

of surface tension (Fig. 1b). This can be visualized in terms of a capillary length — about 3 mm for the air–water interface. The Weber number compares inertia with surface cohesion: $We = (\rho_a u^2 d) / \sigma$, where ρ_a and σ are the air density and surface tension, respectively. Thus, We can be viewed as a squared ratio of drop size and capillary length. Villermaux and Bossa³ arrive at a critical Weber number of six (Fig. 1b), which translates to a critical drop diameter of about 6 mm, consistent with their observations and those of others⁶.

In this respect, reports of very large raindrops of up to 1 cm in diameter in Brazilian and Hawaiian clouds are interesting and puzzling⁷. Surfactants, such as those produced by forest fires, complicate the situation and have been detected in raindrops⁸, but their presence should lower the surface tension and therefore the Weber number. On the other hand, they are likely to promote coalescence of such ‘softer’ raindrops on the way down.

Terminal speed is an important parameter when applying Villermaux and Bossa’s critical Weber numbers to natural rain, particularly because speed increases with drop size. In that respect, the perspective of Villermaux and Bossa is complemented by the recent findings that not all raindrops fall at their terminal speed⁹. Some break the speed limit by an order of magnitude in the immediate aftermath of fragmentation, by ejected smaller droplets that momentarily maintain the momentum of their parent drops. Such ‘superterminal’ drops were caught on camera before they had a chance to relax to their terminal speed (which takes only a fraction of a second), and thereby corroborate the break-up mechanism. This may allow for further study and testing of the Villermaux and Bossa perspective. In particular, it could address longstanding questions about whether break-up is

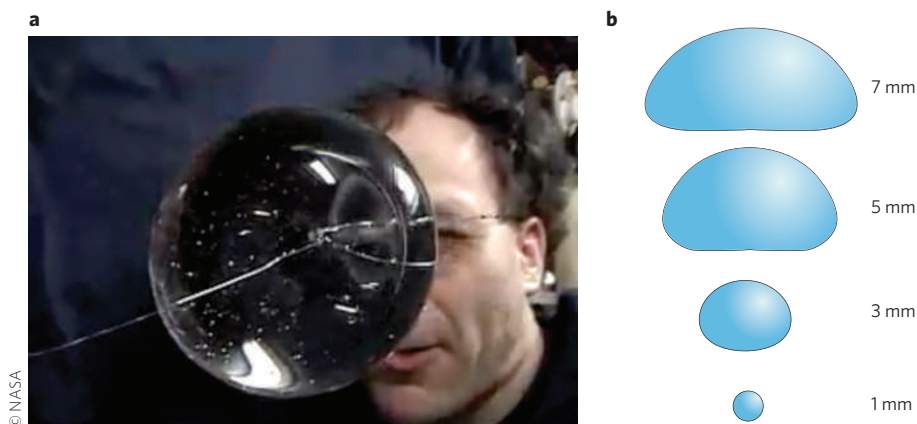


Figure 1 | Raindrops under stress. **a**, Giant raindrops such as the one shown here, held by astronaut Don Pettit in the space shuttle, are not observed in the atmosphere because of the deformation and subsequent instability experienced by a terrestrial raindrop falling through air at its terminal speed, u (reached when the gravitational force is balanced by air resistance). **b**, Deformation of falling drops is determined by competition between surface tension and fluid stresses. As a result, asphericity increases with drop size. The opposition between aerodynamic stress $\rho_a u^2$ and the surface tension σ is captured by the Weber number $We = (\rho_a u^2 d) / \sigma$. The critical We number of six derived by Villermaux and Bossa³, above which spontaneous drop break-up occurs, can be motivated by equating the spherical-drop surface energy density to inertial energy density as $(\sigma \pi d^2) / (\pi d^3 / 6) = \rho_a u^2$.

predominantly spontaneous, as they suggest, or the result of collisions between drops, which is the common view.

Remote sensing has much to contribute here. For example, perhaps drop oscillations between prolate and oblate asphericity (preceding the break-up) are the source of the surprising depolarization of radar or lidar waves that has been observed at vertical incidence¹⁰. Also, returning to radar meteorology, the d^6 dependence of the radar echo suggests that the Villermaux and Bossa³ vision of progressive refinement of the size distribution below the cloud base can be tested by studying radar reflectivity versus height with high spatial resolution. Natural rainfall still has something to teach us, so let it rain. □

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HEAVY ELECTRONS

The gathering storm of data

The nature of the ‘hidden order’ in URu_2Si_2 has resisted characterization for the past twenty-five years. Recent photoemission results report the observation of a narrow heavy-fermion band that sharpens below the mysterious transition and provides new clues about its origins.

P. Chandra and P. Coleman

The electronic applications of tomorrow will probably not be direct extensions of what we know today, but will more likely depend on principles of electron organization in matter, which we

are only beginning to discover. Physicists are just starting to appreciate the rich fabric of the periodic table and the vast diversity of electronic behaviour that can be shown by different compounds. Although practical

applications of emergent phenomena such as superconductivity or magnetism require materials in which these states occur at, or above, room temperature, the unusual electronic correlations and

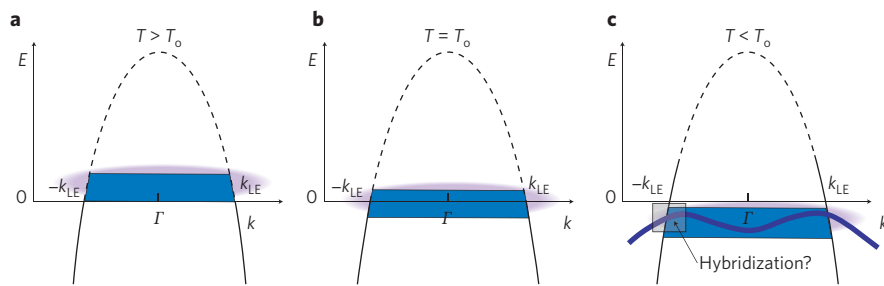


Figure 1 | Schematic of angle-resolved photoemission spectra obtained by Santander-Syro *et al.*². **a**, At temperatures $T > T_0$, a light band of electrons (grey) intersects at wavevectors $\pm k_{LE}$ with a narrow, incoherent heavy band (shaded blue) above the Fermi energy. **b**, At $T = T_0$, the heavy incoherent band moves down to the Fermi level. **c**, At temperatures $T < T_0$, the heavy incoherent band sinks beneath the Fermi energy and develops sharp dispersion, as indicated by the thick blue line. At the intersection of the light and heavy bands, hybridization may develop as part of the hidden-order process.

competing energy scales that drive them are often more readily characterized in materials in which they develop at low energies and temperatures. Such systems are more readily tuned using pressure and magnetic fields, and they also offer a pure environment for low-noise thermal and electric measurements. An example of this is the heavy-electron materials¹. These are metals with large quasiparticle masses and strong magnetic correlations that can induce superconductivity and other unusual forms of order at single-digit transition temperatures. There are many unanswered questions relating to these systems, and the hope is that the solutions may lead to materials that have similarly exotic behaviour at room temperature.

On page 637 of this issue², Santander-Syro *et al.* report on one of the many outstanding problems of heavy-electron physics, the determination of the nature of the underlying order in URu₂Si₂ (ref. 3). This material has a ‘textbook’ mean-field phase transition, signalled by large discontinuities in the specific heat and other bulk properties at $T_0 = 17$ K, which is a precursor to previously unknown superconductivity at a still-lower temperature, $T_c = 1.2$ K. Despite the entropy of condensation being large, the order parameter associated with the transition at T_0 remains undetected after a quarter-century of research.

What exactly is the hidden order in URu₂Si₂? We know that its internal entropy drops precipitously at T_0 , indicating a dramatic growth in the organization of the underlying electrons. Yet knowing that long-range order is present is very different from directly observing it. Most famously, superconductivity was discovered in the laboratory a century ago, but it took fifty more years to link it with the formation of electron pairs. What nags

us about URu₂Si₂ is that we do not yet know whether something very special is hiding in the order that could have important consequences for other strongly correlated materials.

The understanding of unique electronic self-ordering requires new experimental techniques. Many high-resolution probes, developed as part of the study of copper oxide superconductors, are now becoming generally available to characterize other highly correlated metals. These techniques include neutron scattering⁴, scanning tunnelling spectroscopy⁵ and angle-resolved photoemission², together with the possibility of tuning materials to extreme pressures⁶ and magnetic fields⁷. Each reveals different aspects of the correlations inside matter. Today, there is a gathering storm of results as physicists apply these new techniques to URu₂Si₂.

More than one hundred years ago, Einstein used photoemission to identify the discrete nature of light. Today, this same experiment, in which an absorbed photon ejects an electron from matter, has become a state-of-the-art probe; it reveals the energy–momentum distribution of electrons inside a material. Tremendous advances in resolution⁸, and the advent of low-temperature measurement techniques, now permit photoemission to be applied to heavy-electron materials. These developments have led Santander-Syro *et al.*² to ask whether shining light on URu₂Si₂ can somehow illuminate the hidden order.

Using angle-resolved photoemission spectroscopy, Santander-Syro *et al.* are able to directly observe the heavy electrons at the Fermi surface of URu₂Si₂. At temperatures greater than T_0 , they see a narrow, but incoherent, band of electrons just above the Fermi energy (Fig. 1). This band moves through the Fermi surface of URu₂Si₂ at the

hidden-order transition, and as it does so it sharpens into a dispersing band of heavy electrons with an associated mass, m^* , about 20 times that of a free electron, m_e . This value is consistent with that observed in previous thermodynamic measurements. The authors also observe light holes with effective masses of $m^* \approx 1.4m_e$ coexisting with these heavy electrons.

For outgoing electron momenta, k , in the [110] and [111] directions, the heavy-electron band gets closest to the Fermi energy for $k_{LE} = 0.2$ and 0.15 \AA^{-1} , respectively — positions that coincide with the extrapolated Fermi surface of the light band. Indeed, there appears to be a ‘pocket’ of light holes and heavy electrons around the $k = 0$ point in the Brillouin zone (the Γ point in Fig. 1). Quantum theory tells us that when two electron bands cross, they hybridize, developing a gap that prevents them from intersecting. Such hybridization between heavy and light electrons is normally part of the gradual development of coherent heavy-electron bands¹. As Santander-Syro *et al.*² discuss, the current measurements cannot resolve such a hybridization, but the sharpening of the heavy band suggests that it develops at T_0 . What is surprising here is that the process of hybridization seems to occur as an integral part of the hidden-order phase transition, as if the heavy electrons form with the transition.

What are the implications of these observations for the hidden order? For more than 20 years, physicists⁹ have supposed that the hidden order in URu₂Si₂ is some kind of density wave^{10,11} associated with a Fermi-surface reordering¹². Recent neutron measurements⁴ support this idea. However, the data of Santander-Syro *et al.* cannot be easily interpreted in this way, for a density wave would lead to a ‘back-folding’ of the energy spectra in momentum space, which is not observed. One way of reconciling the two observations might be to suppose that the wave vector of the density wave is perpendicular to the surface — a thought that requires further investigation.

Another question posed by these results concerns the relationship between the observed heavy-electron band and the microscopic physics of the uranium atoms. Normally, heavy-electron behaviour is associated with magnetic ions containing an odd number of f electrons. However, recent calculations¹³ based on dynamical mean-field theory predict that the number of f electrons per uranium atom is two. Future experiments that help to resolve this issue and to identify the source of the entropy that is released at the

hidden-order transition are therefore vital. The photoemission data⁴ should be seen in the context of a rising tide of activity, both experimental and theoretical, to explain the mysteries of this material. □

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COMPLEX NETWORKS

Structure comes to random graphs

Incorporating structural features into random-graph calculations should bring theoretical models describing the properties and behaviour of complex networks closer to real-world systems.

Raissa M. D'Souza

Many social, technological and biological systems are best described as a collection of interacting agents, parts or molecules. Insight into their behaviour can be gained through mathematical models of random networks, where we consider a collection of nodes with edges connecting them at random. Random networks — also called random graphs — show phase transitions^{1,2}, such as the sudden emergence of large-scale connectivity as a function of the density of connections. For real-world systems, such behaviour can have important consequences, from enhancing the reach of telecom and transportation networks to increasing the possible extent of a viral outbreak. Also, random-network formulations provide models for studying, for instance, percolation phenomena on networks with differing connectivity properties and the sudden formation of so-called dense *k*-cores (subgraphs of highly connected nodes)^{1,2}, and provide insights into how we might alter the onset of phase transitions³. These random constructions, however, typically assume that all connections between nodes are added independently of one another, thus missing key structural features found in real-world networks. Now, writing in *Physical Review Letters*⁴, Mark Newman shows how to incorporate small-scale structural elements into calculations of random networks, bringing the structures studied mathematically closer to their real-world counterparts. In particular, the refined calculations of critical properties should enable better predictions for real systems.

The importance of structural elements in social networks has long been recognized,

starting with the role of triangles, or triads, connecting together groups of three nodes⁵. Triads reflect transitivity, the property that individuals sharing a common friend are reasonably likely to be direct friends themselves. Consider a network of scientific collaboration in which nodes are scientists and edges connect coauthors. Over time, disconnected scientists sharing a common coauthor often meet, begin collaborating and become direct coauthors, thus gaining an edge. In a now seminal paper⁶, Watts and Strogatz proposed that triangular patterns exist in many contexts, and demonstrated evidence for their presence in the neuronal network of the worm *Caenorhabditis elegans* and in a power-grid network. They defined

a 'small world' network as one showing high 'clustering' (that is, many triangles) together with short paths connecting all pairs of nodes. Many more systems with small-world structures have been identified subsequently.

Watts and Strogatz also proposed a mathematical small-world model that accounts for clustering⁶, but it places unrealistic restrictions on local patterns of connectivity. A more general starting point is the configuration-model approach¹ introduced by Bollobás in 1980 and Molloy and Reed⁷ in 1995, which considers all possible networks that can be constructed such that each node has a specified number of edges. In 2001, Newman, Strogatz and

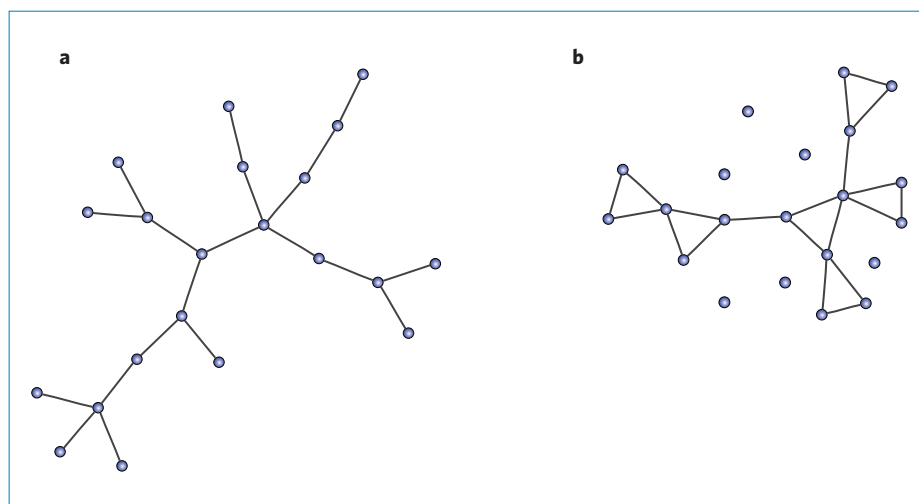


Figure 1 | Triangles limit the extent of connectivity achievable. **a, b**, Both networks consist of 21 nodes and 20 edges, but in **b** six nodes are disconnected. Triangles, however, aid the extent to which an infection can spread over the connected portion of the graph, owing to there being numerous alternative paths connecting nodes.