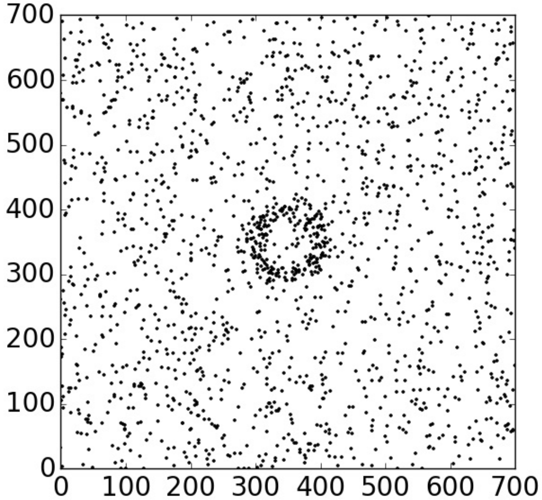


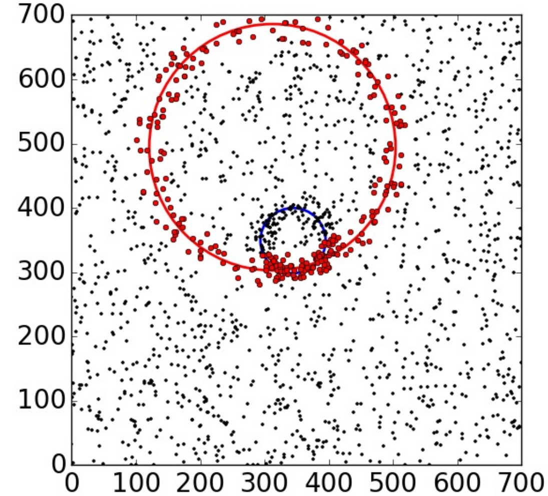
n_{out}=800 smallest 53/100, middle 19/100 too are outliers

exmp: outliers (blue) *before* inliers (cyan)||two inliers interact circle n_{in}=200 n_{out}=1500. Scale estimates correct around ~23.5

$\sigma_g = 10$ Mean shift depends of the radius of the circle.

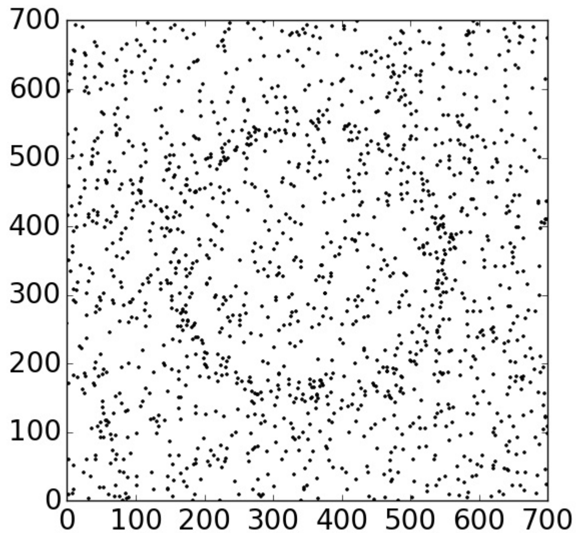
r = 50

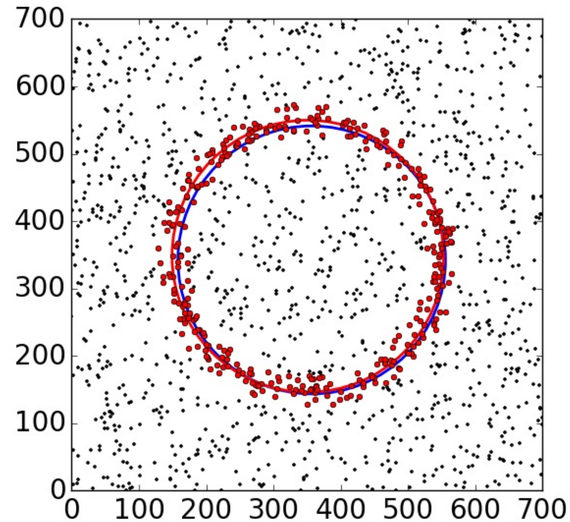




incorrect often

r = 200





correct in 100 tests

MISRE, the Multiple Input Structures with Robust Estimator estimates each structure independently.

MISRE has a simple set-up with the same two constants for every estimation.

MISRE works for inlier structures with vastly different standard deviations too.

Paper in **IEEE PAMI** "Early Access" number **9091905**

Programs at **https://github.com/MISRE**

Thank You